LOW AND MEDIUM β SUPERCONDUCTING CAVITIES AND ACCELERATORS

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Introduction

- There have been increased needs for reduced-beta ($\beta<1$) SRF cavity especially in CW machine (or high duty pulsed machine; duty >10 %)

  - Accelerator driven system (ADS)
    Nuclear transmutation of long-lived radio active waste
    Energy amplifier
    Intense spallation neutron source

  - Nuclear physics
    Radioactive ion acceleration
    Muon/neutrino production

  - Defense applications

- SRF technology → Critical path !!

Introduction

- SRF cavity for CW application or long pulse application
  - efforts for expanding their application regions down to β~0.1,

- Reduced beta Elliptical multi-cell SRF cavity
  - for CW, prototyping by several R&D groups have demonstrated as low as β=0.47
  - for pulsed, SNS β=0.61, 0.81 cavities & ESS

- Elliptical cavity has intrinsic problem as β goes down
  - mechanical problem, multipacting, low RF efficiency

- Spoke cavity; supposed to cover ranges β=0.1~0.5(6), f=300~900 MHz
  - design & prototype efforts in RIA, AAA, EURISOL, XADS, ESS, etc.

For proton β=0.12 corresponds ~7 MeV →all the accelerating structures (except RFQ)

Low and Medium β Superconducting Accelerators

	High Current	Medium/Low Current
CW	Accelerator driven systems waste transmutation energy production	Production of radioactive ions Nuclear Structure
Pulsed	Pulsed spallation sources	

High-current cw accelerators

- Beam: p, H^-, d
- Technical issues and challenges
  - Beam losses (~ 1 W/m)
  - Activation
  - High cw rf power
  - Higher order modes
  - Cryogenics losses
- Implications for SRF technology
  - Cavities with high acceptance
  - Development of high cw power couplers
  - Extraction of HOM power
  - Cavities with high shunt impedance

High-current pulsed accelerators

- Beam: p, H^-
- Technical issues and challenges
  - Beam losses (~ 1 W/m)
  - Activation
  - Higher order modes
  - High peak rf power
  - Dynamic Lorentz detuning
- Implications for SRF technology
  - Cavities with high acceptance
  - Development of high peak power couplers
  - Extraction of HOM power
  - Development of active compensation of dynamic Lorentz detuning

Medium to low current cw accelerators

- Beam; p to U
- Technical issues and challenges
  - Microphonics, frequency control
  - Cryogenic losses
  - Wide charge to mass ratio
  - Multicharged state acceleration
  - Activation
- Implications for SRF technology
  - Cavities with low sensitivity to vibration
  - Development of microphonics compensation
  - Cavities with high shunt impedance
  - Cavities with large velocity acceptance (few cells)
  - Cavities with large beam acceptance (low frequency, small frequency transitions)

Common considerations (I)

- Intermediate velocity applications usually do not require (or cannot afford) very high gradients

- Operational and practical gradients are limited by
  - Cryogenics losses (cw applications)
  - Rf power to control microphonics (low current applications)
  - Rf power couplers (high-current applications)

- High shunt impedance is often more important

- To various degrees, beam losses and activation are a consideration

Common considerations (II)

- Superconducting accelerators in the medium velocity range are mostly used for the production of secondary species
  - Neutrons (spallation sources)
  - Exotic ions (radioactive beam facilities)
- Medium power (100s kW) to high power (~MW) primary impinging on a target
- Thermal properties and dynamics of the target are important considerations in the design of the accelerator (frequency, duration, recovery from beam trips)
- Some implications:
  - Operate cavities sufficiently far from the edge
  - Provide an ample frequency control window

Design considerations

- Low cryogenics losses
  - High QR_s * R_{sh}/Q
  - Low frequency
- High gradient
  - Low E_p/E_{acc}
  - Low B_p/E_{acc}
- Large velocity acceptance
  - Small number of cells
  - Low frequency
- Frequency control
  - Low sensitivity to microphonics
  - Low energy content
  - Low Lorentz coefficient
- Large beam acceptance
  - Large aperture (transverse acceptance)
  - Low frequency (longitudinal acceptance)

A Few Obvious Statements

Low and medium β

- β<1
- Particle velocity will change

The lower the velocity of the particle or cavity β

- The faster the velocity of the particle will change
- The narrower the velocity range of a particular cavity
- The smaller the number of cavities of that β
- The more important it is that the particle achieve design velocity

Be conservative at lower β

Be more aggressive at higher β

A Few More Statements

Two main types of structure geometries

TEM class (QW, HW, Spoke)

TM class (elliptical)

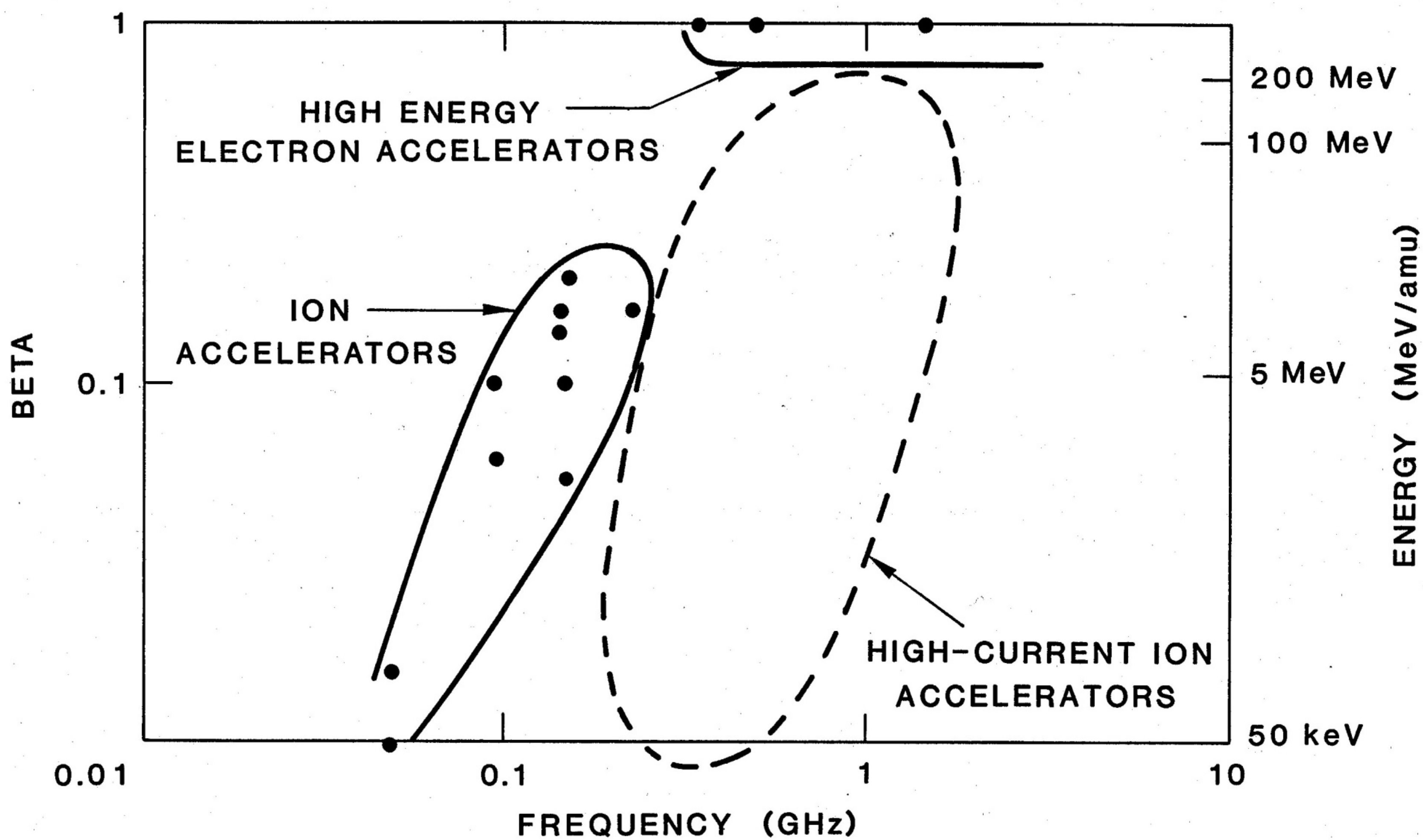
Design criteria for elliptical cavities
Pagani, Barni, Bosotti, Pierini, Ciovati, SRF 2001.

Challenges and the future of reduced beta srf cavity design
Sang-ho Kim, LINAC 2002.

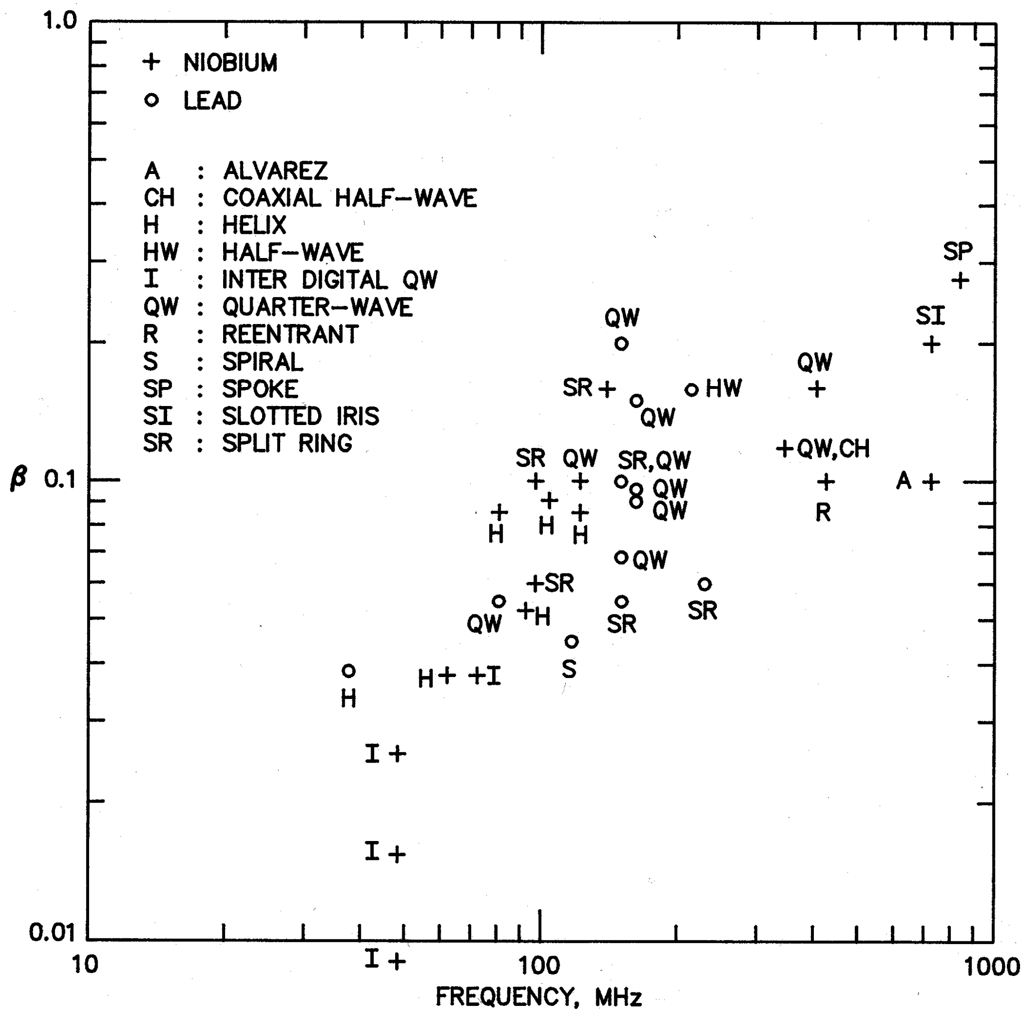
Low and intermediate β cavity design
Jean Delayen, SRF 2003

High-energy ion linacs based on superconducting spoke cavities
K. W. Shepard, P. N. Ostroumov, J. R. Delayen, PRSTAB **6**, 080101 (2003)

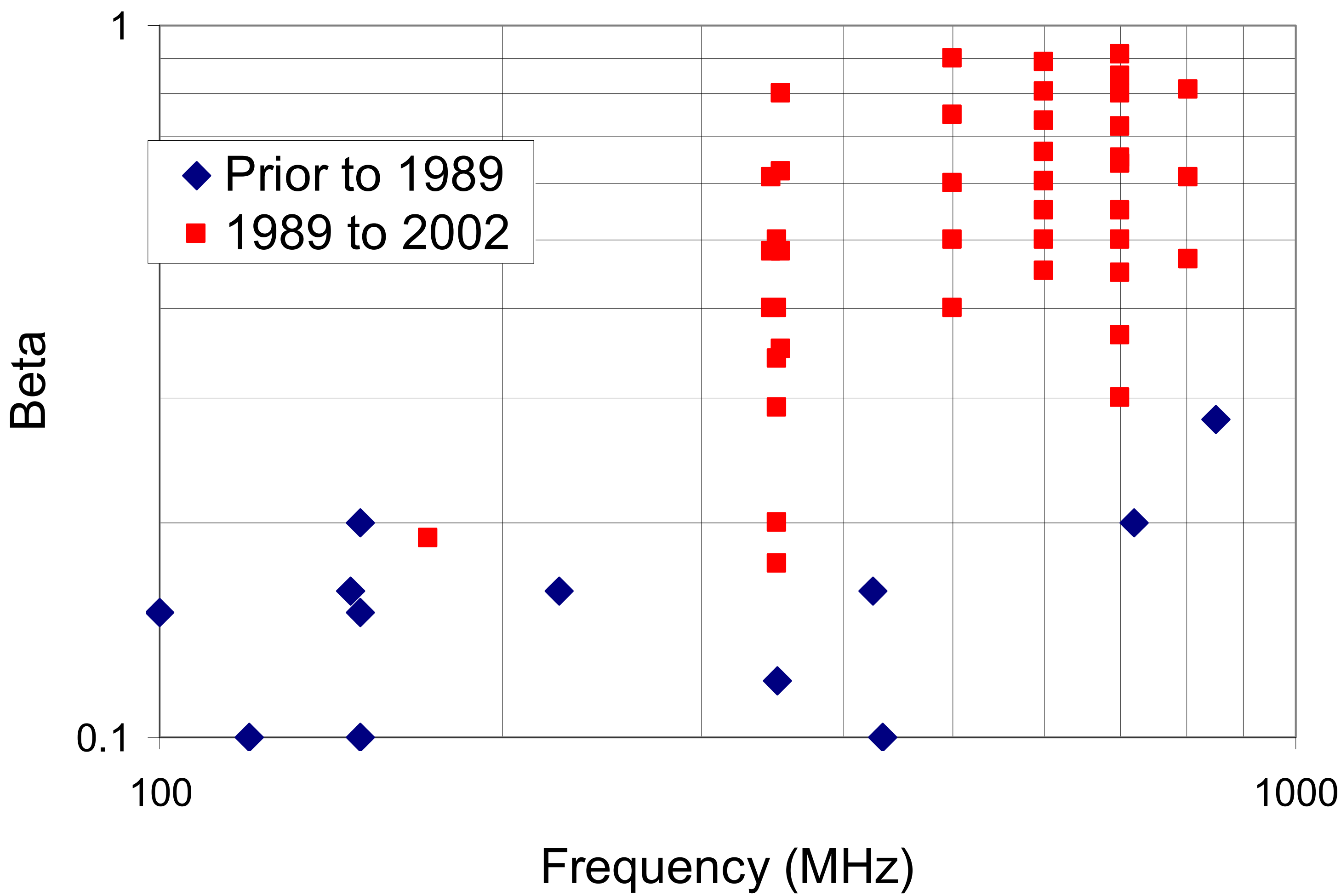
Superconducting Structures – Circa 1987



β<1 Superconducting Structures – Circa 1989



β<1 Superconducting Structures – 2002..



Basic Structure Geometries

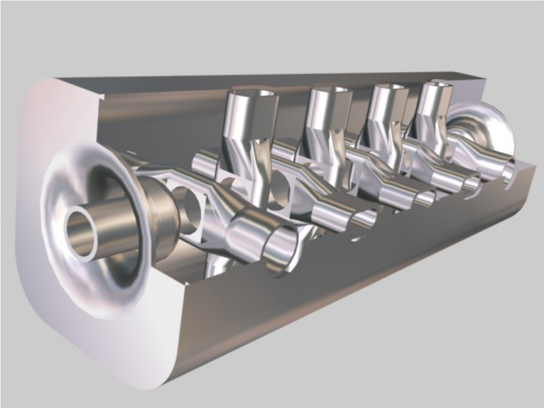
Resonant Transmission Lines

- $\lambda/4$
  - Quarter-wave
  - Split-ring
  - Twin quarter-wave
  - Lollipop
- $\lambda/2$
  - Coaxial half-wave
  - Spoke
  - H-types

- **TM**
  - Elliptical
  - Reentrant
- **Other**
  - Alvarez
  - Slotted-iris

A Word on Design Tools

TEM-class cavities are essentially 3D geometries



3D electromagnetic software is available

MAFIA, Microwave Studio, HFSS, etc.

3D software is usually very good at calculating frequencies

Not quite as good at calculating surface fields

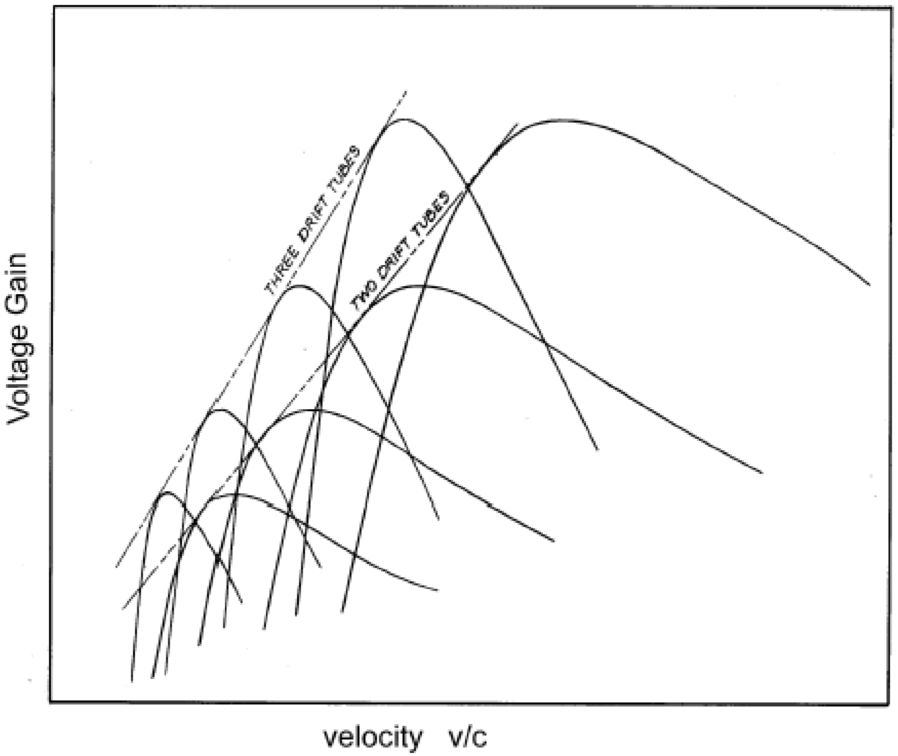
Use caution, vary mesh size

Remember Electromagnetism 101

Design Tradeoffs

Number of cells
- Voltage gain
- Velocity acceptance

Frequency
- Size
- Voltage gain
- Rf losses
- Energy content, microphonics, rf control
- Acceptance, beam quality and losses



Energy Gain
Transit Time Factor - Velocity Acceptance

$$\Delta W = q \int_{-\infty}^{+\infty} E(z) \cos(\omega t + \phi)\, dz$$

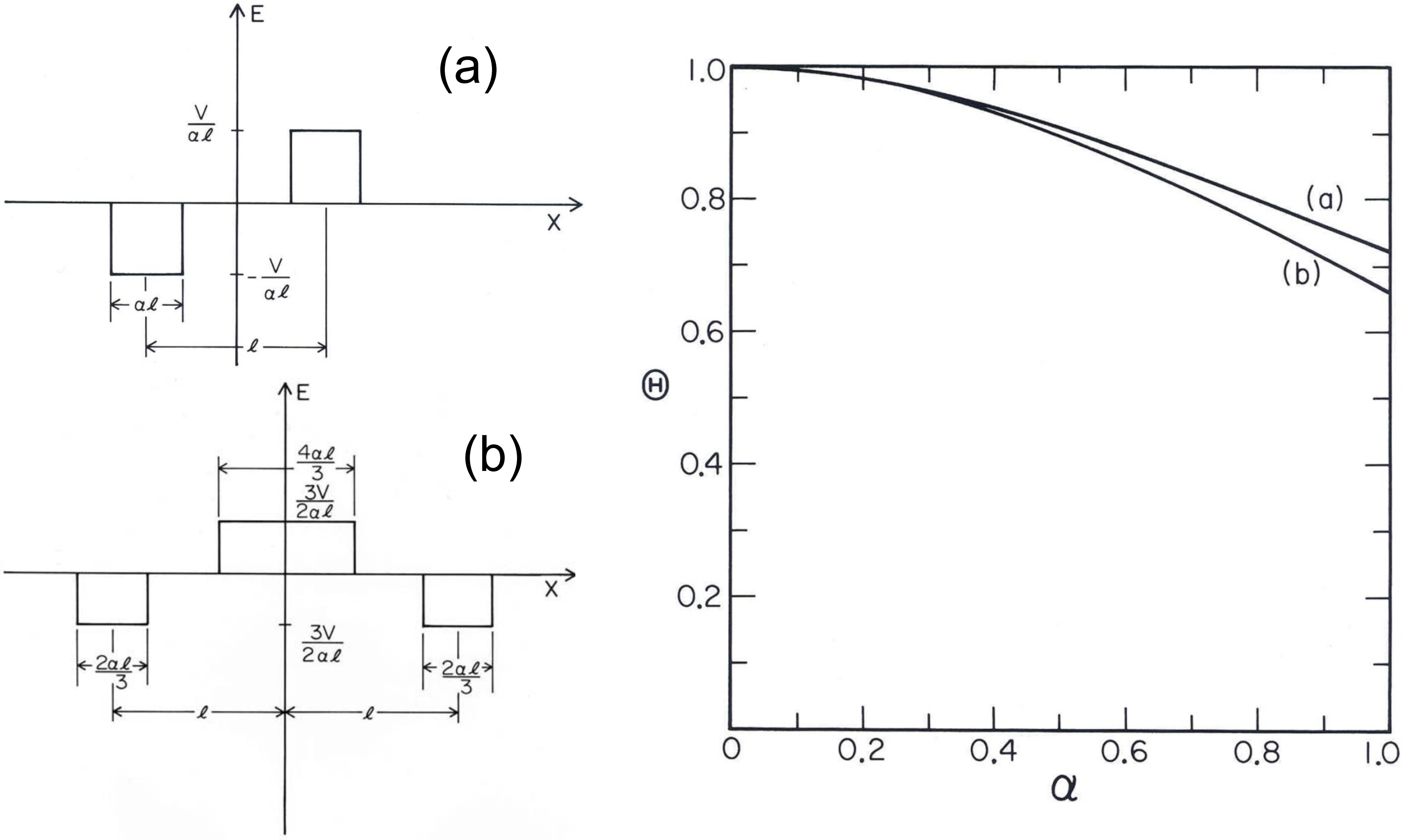
Assumption: constant velocity

$$\Delta W = q \ \cos\phi \ \ \Delta W_0 \ \ T(\beta) \qquad\qquad \Delta W_0 = \Theta \int_{-\infty}^{+\infty} \left| E(z) \right| dz$$

$$\Theta = \frac{\text{Max} \int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}{\int_{-\infty}^{+\infty} \left| E(z) \right| dz}$$ Transit Time Factor

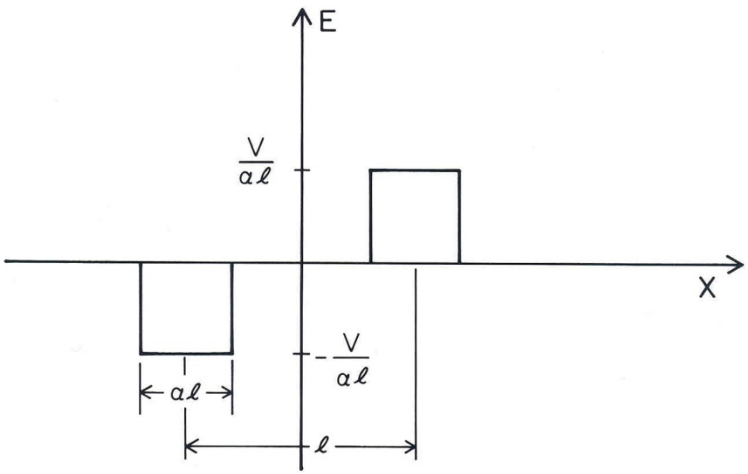
$$T(\beta) = \frac{\int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}{\text{Max} \int_{-\infty}^{+\infty} E(z) \cos\left(\frac{\omega z}{\beta c}\right) dz}$$ Velocity Acceptance

Transit Time Factor

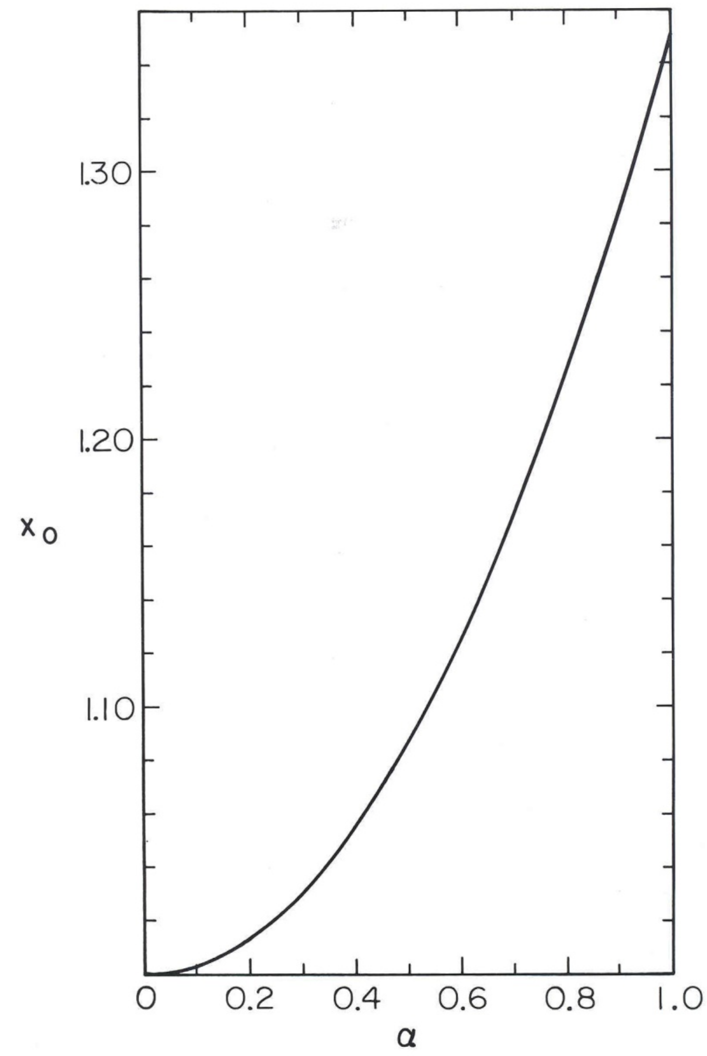


Velocity Acceptance for 2-Gap Structures

$$T(\beta)=\frac{\beta}{\beta_0}\frac{\sin\left(\frac{\pi\alpha}{2x_0}\frac{\beta_0}{\beta}\right)\sin\left(\frac{\pi}{2x_0}\frac{\beta_0}{\beta}\right)}{\sin\left(\frac{\pi\alpha}{2x_0}\right)\sin\left(\frac{\pi}{2x_0}\right)}$$

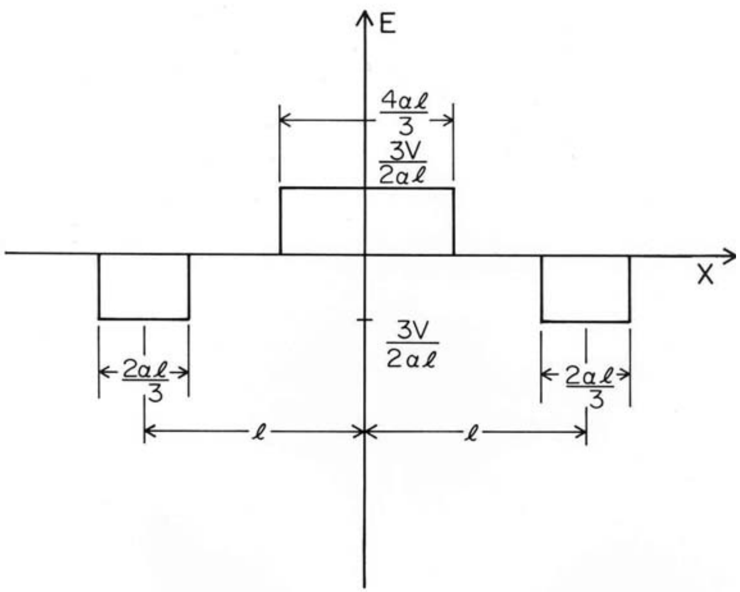


$$x_0=\frac{\beta_0\lambda}{2l}$$

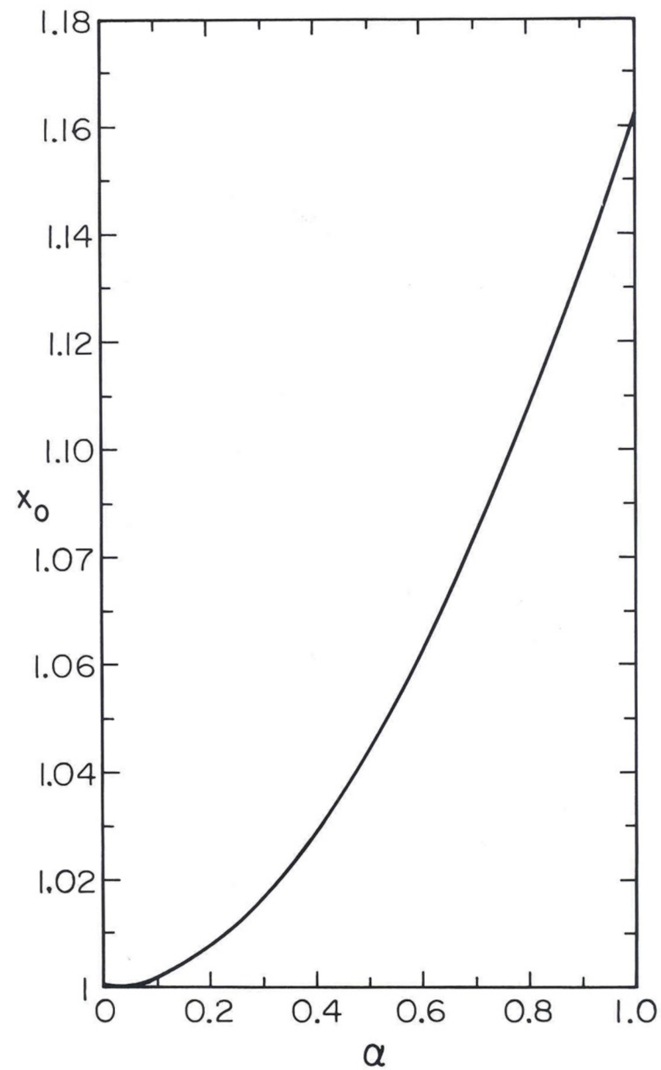


Velocity Acceptance for 3-Gap Structures

$$T(\beta)=\frac{\beta}{\beta_0}\frac{\sin\left(\frac{\pi\alpha}{3x_0}\frac{\beta_0}{\beta}\right)\left[\cos\left(\frac{\pi\alpha}{3x_0}\frac{\beta_0}{\beta}\right)-\cos\left(\frac{\pi}{x_0}\frac{\beta_0}{\beta}\right)\right]}{\sin\left(\frac{\pi\alpha}{3x_0}\right)\left[\cos\left(\frac{\pi\alpha}{3x_0}\right)-\cos\left(\frac{\pi}{x_0}\right)\right]}$$



$$x_0=\frac{\beta_0\lambda}{2l}$$

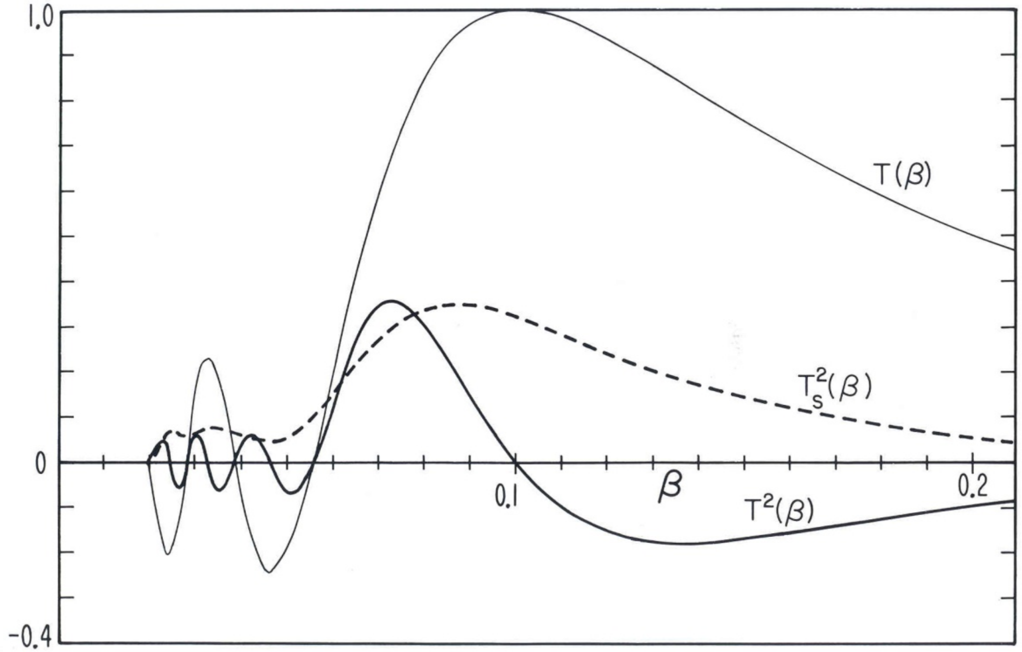


Higher-Order Effects

$$\Delta W = q\cos\phi\,\Delta W_0\,T(\beta) + \frac{(q\Delta W_0)^2}{W}\left[T^{(2)}(\beta) + \sin 2\phi\,T_s^{(2)}(\beta)\right]$$

$$T^{(2)}(k) = -\frac{k}{4}T(k)\frac{d}{dk}T(k) \qquad\qquad k = \omega/\beta c$$

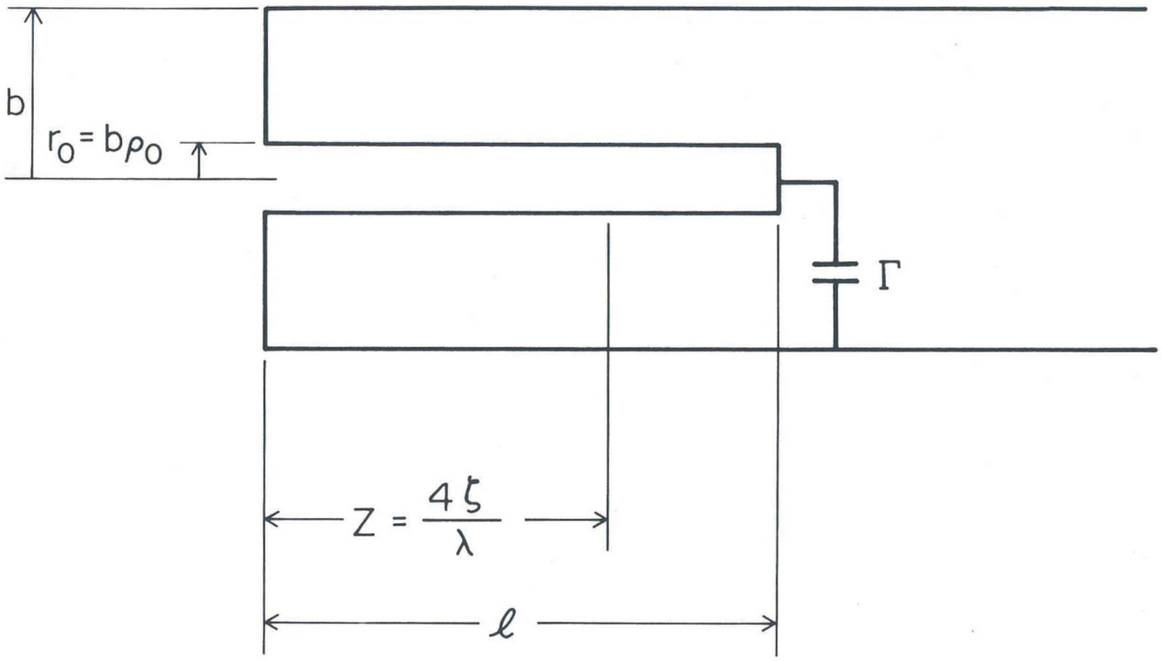
$$T_s^{(2)}(k) = -\frac{k}{4\pi}\int_0^\infty \frac{T(k+k')T(k-k') - T(k)T(k)}{k'^2}\,dk'$$



A Simple Model:
Loaded Quarter-wavelength Resonant Line

If characteristic length <<λ (β<0.5), separate the problem in two parts:

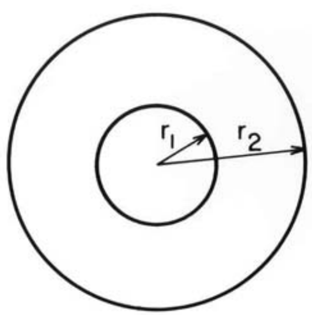
- Electrostatic model of high voltage region
- Transmission line



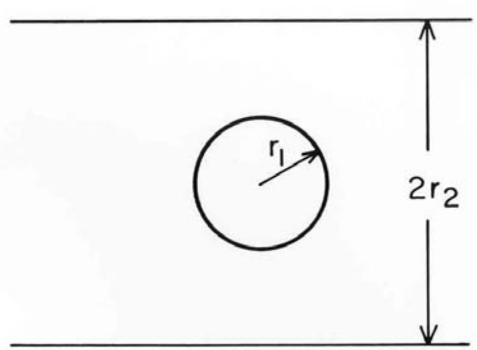
Basic Electrostatics

a: concentric spheres
b: sphere in cylinder
c: sphere between 2 planes
d: coaxial cylinders
e: cylinder between 2 planes

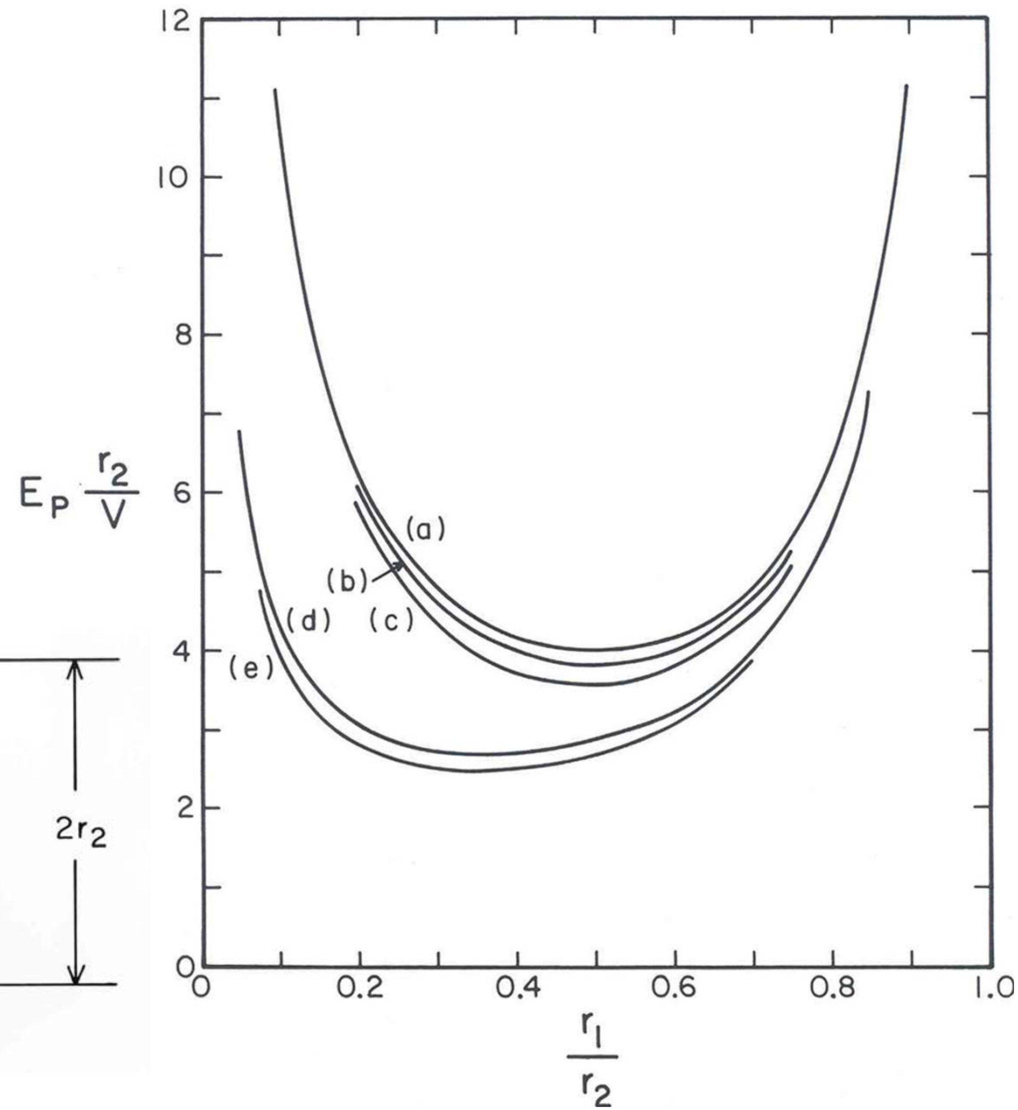
V_p : Voltage on center conductor
Outer conductor at ground
E_p: Peak field on center conductor



(a), (b), (d)



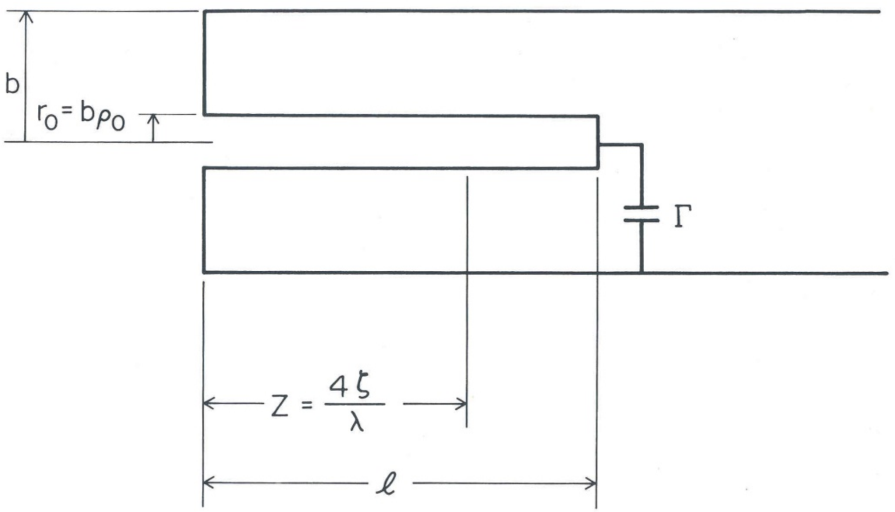
(c), (e)



Loaded Quarter-wavelength Resonant Line

Capacitance per unit length

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{r_0}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{1}{\rho_0}\right)}$$



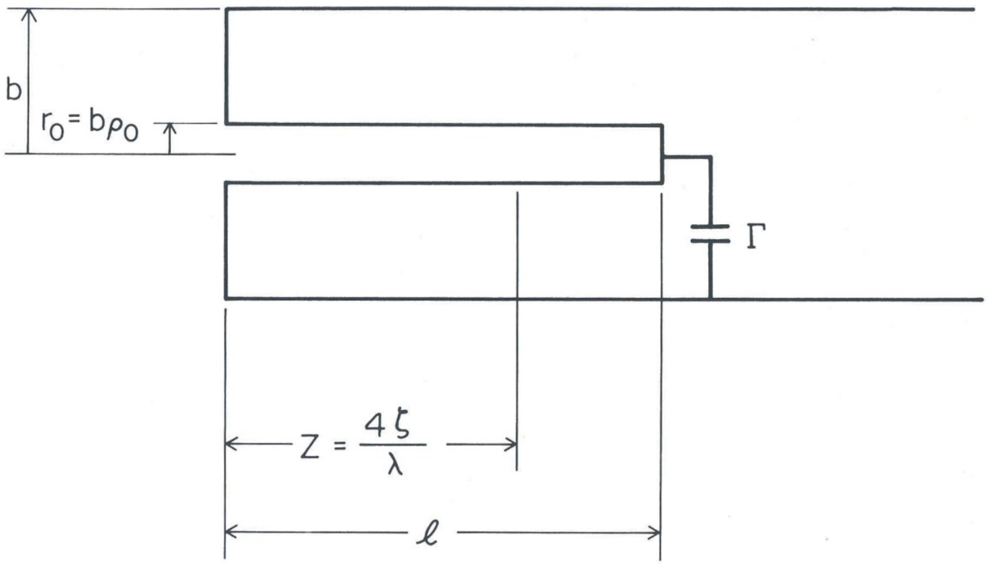
Inductance per unit length

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{r_0}\right) = \frac{\mu_0}{2\pi}\ln\left(\frac{1}{\rho_0}\right)$$

Loaded Quarter-wavelength Resonant Line

Center conductor voltage

$$V(z) = V_0 \sin\left(\frac{2\pi}{\lambda} z\right)$$



Center conductor current

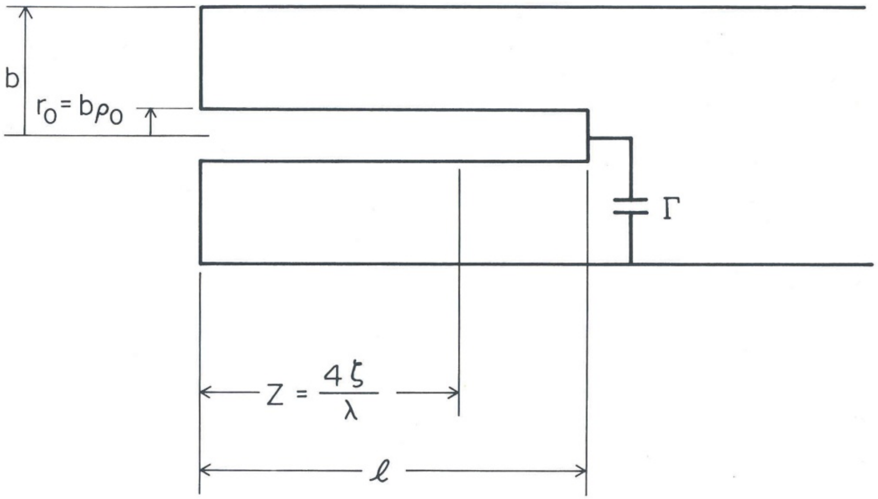
$$I(z) = I_0 \cos\left(\frac{2\pi}{\lambda} z\right)$$

Line impedance

$$Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi} \ln\left(\frac{1}{\rho_0}\right), \qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 377\Omega$$

Loaded Quarter-wavelength Resonant Line

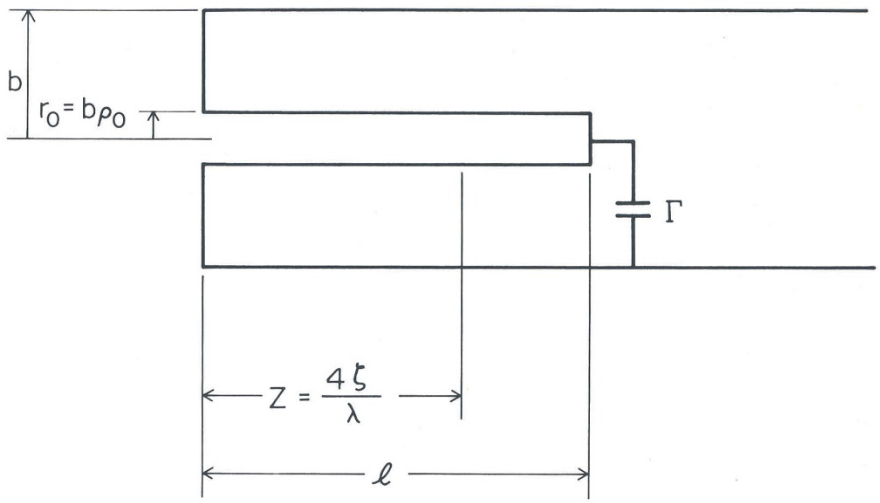
Loading capacitance



$$\Gamma(z) = \lambda\varepsilon \frac{\operatorname{cotan}\left(\frac{2\pi}{\lambda} z\right)}{\ln\left(1/r_0\right)} = \lambda\varepsilon \frac{\operatorname{cotan}\left(\frac{\pi}{2}\zeta\right)}{\ln\left(1/\rho_0\right)}$$

$$l = \frac{\lambda}{2\pi} \operatorname{Arctan}\left[\frac{\lambda\varepsilon}{\Gamma \ln\left(1/\rho_0\right)}\right]$$

Loaded Quarter-wavelength Resonant Line



Peak magnetic field

$$\frac{V_p}{b} = \begin{Bmatrix} \eta & H \\ c & B \\ 300 & B \end{Bmatrix} \rho_0 \ln\left(\frac{1}{\rho_0}\right) \sin\left(\frac{\pi}{2}\zeta\right) \qquad \begin{Bmatrix} \text{m, A/m} \\ \text{m, T} \\ \text{cm, G} \end{Bmatrix}$$

V_p: Voltage across loading capacitance

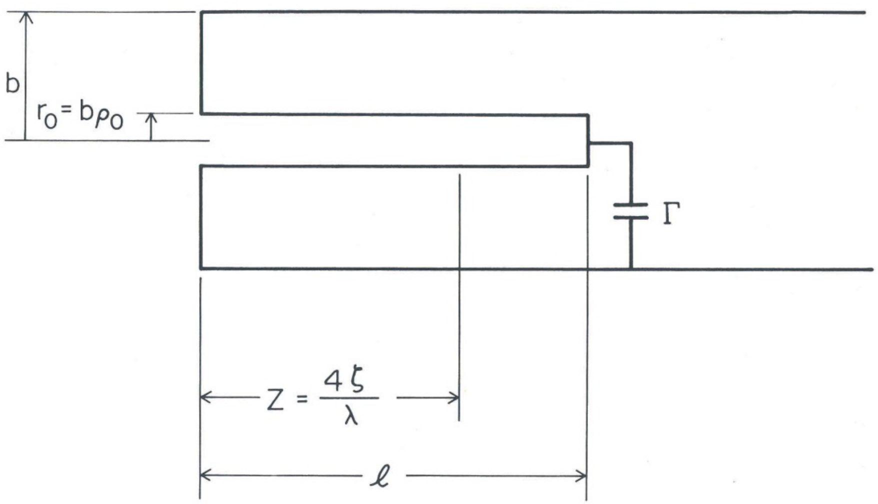
$B \simeq 9$ mT at 1 MV/m

Loaded Quarter-wavelength Resonant Line

Power dissipation (ignore losses in the shorting plate)

$$P = V_p^2 \frac{\pi}{8} \frac{R_s}{\eta^2} \frac{\lambda}{b} \frac{1+1/\rho_0}{\ln^2 \rho_0} \frac{\zeta + \frac{1}{\pi}\sin \pi\zeta}{\sin^2 \frac{\pi}{2}\zeta}$$

$$P \propto \frac{R_s}{\eta^2} E^2 \beta \lambda^2$$

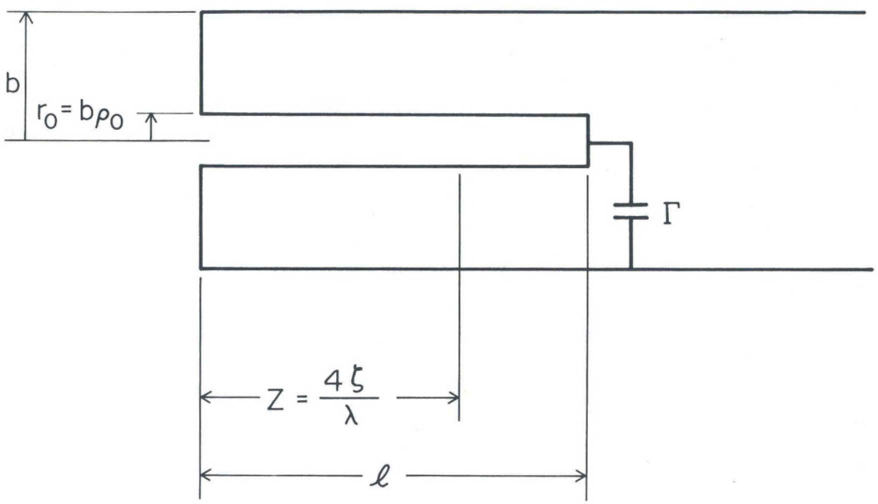


Loaded Quarter-wavelength Resonant Line

Energy content

$$U = V_p^2 \frac{\pi \varepsilon_0}{8} \lambda \frac{1}{\ln\left(1/\rho_0\right)} \frac{\zeta + \frac{1}{\pi}\sin \pi\zeta}{\sin^2 \frac{\pi}{2}\zeta}$$

$$U \propto \varepsilon_0 E^2 \beta^2 \lambda^3$$

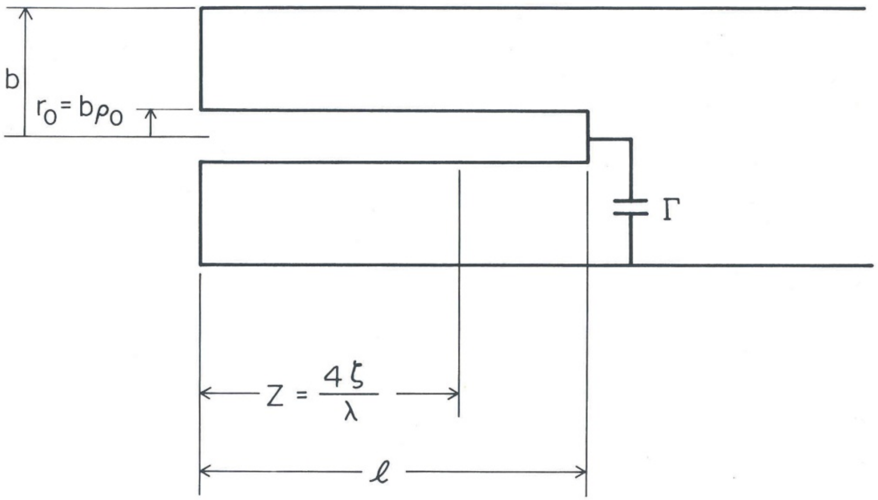


Loaded Quarter-wavelength Resonant Line

Geometrical factor

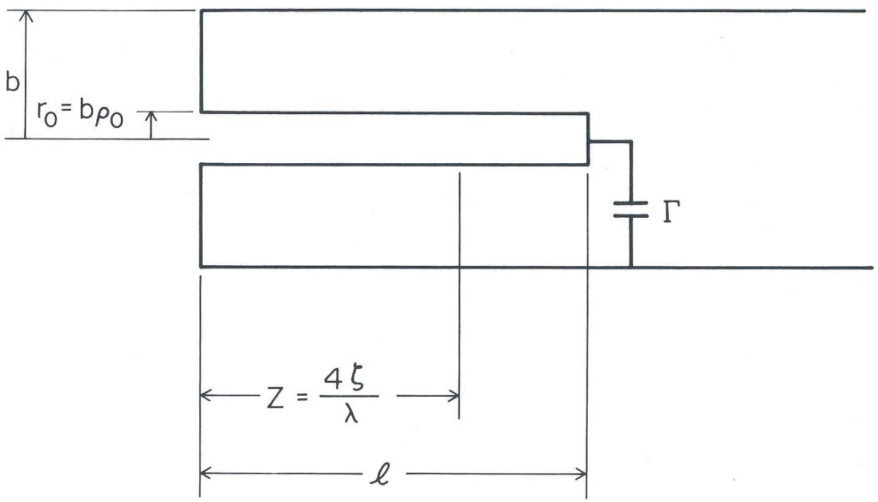
$$G = QR_s = 2\pi\,\eta\,\frac{b}{\lambda}\,\frac{\ln(1/\rho_0)}{1+1/\rho_0}$$

$$G \propto \eta\,\beta$$



Loaded Quarter-wavelength Resonant Line

Shunt impedance $\left(4V_p^2 / P\right)$



$$R_{sh} = \frac{\eta^2}{R_s}\frac{32}{\pi}\frac{b}{\lambda}\frac{\ln^2 \rho_0}{1+1/\rho_0}\frac{\sin^2 \frac{\pi}{2}\zeta}{\zeta + \frac{1}{\pi}\sin \pi\zeta}$$

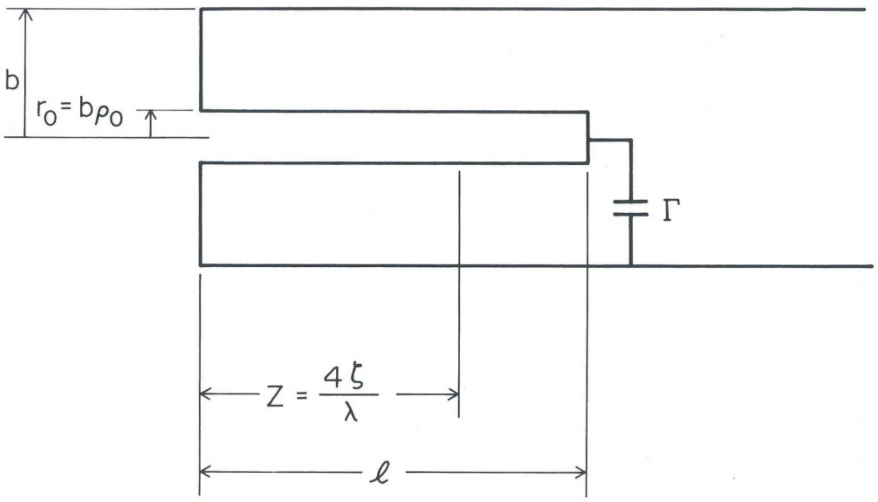
$$R_{sh}\ R_s \propto \eta^2\ \beta$$

Loaded Quarter-wavelength Resonant Line

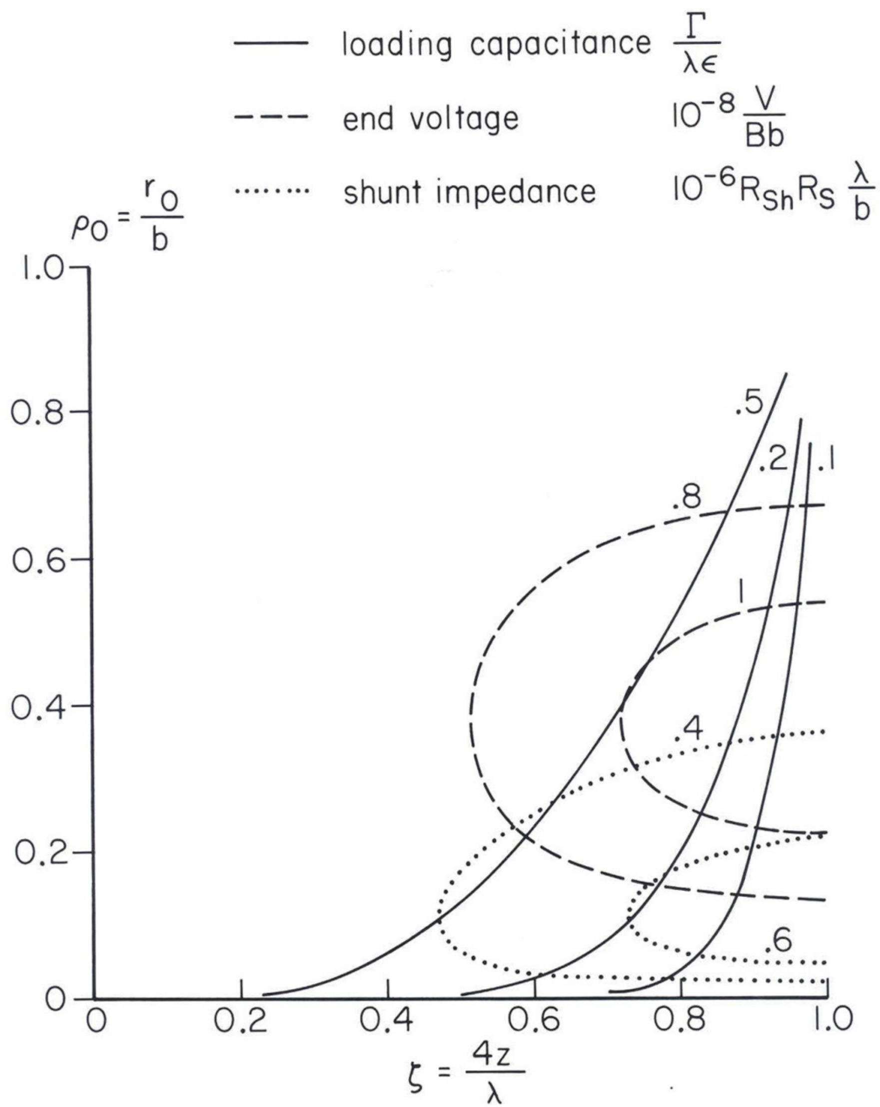
R/Q

$$\frac{R_{sh}}{Q} = \frac{16}{\pi^2}\eta \ln\left(1/\rho_0\right)\frac{\sin^2\frac{\pi}{2}\zeta}{\zeta + \frac{1}{\pi}\sin\pi\zeta}$$

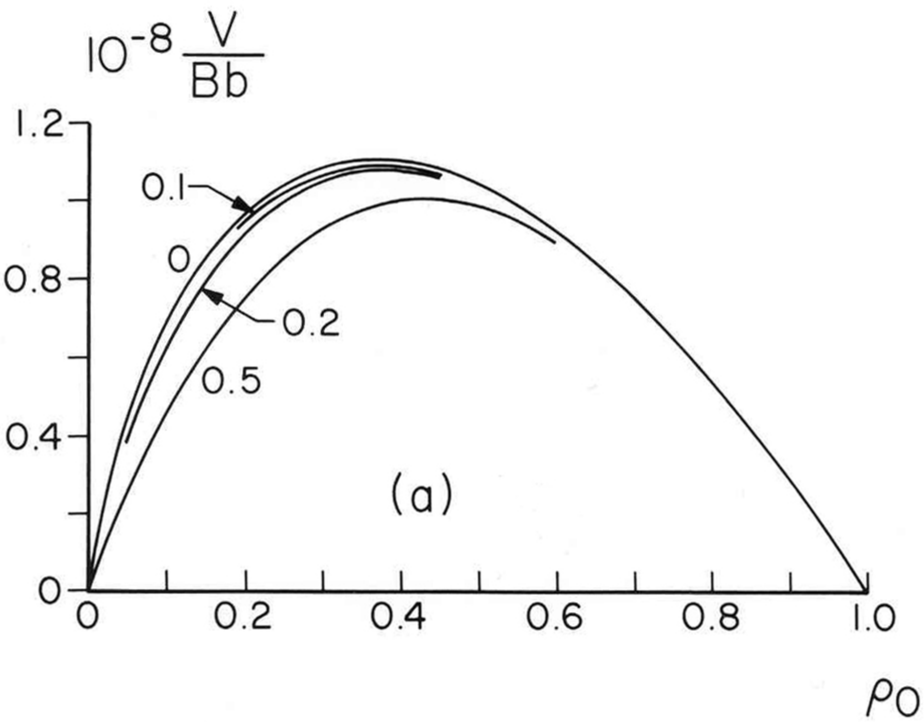
$$\frac{R_{sh}}{Q} \propto \eta$$

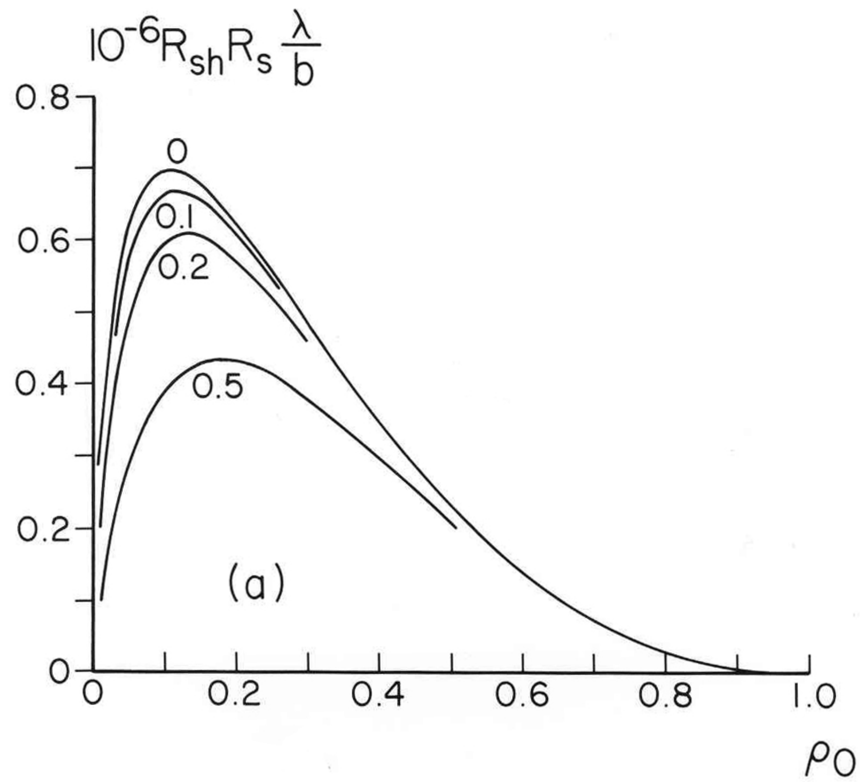


Loaded Quarter-wavelength Resonant Line



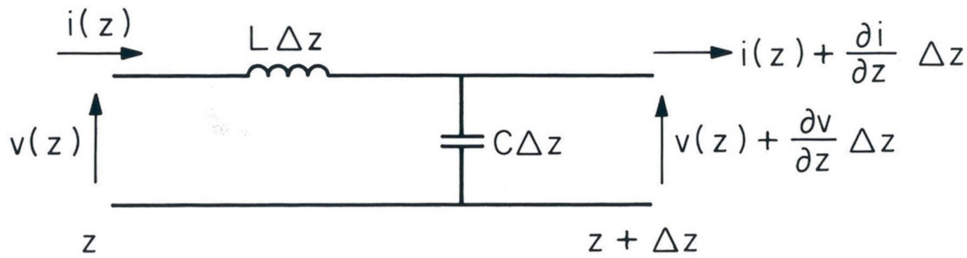
Loaded Quarter-wavelength Resonant Line





MKS units, lines of constant normalized loading capacitance $\Gamma/\lambda\varepsilon_0$

More Complicated Center Conductor Geometries



$$\frac{d^2v}{d\zeta^2} - \frac{1}{\rho \ln \rho}\frac{d\rho}{d\zeta}\frac{dv}{d\zeta} + \frac{\pi^2}{4}v = 0$$

$$\frac{d^2i}{d\zeta^2} + \frac{1}{\rho \ln \rho}\frac{d\rho}{d\zeta}\frac{di}{d\zeta} + \frac{\pi^2}{4}i = 0$$

$$\Gamma(z) = -C(z)\frac{i(z)}{di/dz}$$

More Complicated Center Conductor Geometries

Constant logarithmic derivative of line capacitance

Good model for linear taper

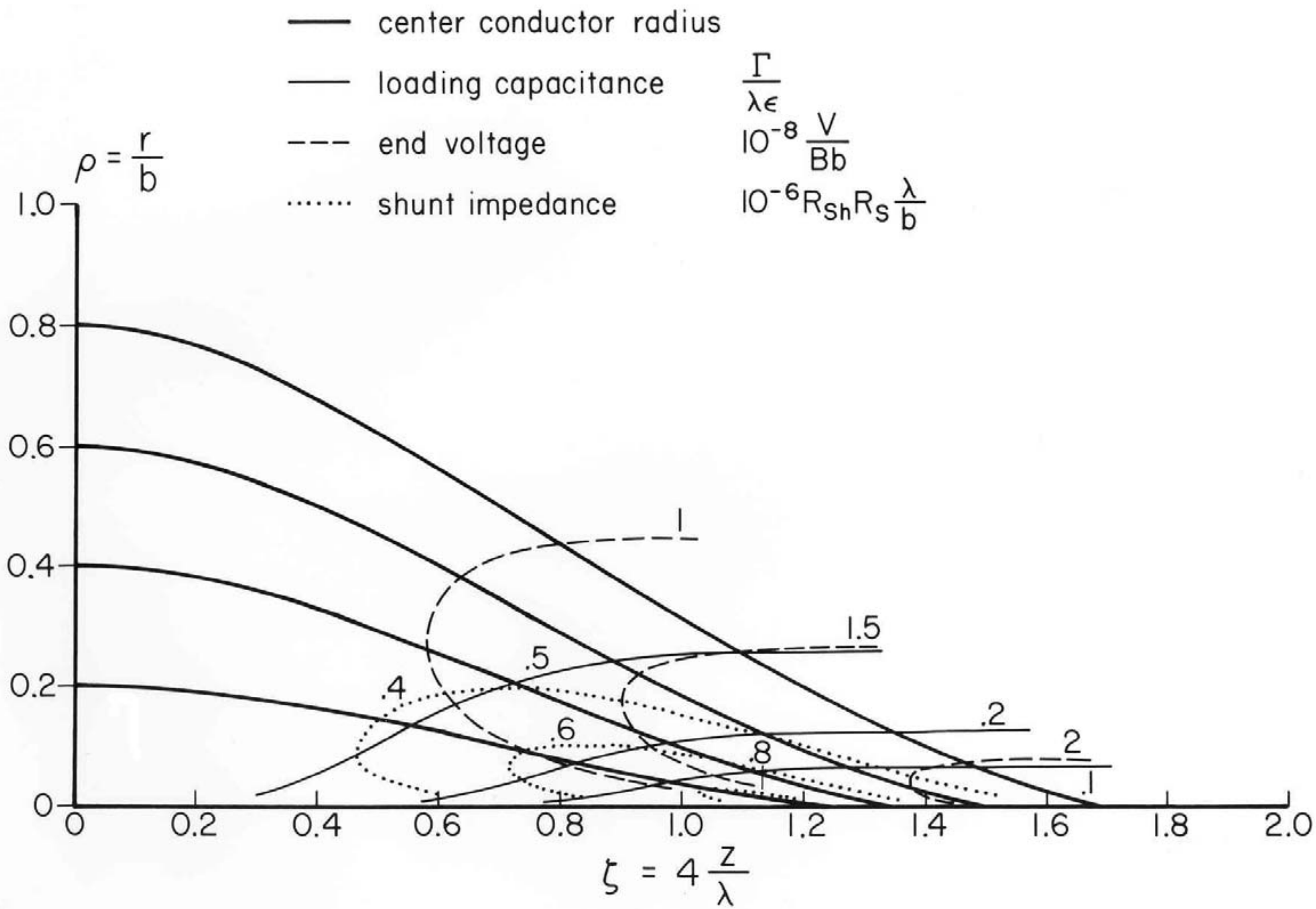
$$\frac{1}{C}\frac{dC}{dz} = -\frac{1}{d} \qquad r(z) = b\left(\frac{r_0}{b}\right)^{\exp(z/d)}$$

Constant surface magnetic field

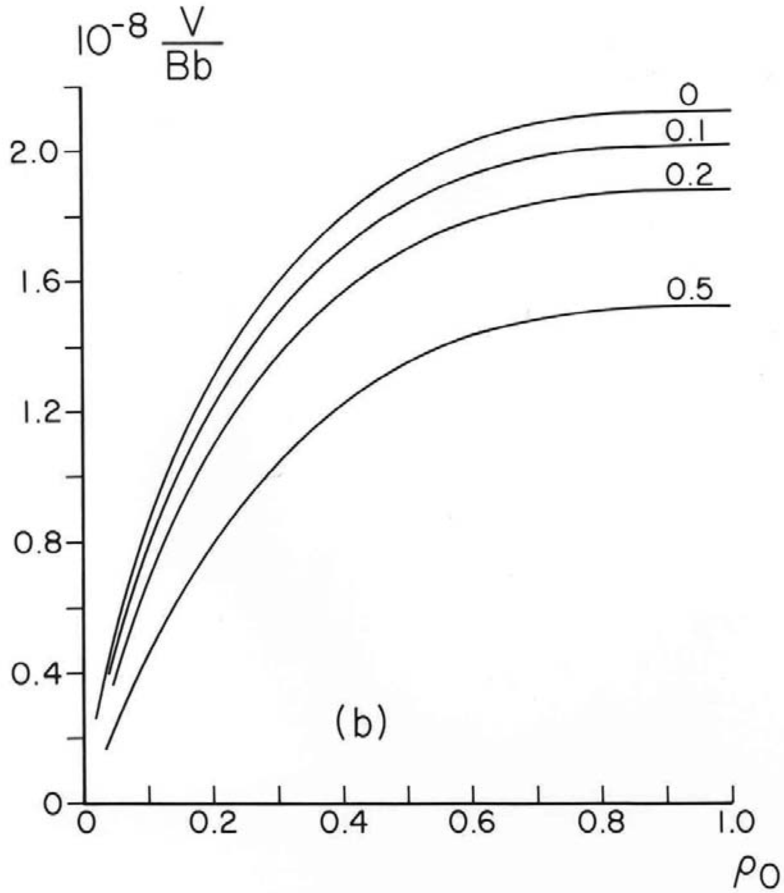
$$i(z) \propto r(z)$$

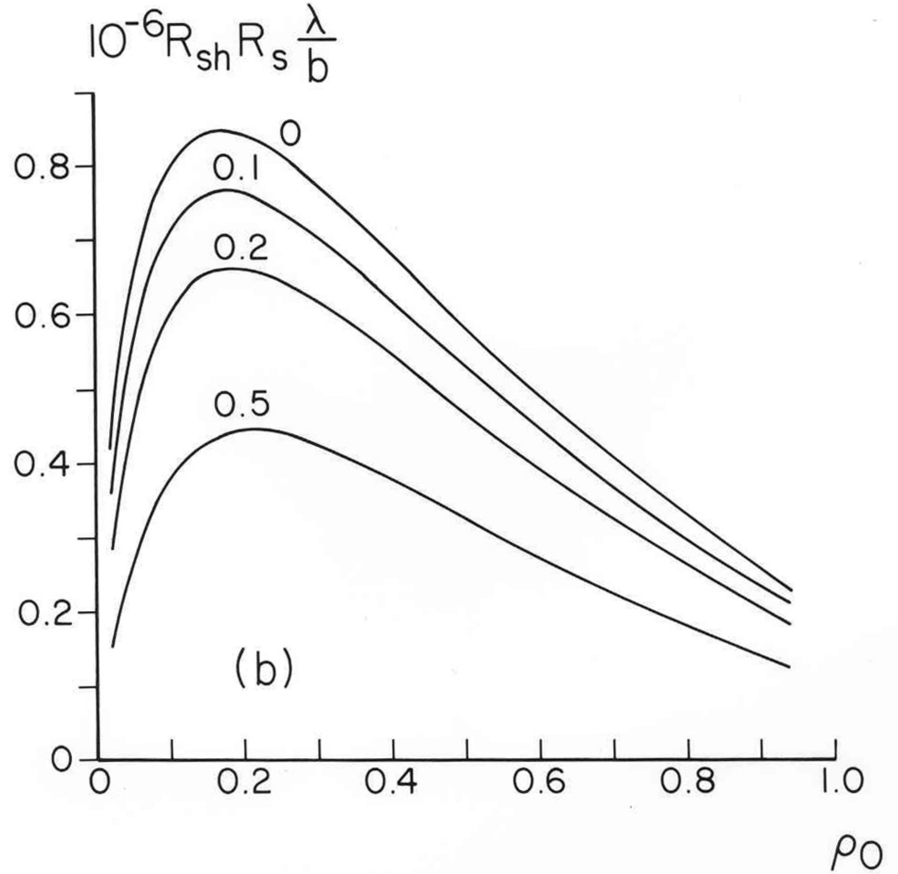
$$\frac{d^2 r}{dz^2} - \frac{1}{r\ln(b/r)}\left(\frac{dr}{dz}\right)^2 + \frac{4\pi^2}{\lambda^2} r = 0$$

Profile of Constant Surface Magnetic Field



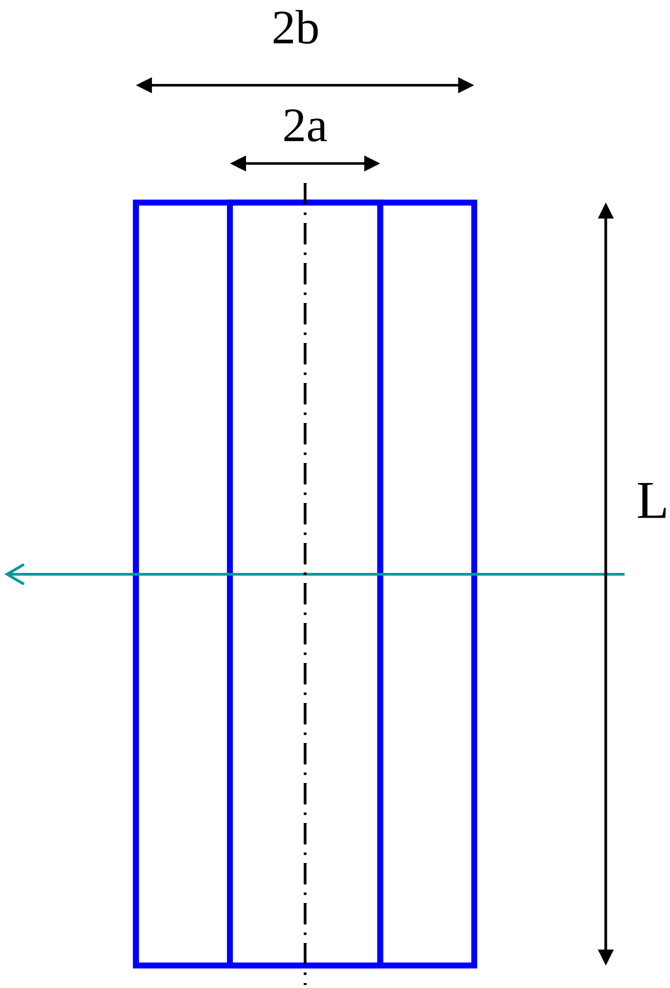
Profile of Constant Surface Magnetic Field

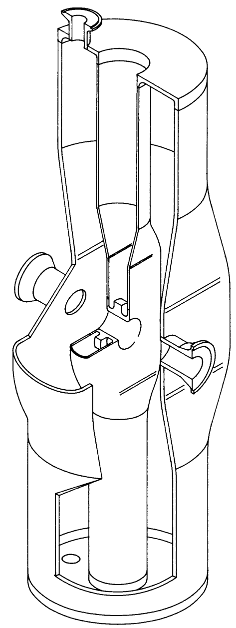


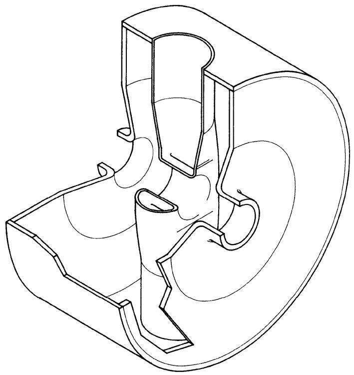


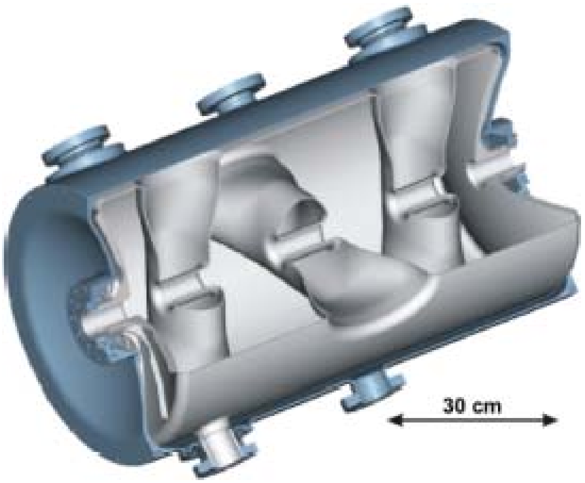
MKS units, lines of constant normalized loading capacitance $\Gamma/\lambda\varepsilon_0$

Another Simple Model: Coaxial Half-wave Resonator









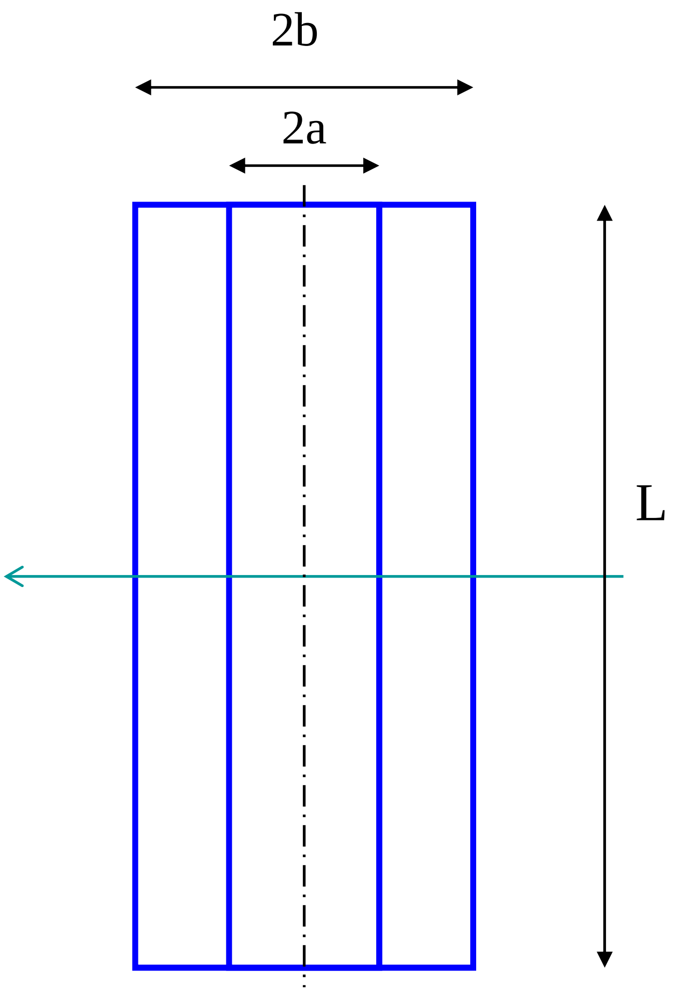
Coaxial Half-wave Resonator

Capacitance per unit length

$$C = \frac{2\pi\varepsilon_0}{\ln\left(\frac{b}{a}\right)} = \frac{2\pi\varepsilon_0}{\ln\left(\frac{1}{\rho_0}\right)}$$

Inductance per unit length

$$L = \frac{\mu_0}{2\pi}\ln\left(\frac{b}{r_0}\right) = \frac{\mu_0}{2\pi}\ln\left(\frac{1}{\rho_0}\right)$$



Coaxial Half-wave Resonator

Center conductor voltage

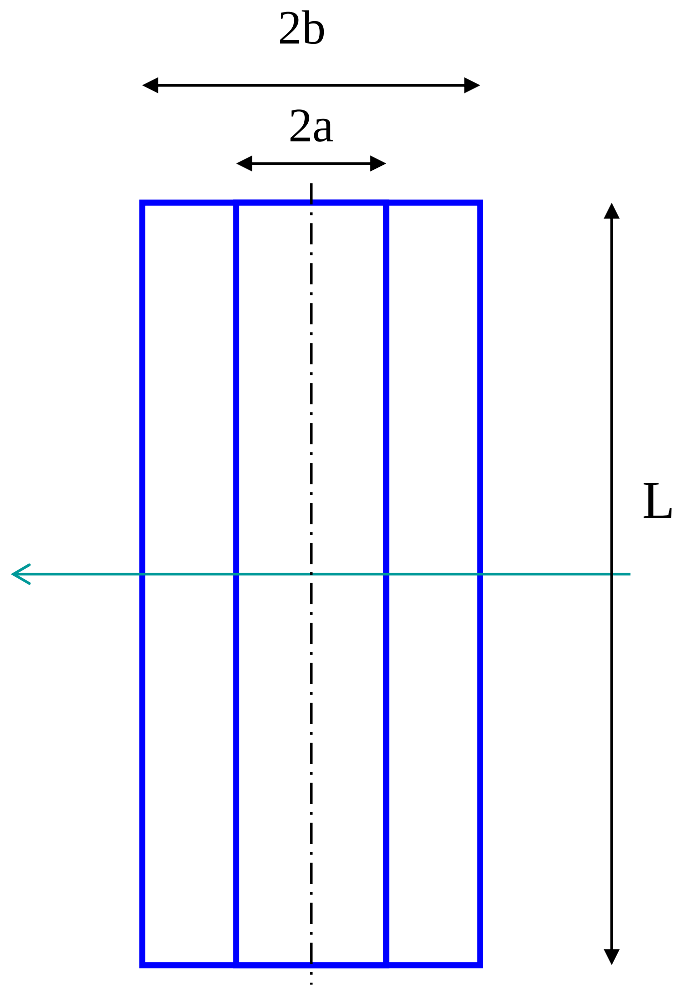
$$V(z) = V_0 \cos\left(\frac{2\pi}{\lambda} z\right)$$

Center conductor current

$$I(z) = I_0 \sin\left(\frac{2\pi}{\lambda} z\right)$$

Line impedance

$$Z_0 = \frac{V_0}{I_0} = \frac{\eta}{2\pi} \ln\left(\frac{1}{\rho_0}\right), \qquad \eta = \sqrt{\frac{\mu_0}{\varepsilon_0}} \simeq 377\Omega$$

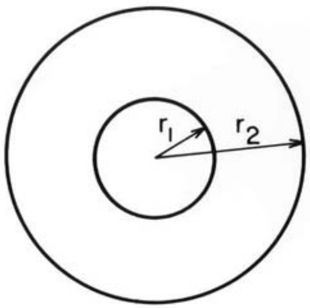


Coaxial Half-wave Resonator

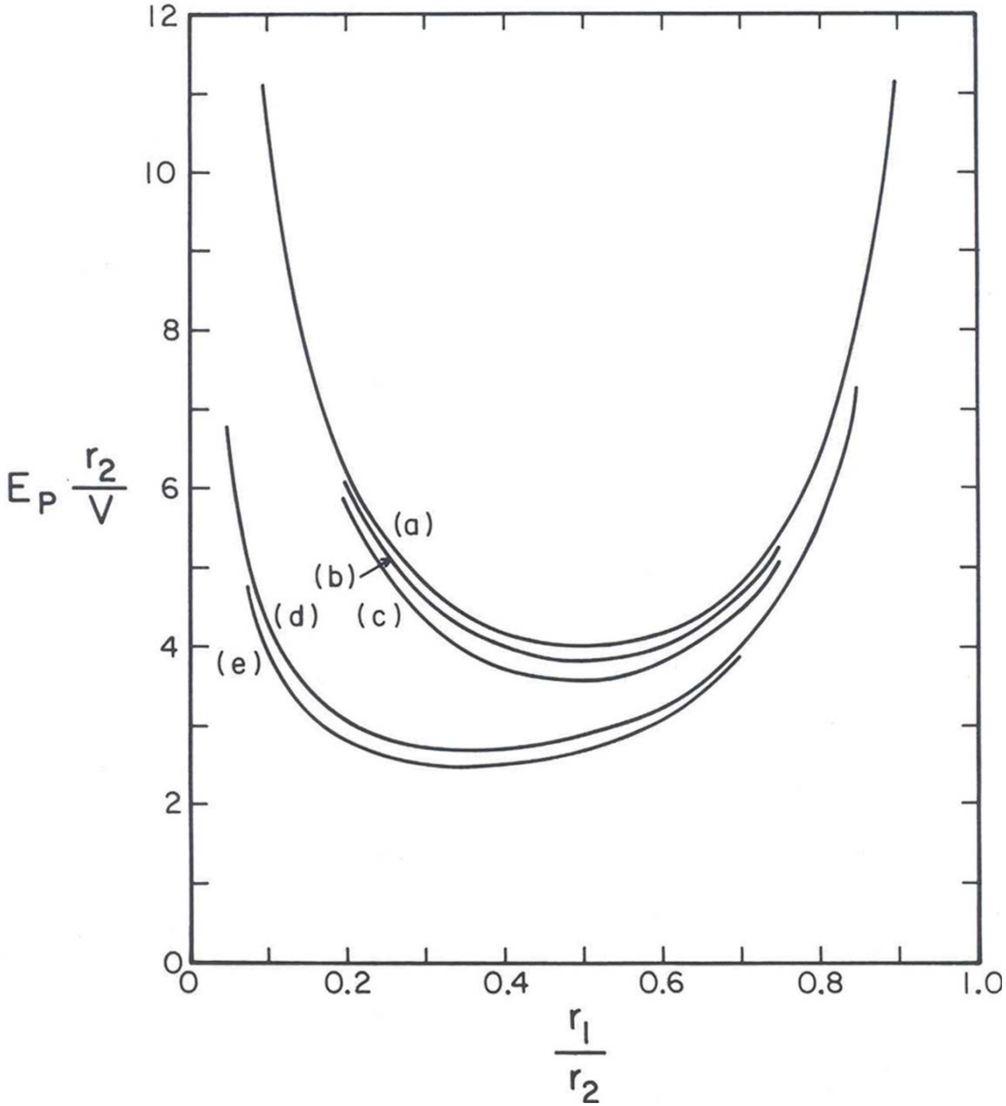
Peak Electric Field

d: coaxial cylinders

V_p : Voltage on center conductor
Outer conductor at ground
E_p: Peak field on center conductor



(a), (b), (d)



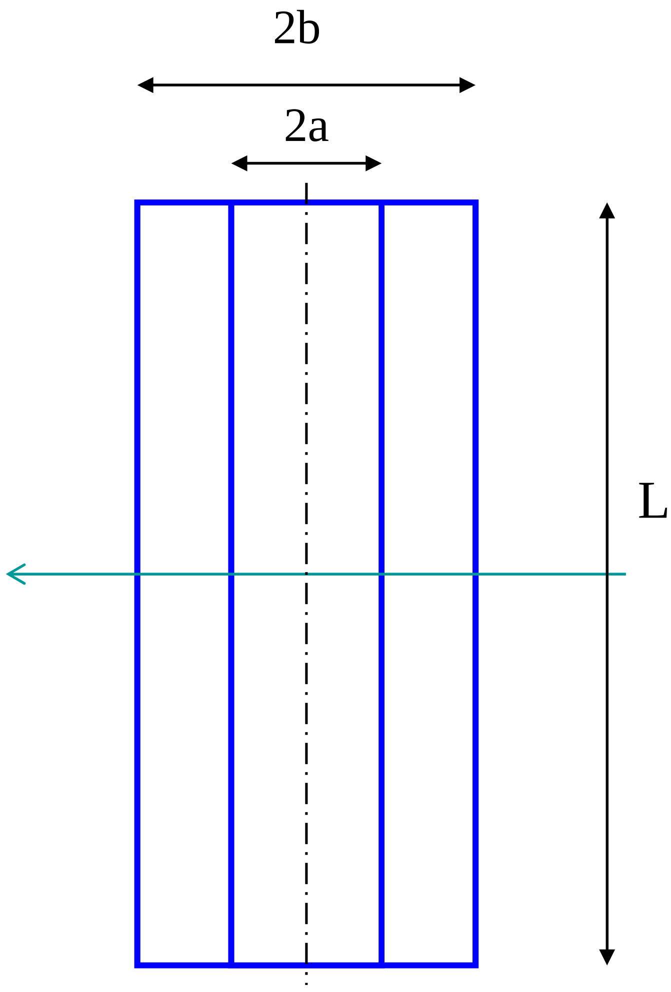
Coaxial Half-wave Resonator

Peak magnetic field

$$\frac{V_p}{b} = \begin{Bmatrix} \eta & H \\ c & B \\ 300 & B \end{Bmatrix} \rho_0 \ln\left(\frac{1}{\rho_0}\right) \qquad \begin{Bmatrix} \text{m, A/m} \\ \text{m, T} \\ \text{cm, G} \end{Bmatrix}$$

V_p: Voltage across loading capacitance

$B \simeq 9$ mT at 1 MV/m

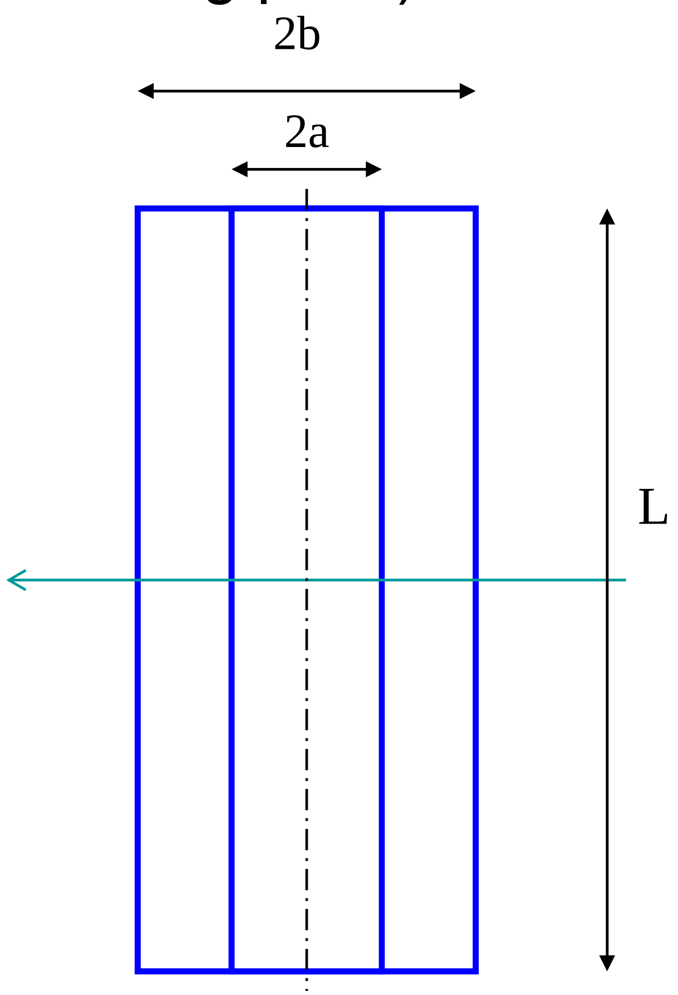


Coaxial Half-wave Resonator

Power dissipation (ignore losses in the shorting plate)

$$P = V_p^2 \frac{\pi}{4} \frac{R_s}{\eta^2} \frac{\lambda}{b} \frac{1+1/\rho_0}{\ln^2 \rho_0}$$

$$P \propto \frac{R_s}{\eta^2} E^2 \beta \lambda^2$$

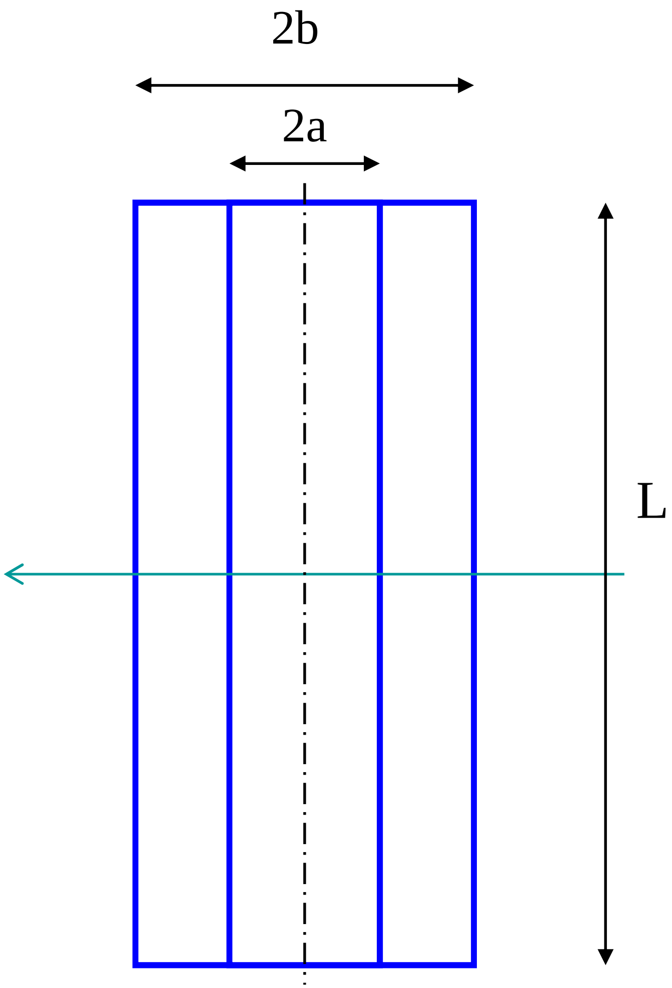


Coaxial Half-wave Resonator

Energy content

$$U = V_p^2 \frac{\pi \varepsilon_0}{4} \lambda \frac{1}{\ln\left(1/\rho_0\right)}$$

$$U \propto \varepsilon_0 E^2 \beta^2 \lambda^3$$

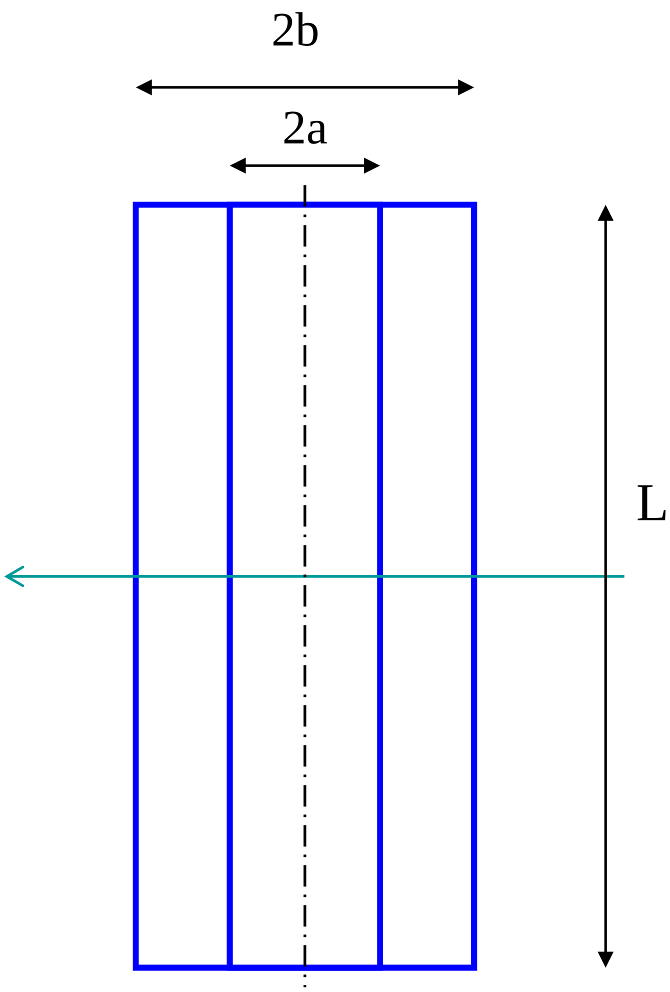


Coaxial Half-wave Resonator

Geometrical factor

$$G = QR_s = 2\pi\,\eta\,\frac{b}{\lambda}\,\frac{\ln\left(1/\rho_0\right)}{1+1/\rho_0}$$

$$G \propto \eta\,\beta$$

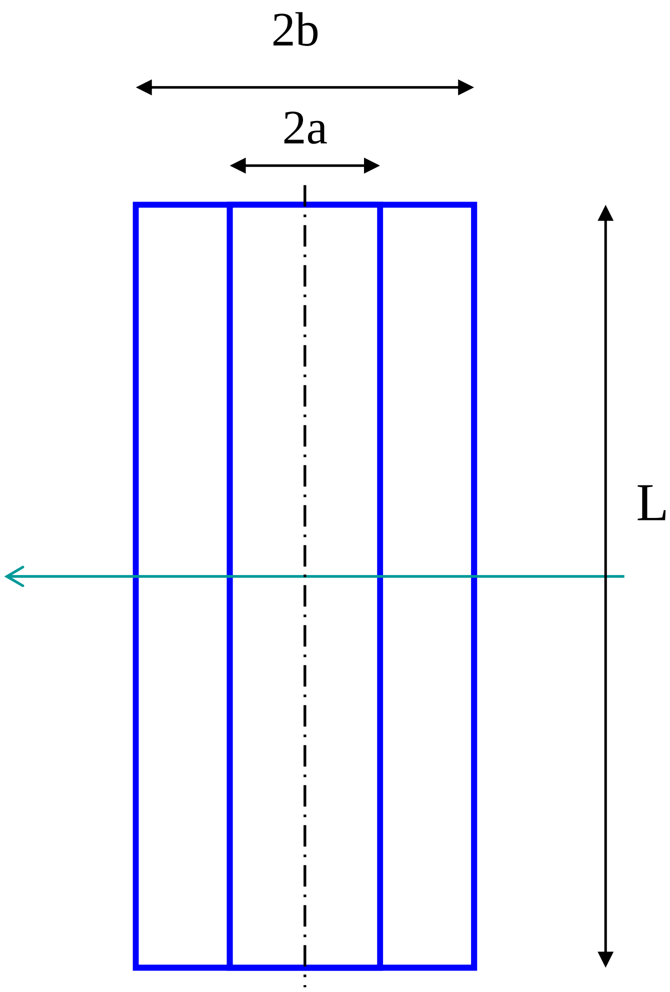


Coaxial Half-wave Resonator

Shunt impedance $\left(4V_p^2 / P\right)$

$$R_{sh} = \frac{\eta^2}{R_s}\frac{16}{\pi}\frac{b}{\lambda}\frac{\ln^2 \rho_0}{1+1/\rho_0}$$

$$R_{sh}\ R_s \propto \eta^2\ \beta$$

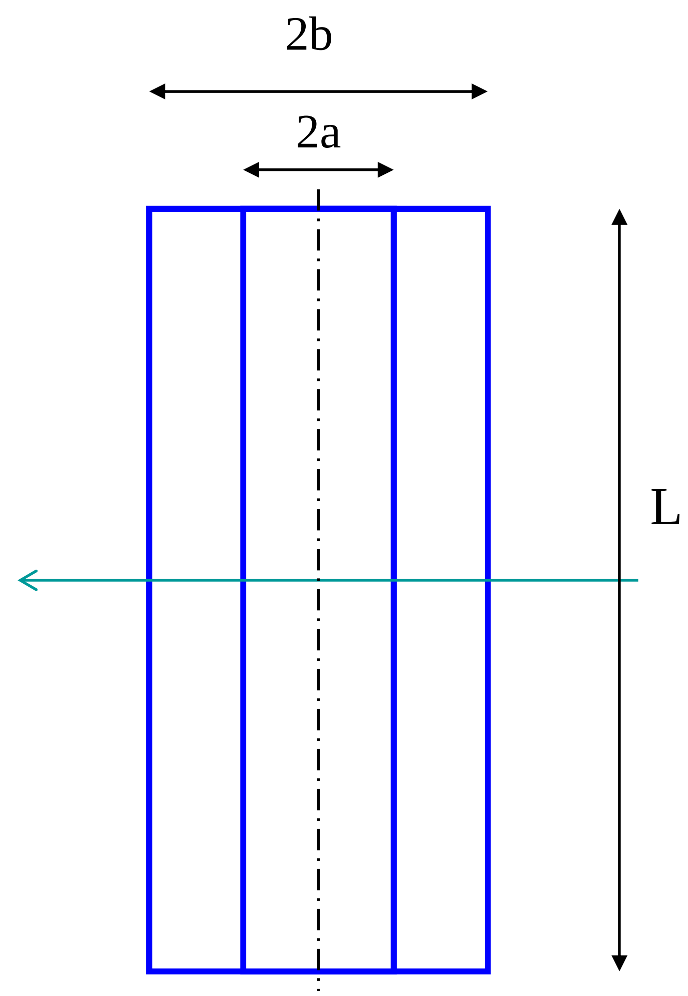


Coaxial Half-wave Resonator

R/Q

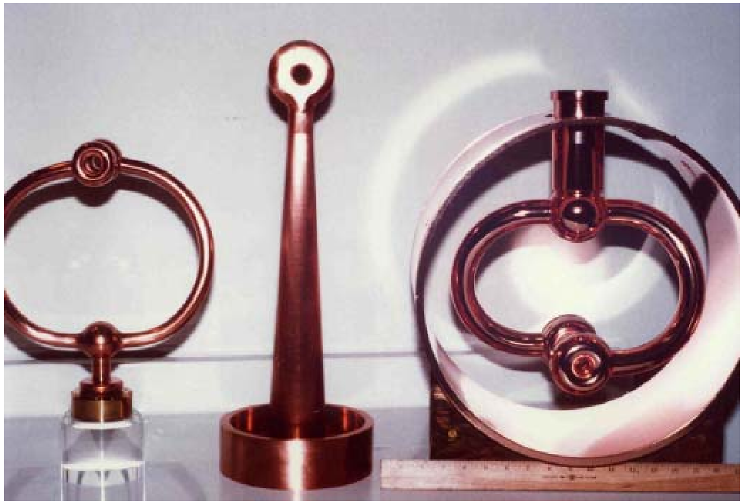
$$\frac{R_{sh}}{Q} = \frac{8}{\pi^2}\eta \ln\left(1/\rho_0\right)$$

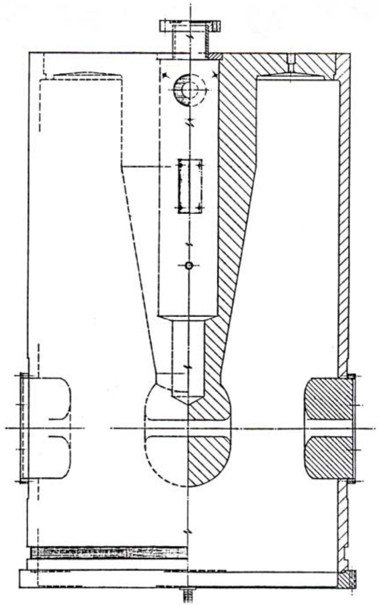
$$\frac{R_{sh}}{Q} \propto \eta$$



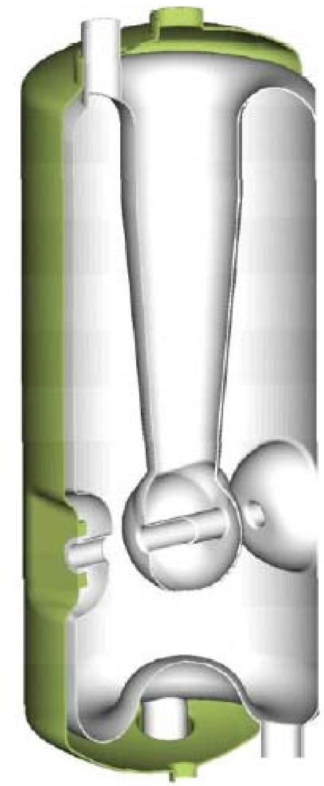
Some Real Geometries (λ/4)

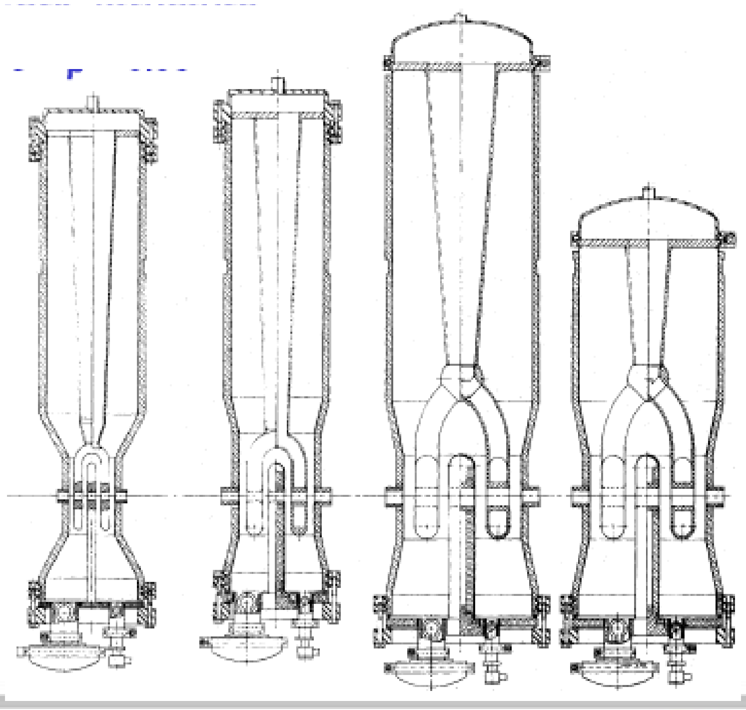


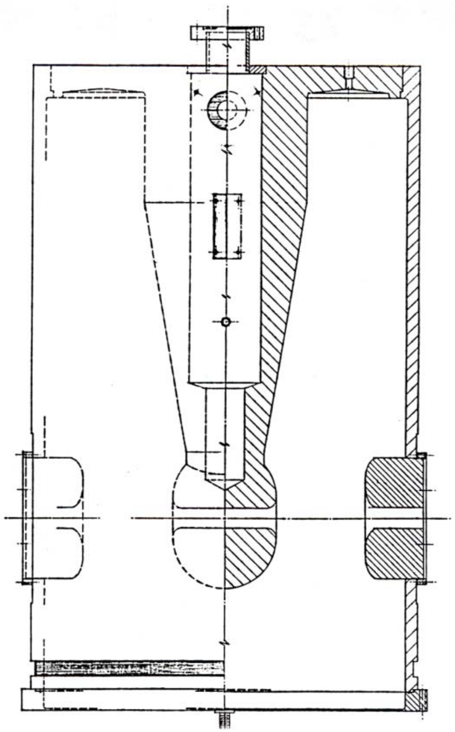


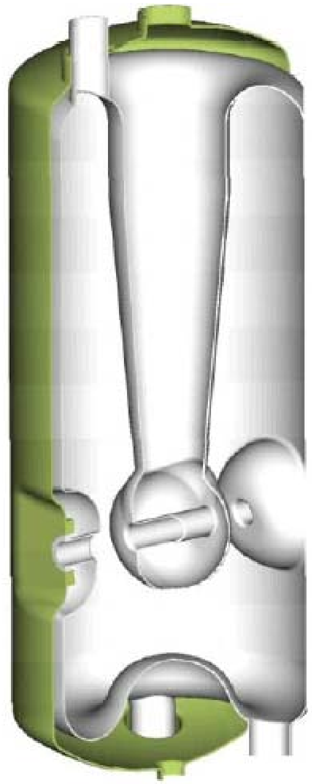
Some Real Geometries (λ/4)

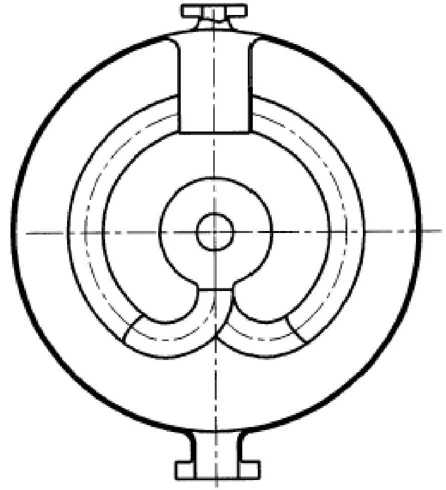


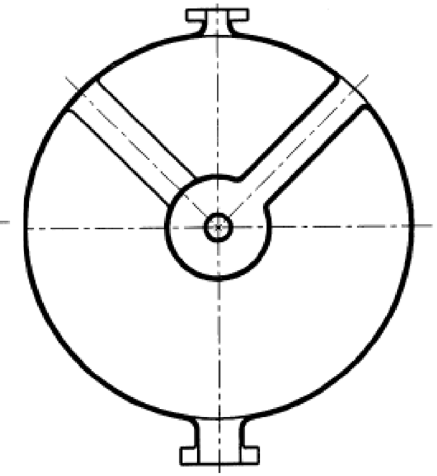


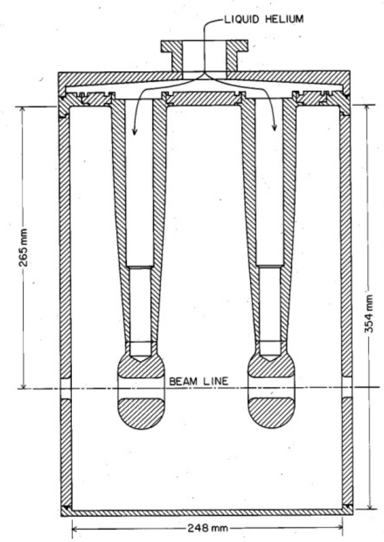
λ/4 Resonant Lines

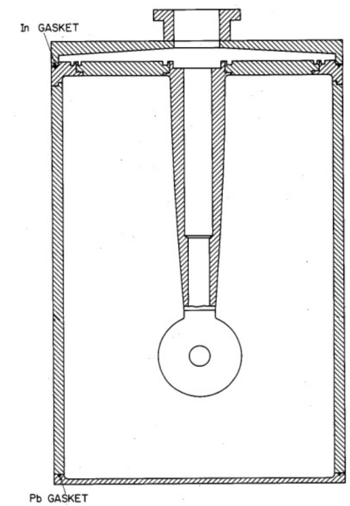




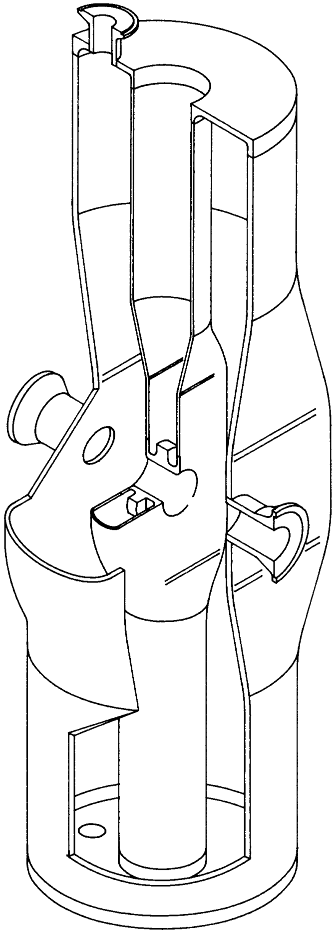


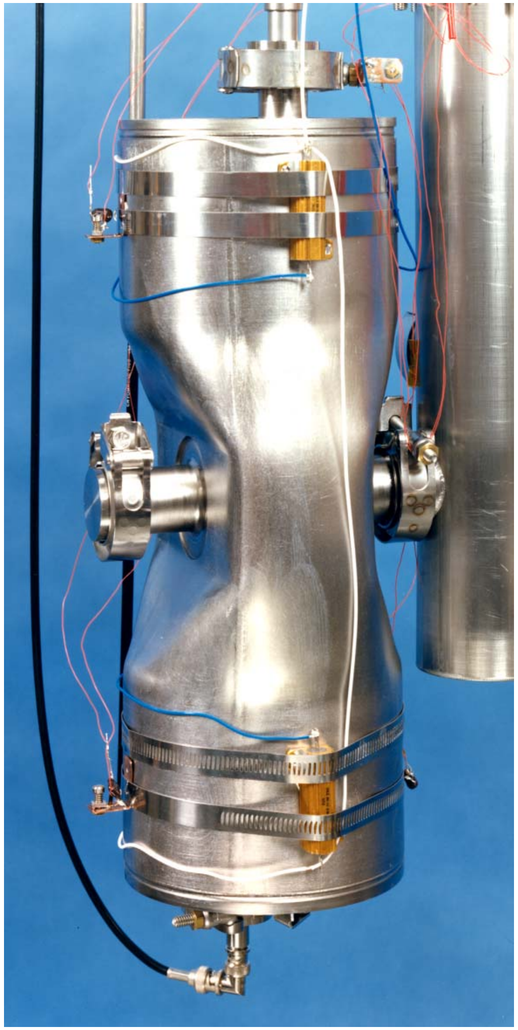


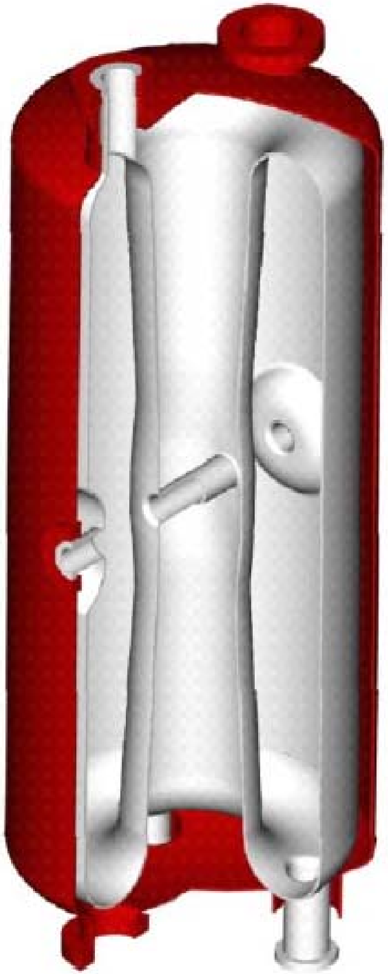




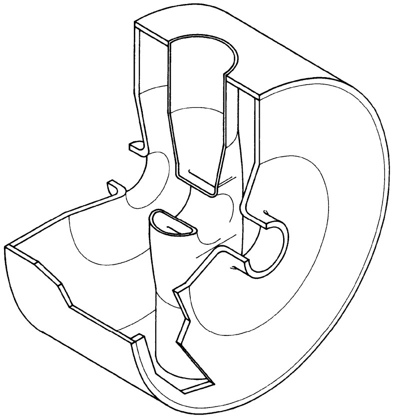
λ/2 Resonant Lines

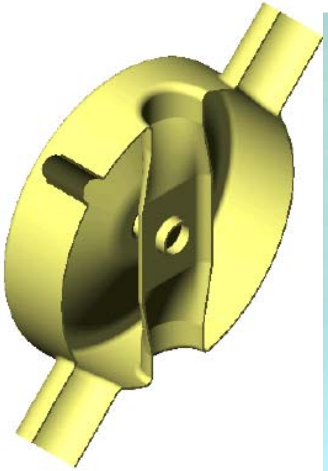


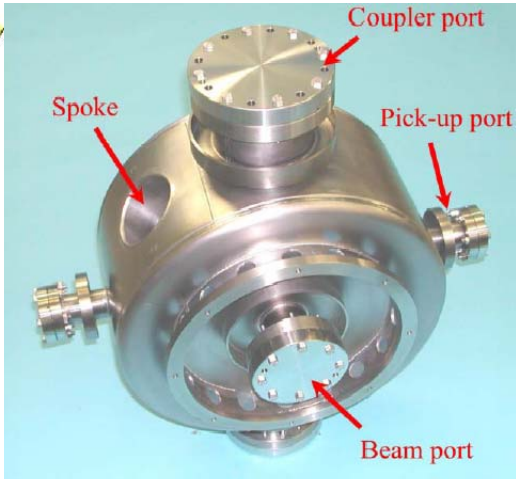




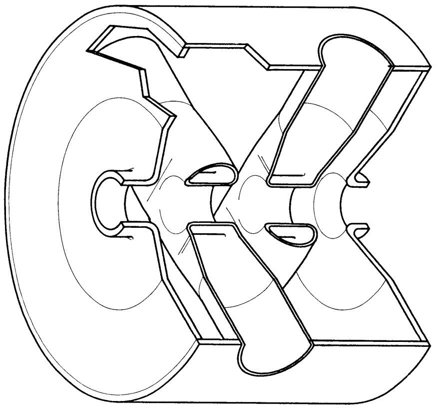
λ/2 Resonant Lines – Single-Spoke

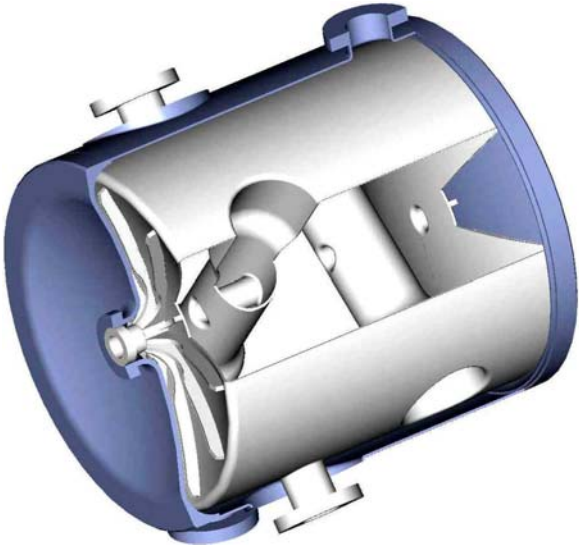


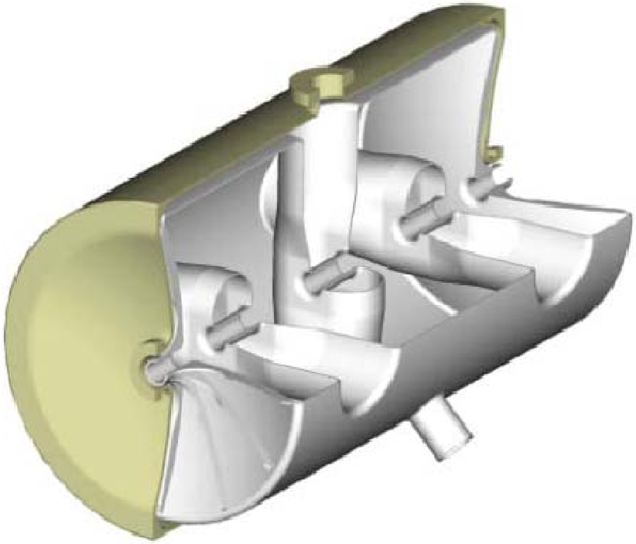




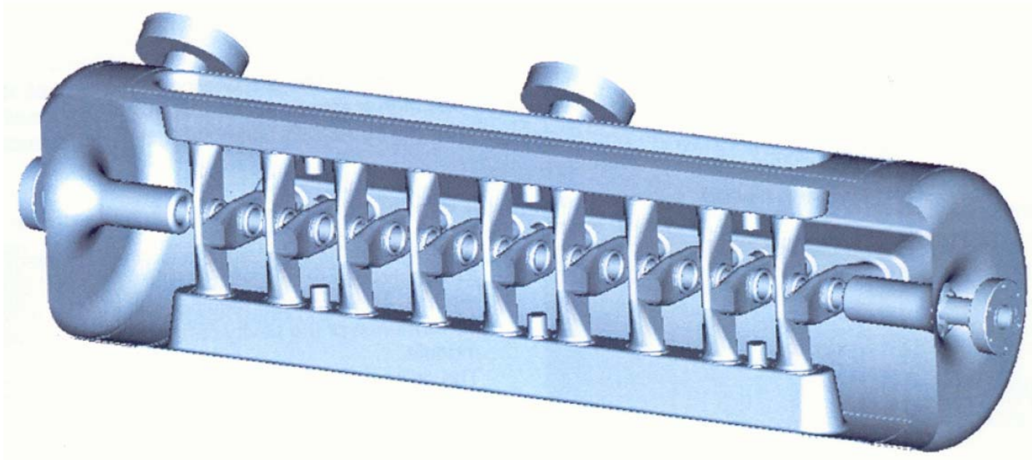
λ/2 Resonant Lines – Double and Triple-Spoke

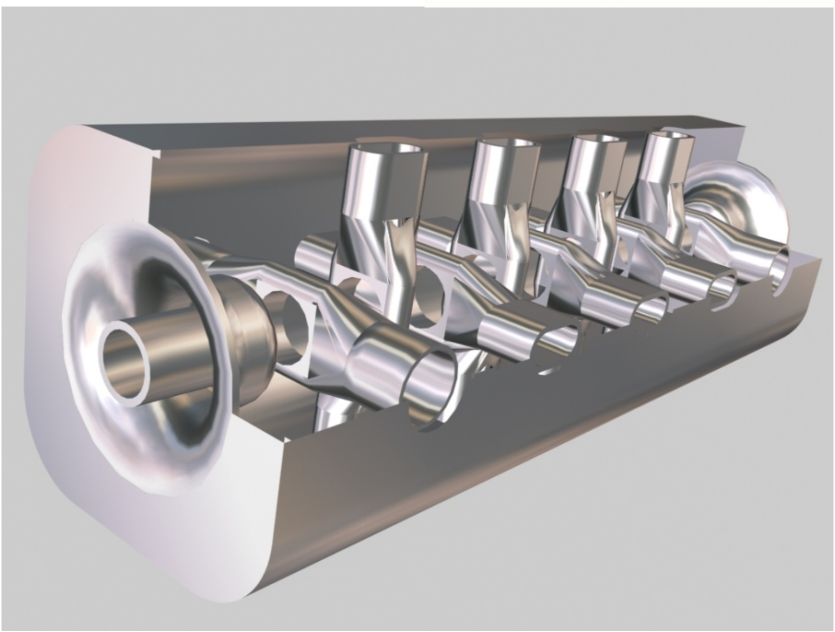


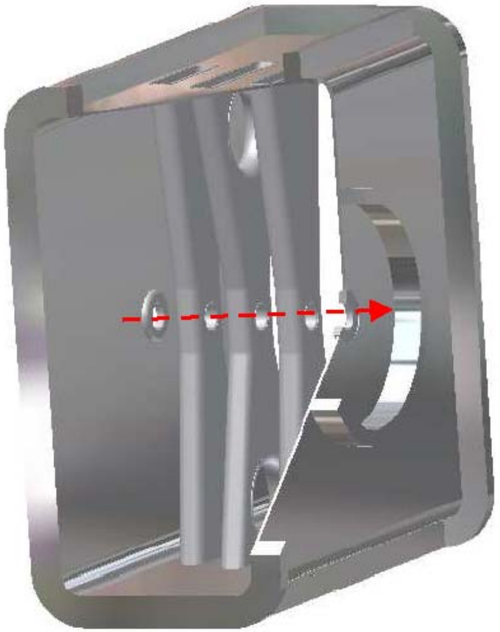




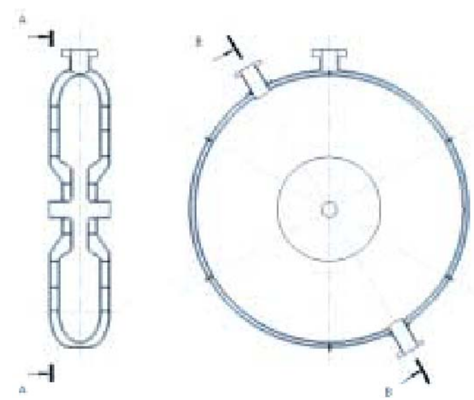
λ/2 Resonant Lines – Multi-Spoke







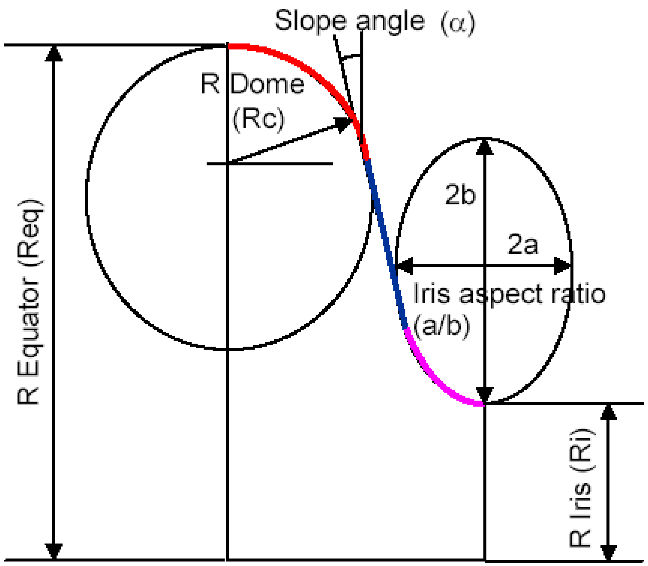
TM Modes



Design Considerations

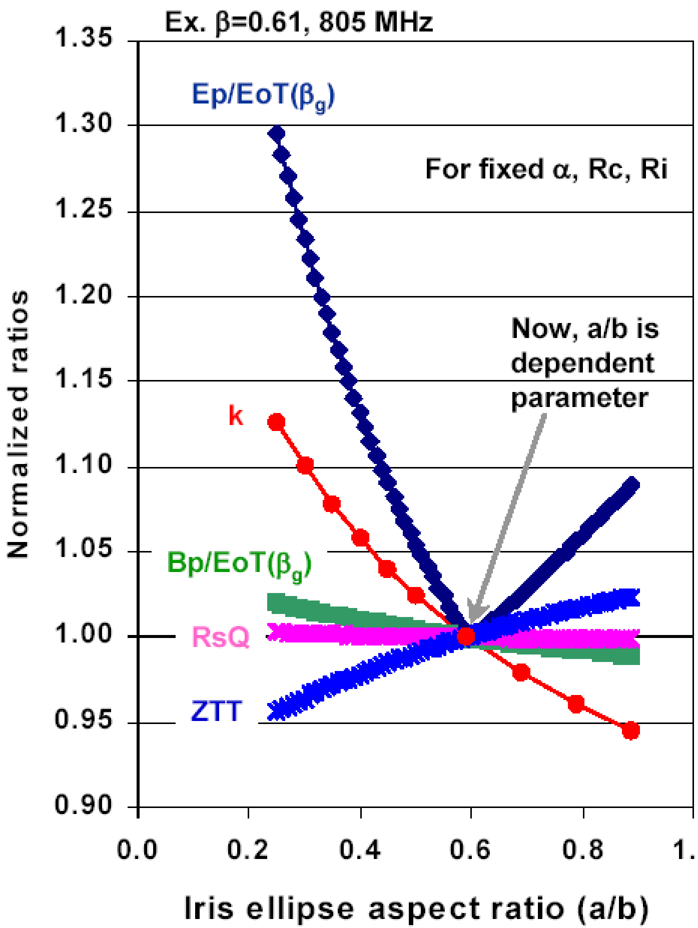
- Minimize the peak surface fields
  Bp; approaches to theoretical limit (190 mT)
  ← high RRR, defect control, better surface treatment (~170 mT)
  Ep; fields exceed 80 MV/m ← improved surface cleaning tech.
- Reasonable Inter-cell coupling between cells in Elliptical cavity
- Spoke cavity intrinsically has big coupling constant
- Provide required external Q
- In CW, higher shunt impedance (mainly determined by the cavity type)
- Reasonable mechanical stiffness
  common; reasonable tuning force, mechanical stability under vacuum pressure (test~2 atm), stable against microphonics
  pulsed; affordable dynamic Lorentz force detuning
- Safe from Multipacting
- Verify HOM and related issues
- Coupled field problems are common between RF, mechanical, thermal..
  → strong interfaces are needed

RF Geometry Optimization (elliptical cavity)



For circular dome
(Elliptical dome cases are same)

Rc, Ri, α, one of (**a/b**, a, b)
; 4 controllable parameters
Req (for tuning)



Elliptical cell geometry and dependencies of RF parameters on the ellipse aspect ratio (a/b) at the fixed slope angle, dome radius and bore radius.

RF Geometry Optimization (Spoke Cavity)

- There have been extensive efforts for design optimization especially to reduce the ratios of Ep/Eacc and Bp/Eacc.
  - Controlling A/B (Ep/Eacc) and C/D (Bp/Eacc) → Shape optimization
  - Flat contacting surface at spoke base will help in another minimization of Bp/Eacc
  - For these cavities:

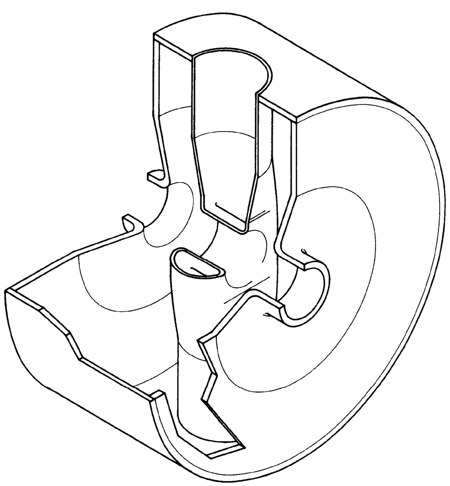
    Calculations agree well→ Ep/Eacc~3, Bp/Eacc~(7~8) mT/(MV/m), though it is tricky to obtain precise surface field information from the 3D simulation.

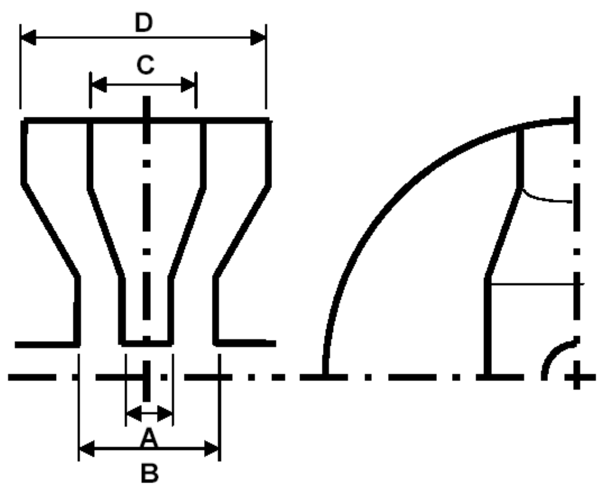
    Intrinsically have very strong RF coupling in multi-gap cavity.

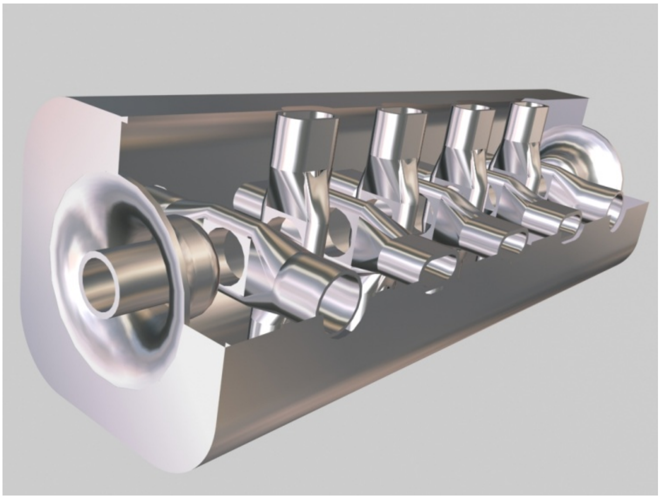
    Have rigid nature against static and dynamic vibrations.

    Beta dependency is quite small.

    Diameter~half of elliptical cavity.







Velocity Acceptance

- **Energy gain** $\Delta W = q\ V\ T(x)\ \ \Phi(x)\ \ \cos\varphi$

$$x = \frac{\beta\lambda}{2l}$$

$T(x)$ **Transit time factor for single cell**

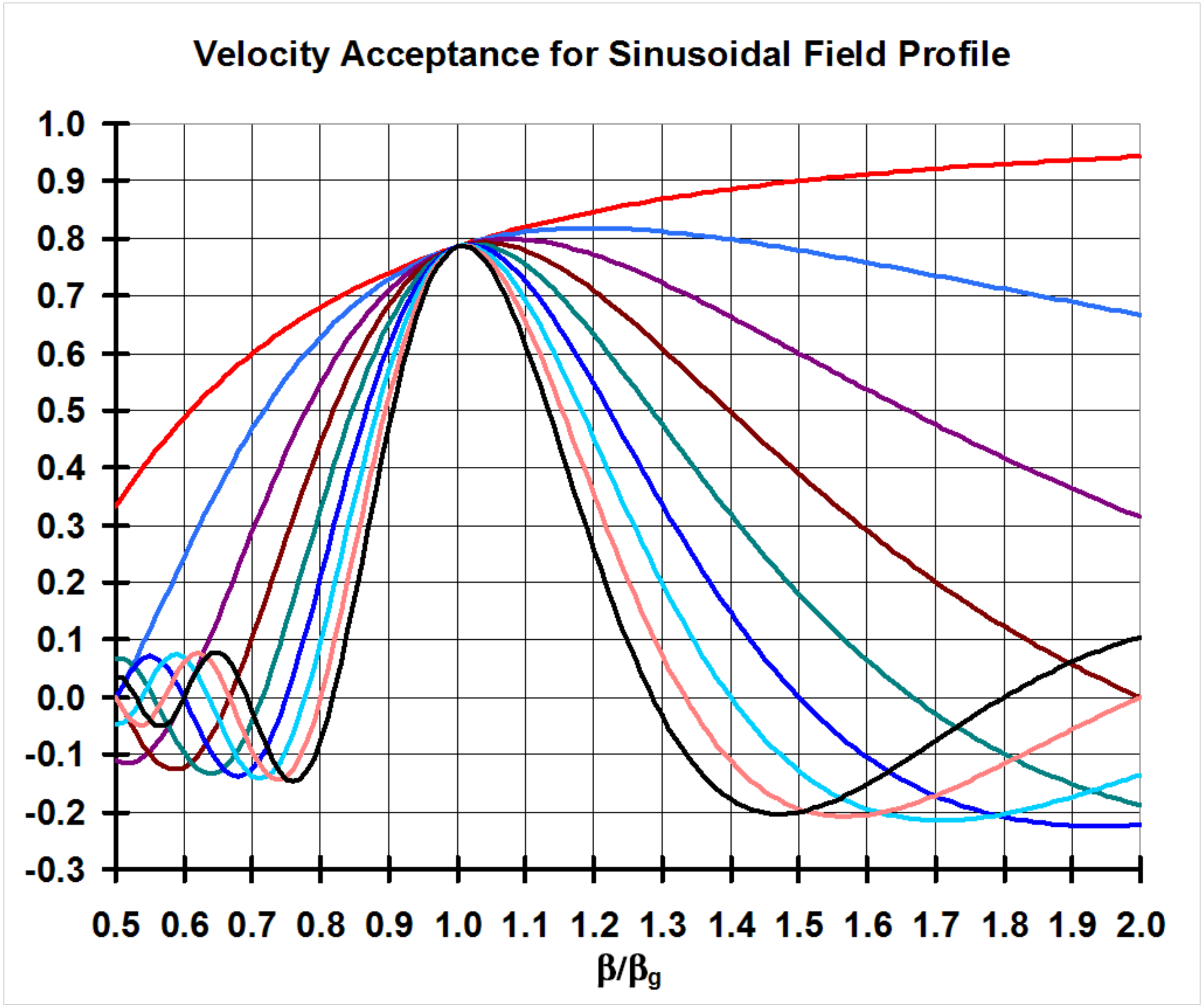
Depends on field profile in cell

$\Phi(x)$ **Phasing factor in multicell cavities**

Depends on cell spacing and field amplitude in cells

Does not depend on field profile in cells (assumed to be identical)

Velocity Acceptance

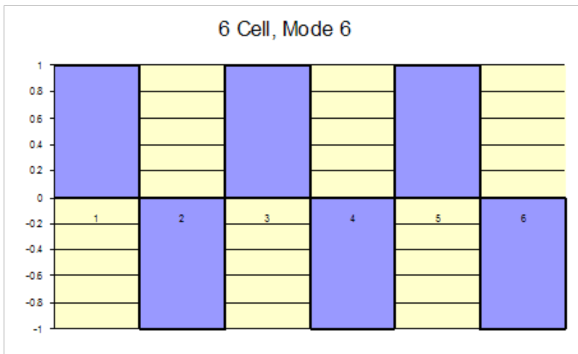


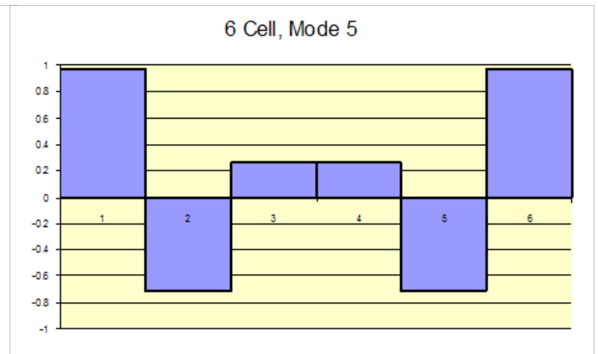
Voltage in Cells

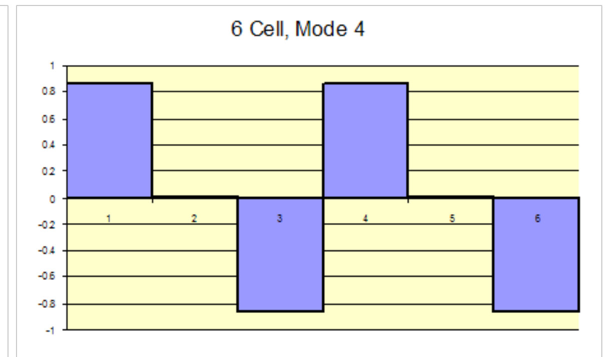
Voltage in j^th^ cell

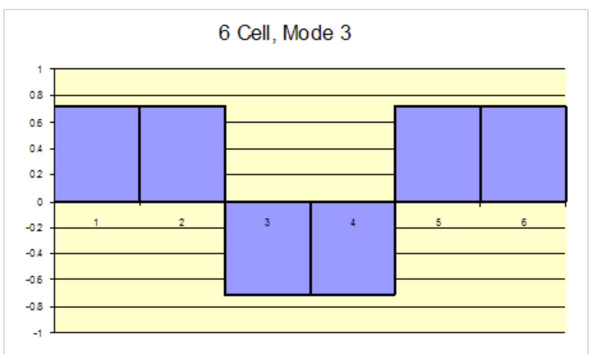
$$V_j^M = \sin\left(\pi M \frac{(2j-1)}{2N}\right)$$

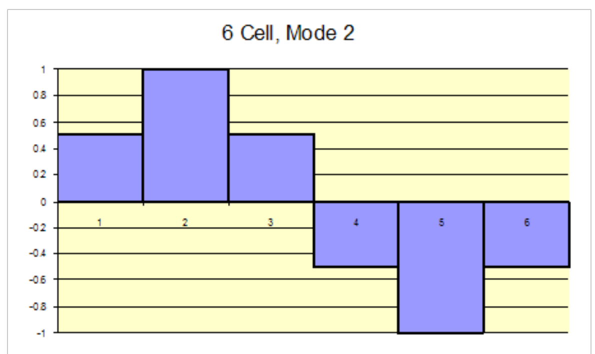
N: Number of cells, M: Mode number

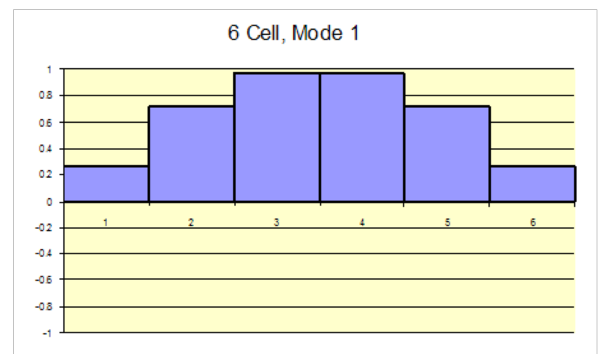












Phasing Factor

For fundamental (π) **mode:** $\Phi(x) = \dfrac{1}{\cos\left(\dfrac{\pi}{2x}\right)} \begin{cases} (-1)^{n+1}\sin\left(\dfrac{N\pi}{2x}\right), & N = 2n \\ (-1)^{n}\cos\left(\dfrac{N\pi}{2x}\right), & N = 2n+1 \end{cases}$

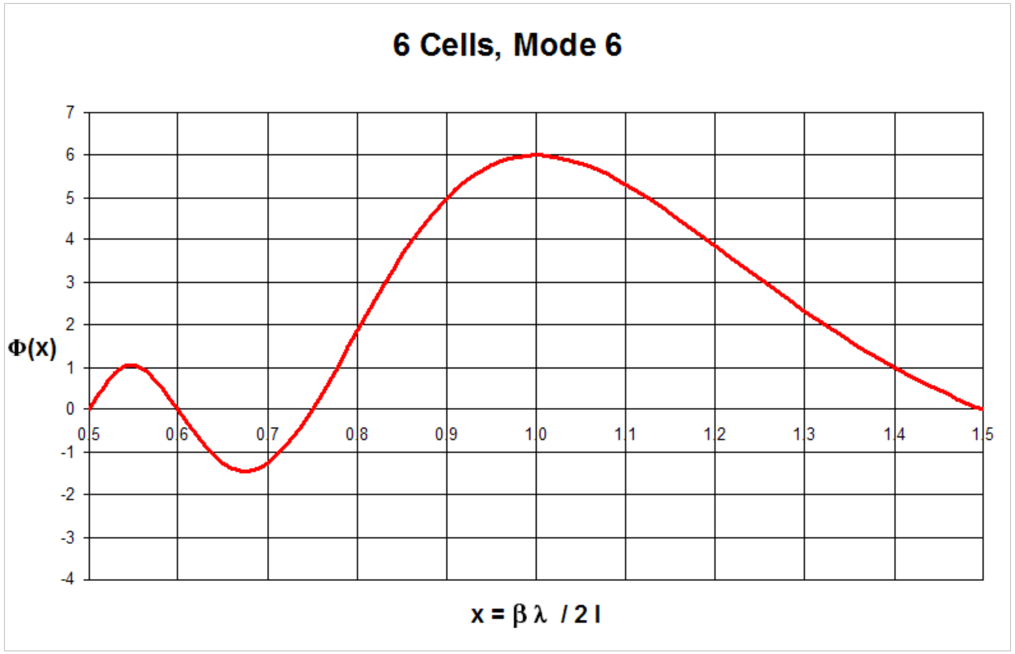
For all modes:

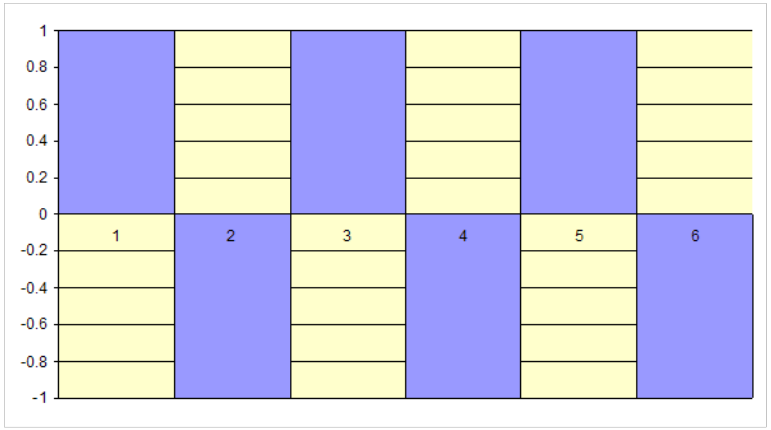
$$\Phi(x) = \frac{1}{2}\left(\frac{\sin\left[\dfrac{N\pi}{2}\left(\dfrac{M}{N} - \dfrac{1}{x}\right)\right]}{\sin\left[\dfrac{\pi}{2}\left(\dfrac{M}{N} - \dfrac{1}{x}\right)\right]} + (-1)^{M+1} \frac{\sin\left[\dfrac{N\pi}{2}\left(\dfrac{M}{N} + \dfrac{1}{x}\right)\right]}{\sin\left[\dfrac{\pi}{2}\left(\dfrac{M}{N} + \dfrac{1}{x}\right)\right]} \right)$$

If M=N, recover previous formula

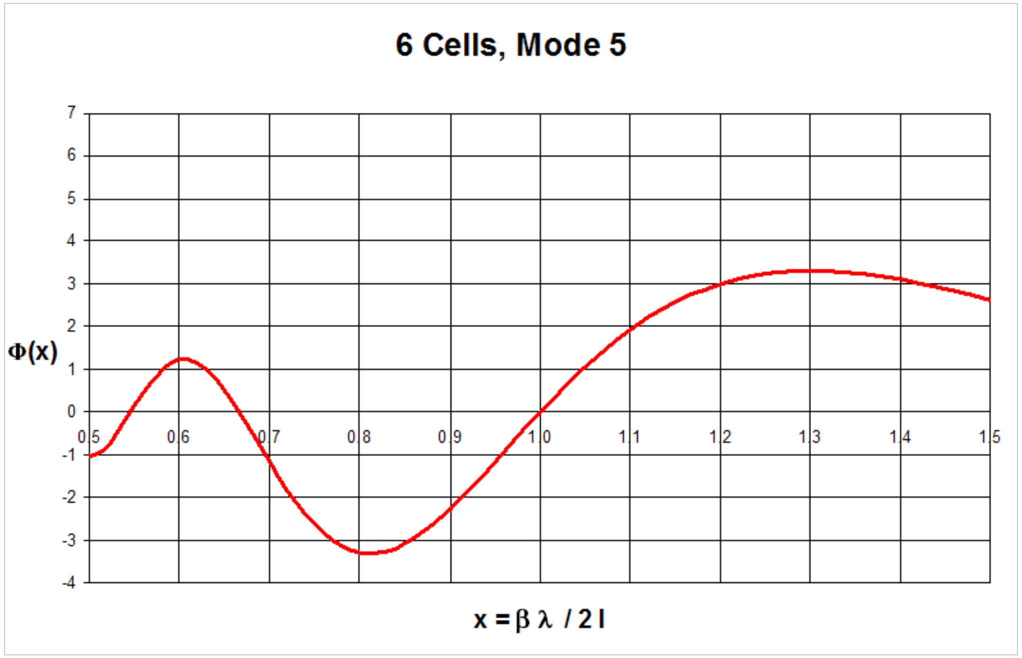
If x=1 $\Phi(x) = N\delta_{MN}$

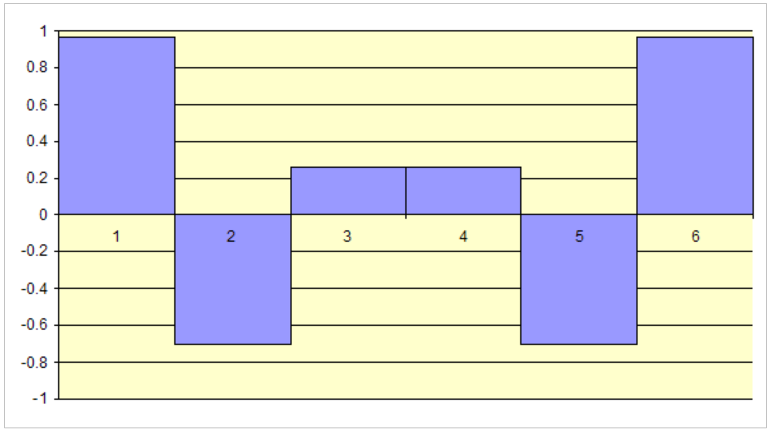
Phasing Factor



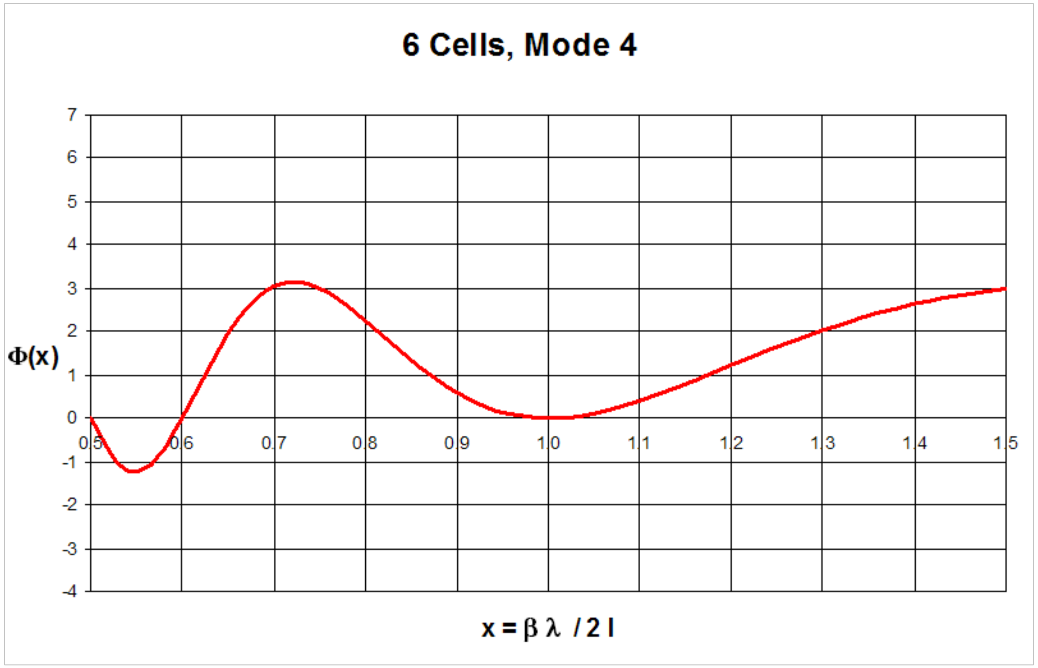


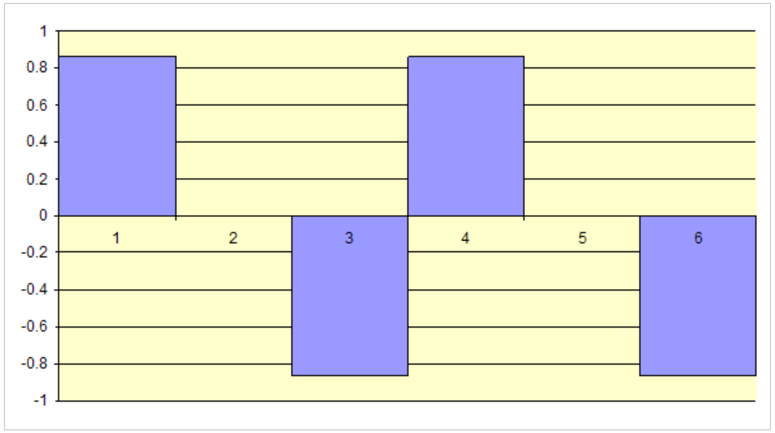
Phasing Factor





Phasing Factor





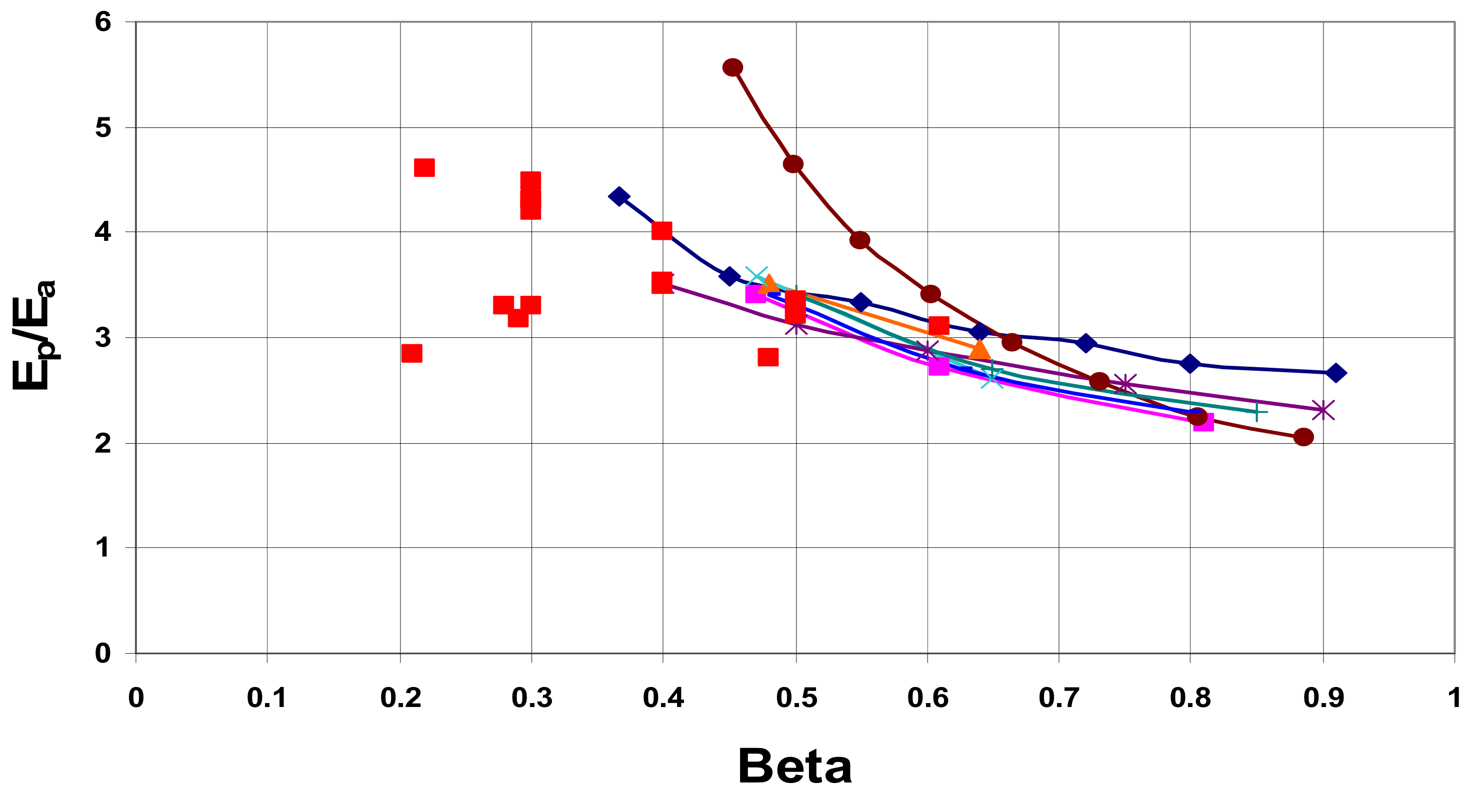
Surface Electric Field

- TM_{010} elliptical structures
  - $E_p/E_a \sim 2$ for $\beta = 1$
  - Increases slowly as β decreases

- $\lambda/2$ structures:
  - Sensitive to geometrical design
  - Electrostatic model of an “shaped geometry” gives $E_p/E_a \sim 3.3$, independent of β

Surface Electric Field

- **Lines: Elliptical** **Squares: Spoke**



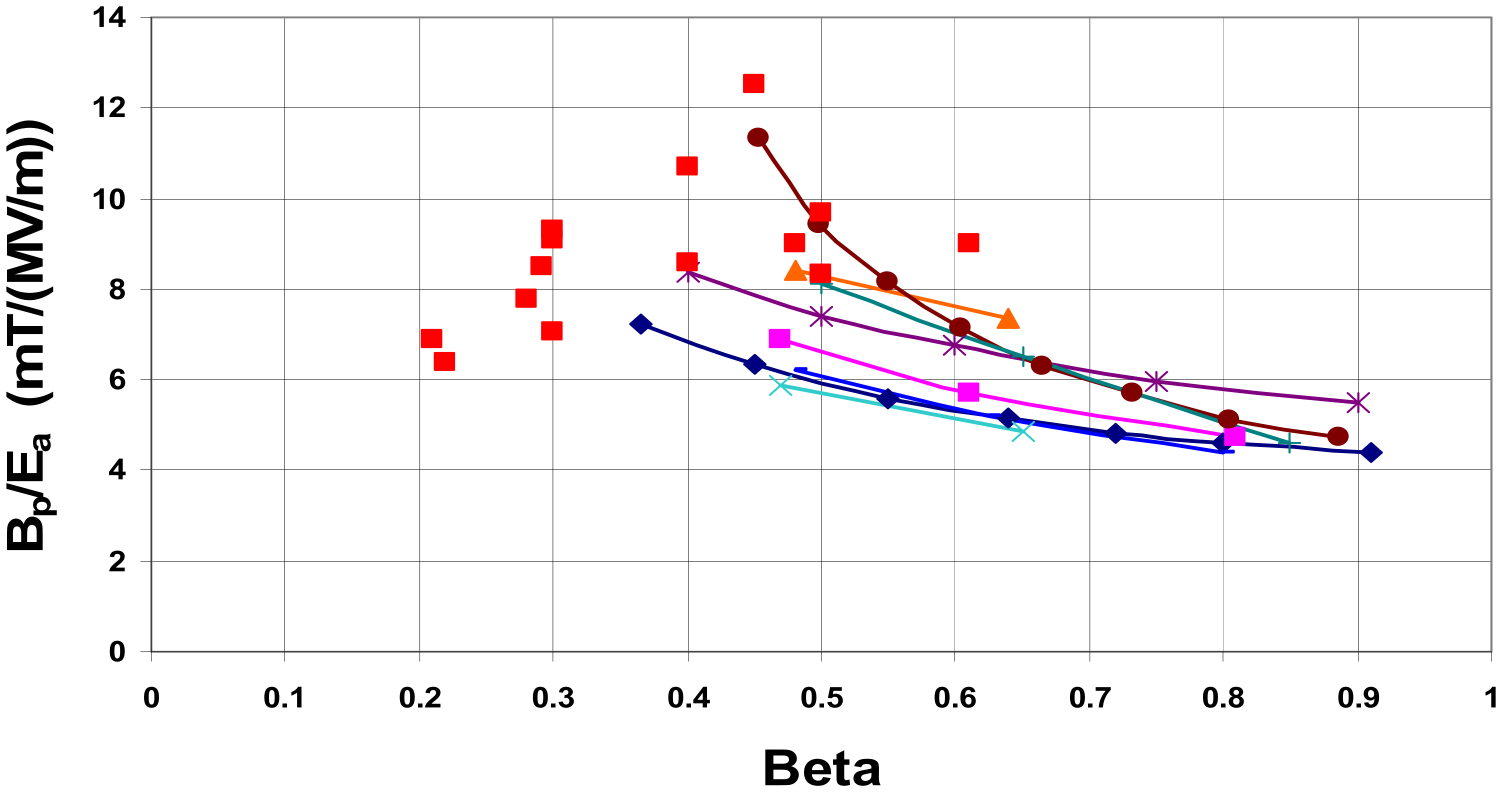
Surface Magnetic Field

- TM_{010} elliptical cavities:
  - $B/E_a \sim 4$ mT/(MV/m) for β=1
  - Increases slowly as β decreases

- λ/2 structures:
  - Sensitive to geometrical design
  - Transmission line model gives $B/E_a \sim 8$ mT/(MV/m), independent of β

Surface Magnetic Field

- **Lines: Elliptical** **Squares: Spoke**



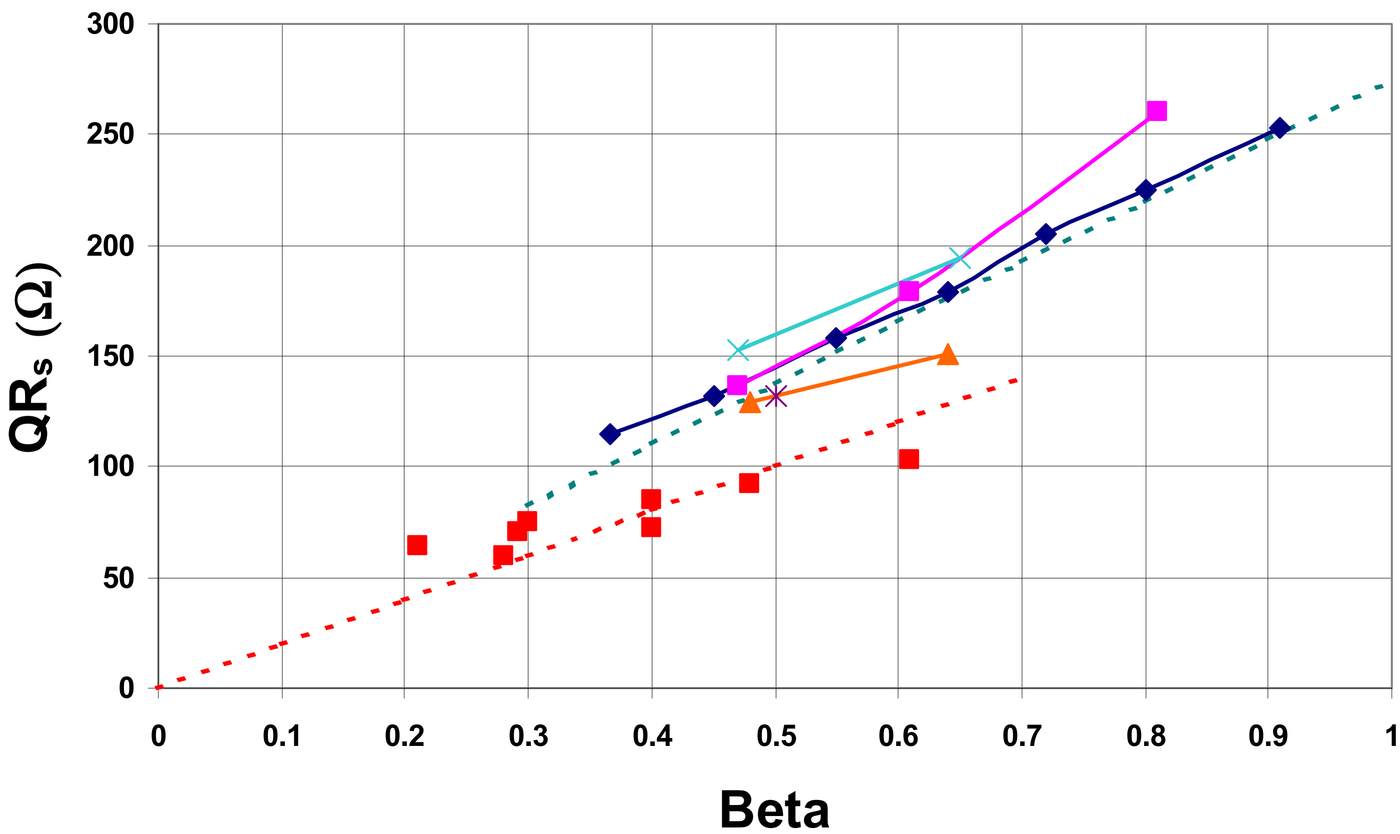
Geometrical Factor (QR_s)

- **TM_{010} elliptical cavities:**
  - **Simple scaling: $QR_s \sim 275\,\beta \ (\Omega)$**

- **$\lambda/2$ structures:**
  - **Transmission line model: $QR_s \sim 200\,\beta \ (\Omega)$**

Geometrical Factor (QR_s)

- Lines: Elliptical Squares: Spoke



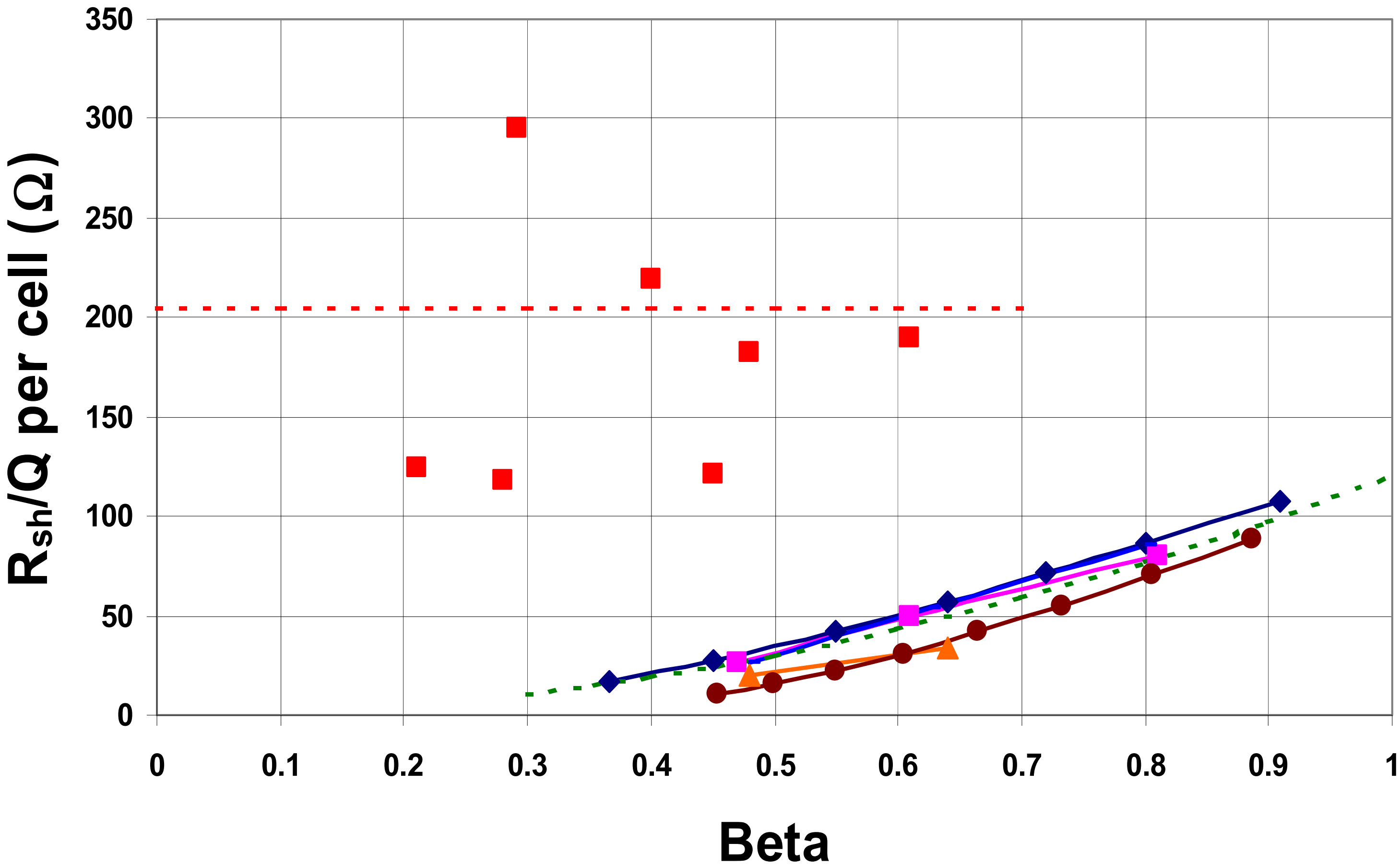
R_{sh}/Q per Cell or Loading Element

- $R_{sh} = V^2/P$
- TM_{010} elliptical cavities:
  - Simple-minded argument, ignoring effect of beam line aperture, gives: $R_{sh} / Q \propto \beta$
  - When cavity length becomes comparable to beam line aperture : $R_{sh} / Q \propto \beta^2$
  - $R_{sh}/Q \sim 120\ \beta^2\quad(\Omega)$

- $\lambda/2$ structures:
  - Transmission line model gives: $R_{sh}/Q \sim 205\ \Omega$
  - Independent of β

R_{sh}/Q per Cell or Loading Element

Lines: Elliptical **Squares: Spoke**



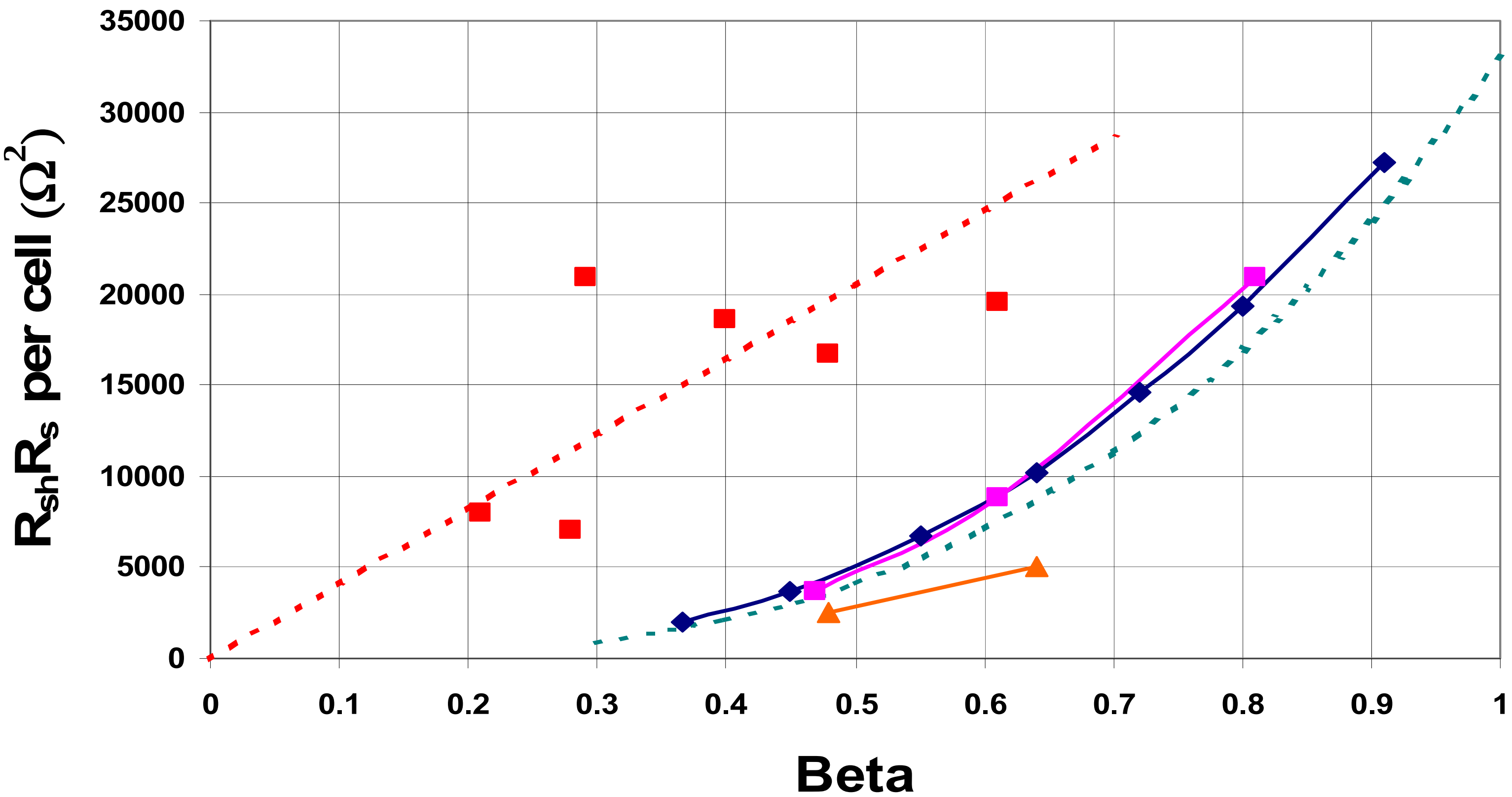
Shunt Impedance R_{sh}
(R_{sh}/Q QR_s per Cell or Loading Element)

- TM$_{010}$ elliptical cavities:
  - $R_{sh}\,R_s \sim 33000\,\beta^3 \quad (\Omega^2)$

- $\lambda/2$ structures:
  - $R_{sh}\,R_s \sim 40000\,\beta \quad (\Omega^2)$

Shunt Impedance R_{sh}
($R_{sh}/Q \; QR_s$ per cell or loading element)

- **Lines: Elliptical** **Squares: Spoke**



Energy Content per Cell or Loading Element

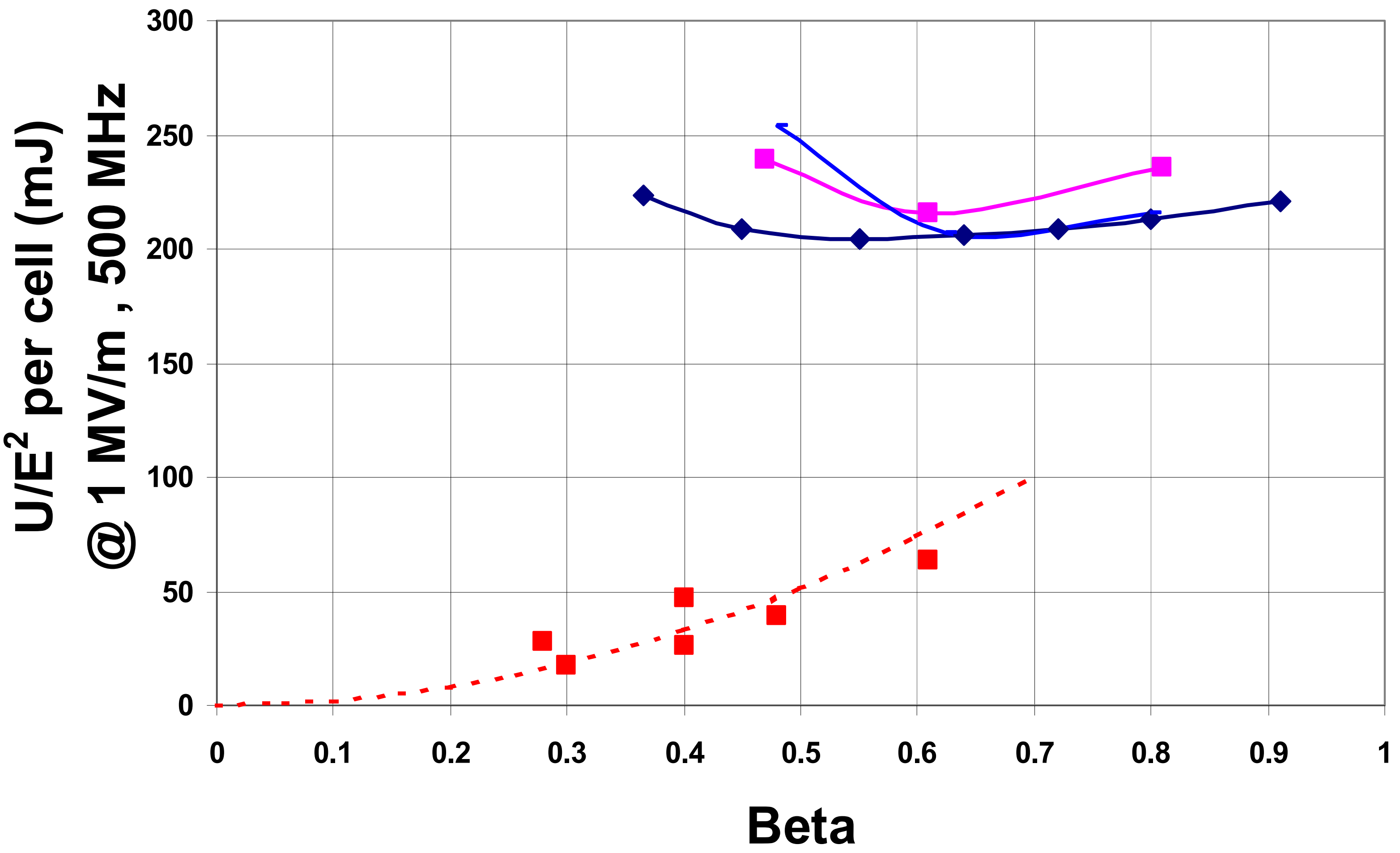
Proportional to $E^2\lambda^3$

At 1 MV/m, normalized to 500 MHz:

- **TM_{010} elliptical cavities:**
  - **Simple-minded model gives** $U / E^2 \propto \beta$
  - **In practice:** U/E^2 **~ 200-250 mJ**
  - **Independent of β (seems to increase when β <0.5 – 0.6)**

- **$\lambda/2$ structures:**
  - **Sensitive to geometrical design**
  - **Transmission line model gives** U/E^2 **~ 200 β^2 (mJ)**

Energy Content per Cell or Loading Element

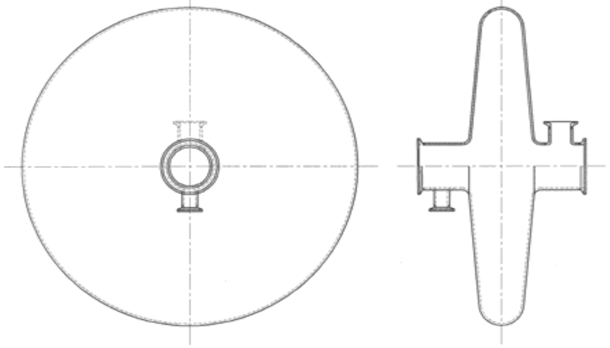


Size & Cell-to-Cell Coupling

TM_{010} Structures

Dia ~ 0.88 – 0.92 λ

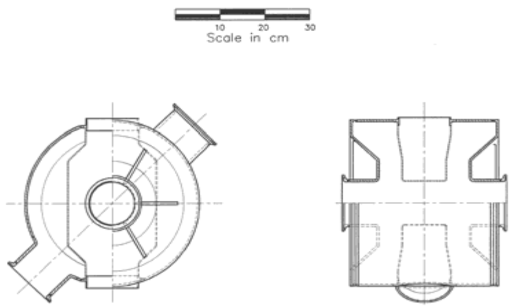
Coupling ~ 2%



λ /2 Structures

Dia~ 0.46 – 0.51 λ

Coupling ~ 20 - 30%



Example : 350 MHz, β= 0.45

Multipacting

- TM$_{010}$ elliptical structures
  - Can reasonably be modeled and predicted/avoided
  - Modeling tools exist

- $\lambda/2$ Structures
  - Much more difficult to model
  - Reliable modeling tools do not exist
  - Multipacting "always" occurs
  - "Never" a show stopper

TM Structures – Positive Features

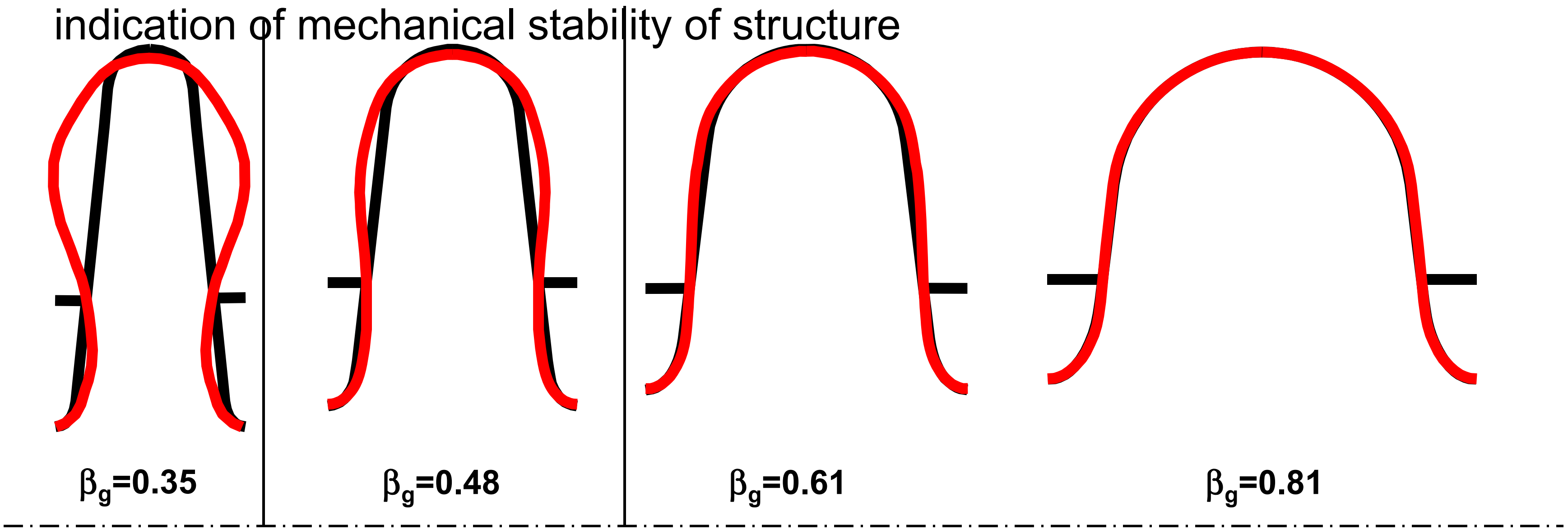
- Geometrically simple
- Familiar
- Large knowledge base
- Good modeling tools
- Low surface fields at high β
- Small number of degrees of freedom

$\lambda/2$ Structures – Positive Features

- **Compact, small size**
- **High shunt impedance**
- **Robust, stable field profile (high cell-to-cell coupling)**
- **Mechanically stable, rigid (low Lorentz coefficient, microphonics)**
- **Small energy content**
- **Low surface fields at low β**
- **Large number of degrees of freedom**

How Low Can We Go with β_g in TM Cavities ?

- Static Lorentz force detuning (LFD) at EoT(β_g)=10 MV/m, 805 MHz (Magnification; 50,000)
- In CW application LFD is not an issue, but static LFD coeff. provides some indication of mechanical stability of structure



RF efficiency; x
Mechanical Stability; x
Multipacting;
Strong possibility

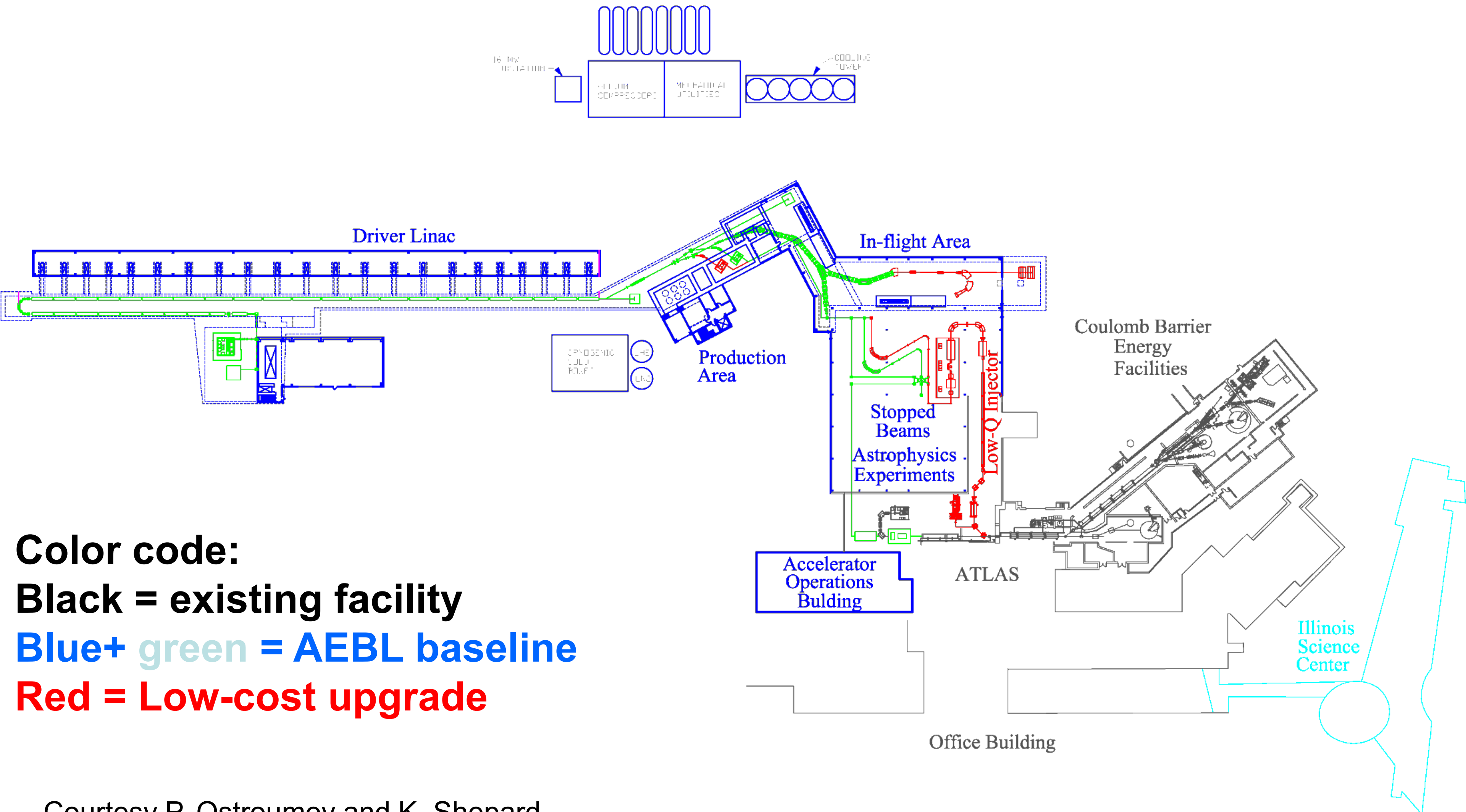
Will work in CW
Pessimistic in Pulsed application
Would be a competing Region with spoke cavity

Suitable for all CW & pulsed applications
Recent test results of SNS prototype cryomodule, β_g=0.61;
quite positive; piezo compensation will work

How High Can We Go with β_g in Spoke Cavities?

- What are their high-order modes properties?
  - Spectrum
  - Impedances
  - Beam stability issues
- Is there a place for spoke cavities in high-high-current applications?
  - FELs, ERLs
  - Higher order modes extraction

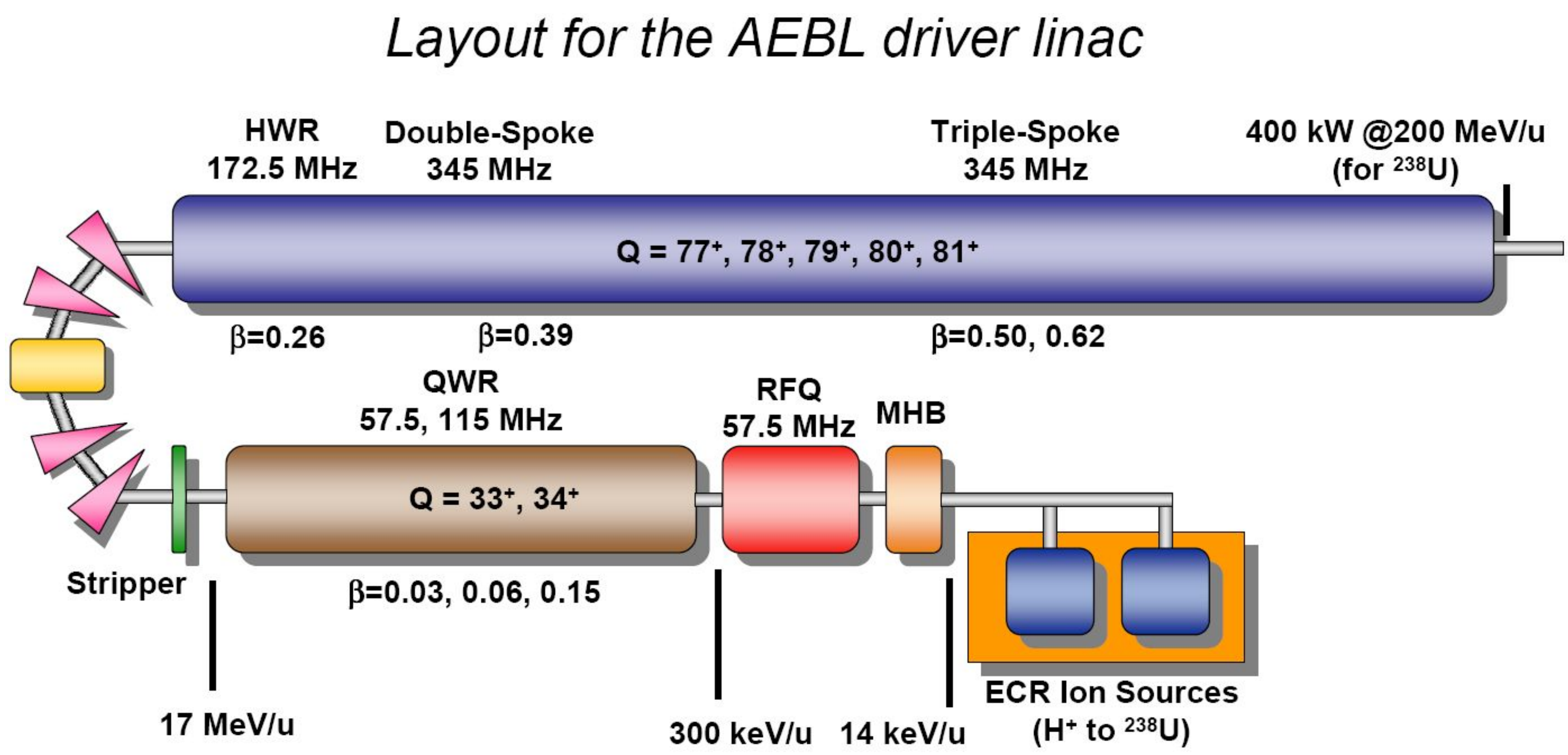
Layout of the AEBL at ANL – 200 MeV/u, 400 kW



Color code:
Black = existing facility
Blue+ green = AEBL baseline
Red = Low-cost upgrade

Courtesy P. Ostroumov and K. Shepard

Driver linac



Courtesy P. Ostroumov and K. Shepard

Advanced Exotic Beam Laboratory

AEBL Driver Linac - SC Resonator Configuration

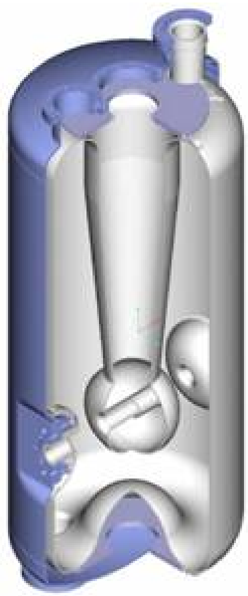
- Input of uranium 33+ and 34+ at beta = .0254

Beta	Type	Freq	Length	Esurf	Eacc	# Cav
		MHz	cm	MV/m	MV/m	
0.031	FORK	57.5	25	22.4	5.60	3
0.061	QWR	57.5	20	27.5	9.29	21
0.151	QWR	115.0	25	27.5	8.68	48
	STRIPPER				Subtotal	72
0.263	HWR	172.5	30	27.5	9.45	40
0.393	2SPOKE	345.0	38.1	27.5	9.17	16
0.500	3SPOKE	345.0	65.2	27.5	9.55	54
0.620	3SPOKE	345.0	80.9	27.5	9.26	24
					Subtotal	134

Courtesy P. Ostroumov and K. Shepard

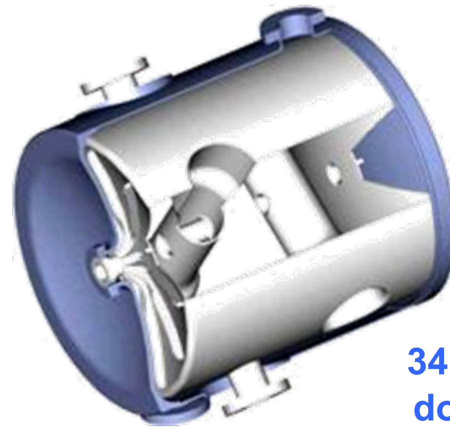
Total Cavity Count = 206

SC cavities covering the velocity range 0.12 < < 0.8 developed for the RIA driver linac and will be used in AEBL

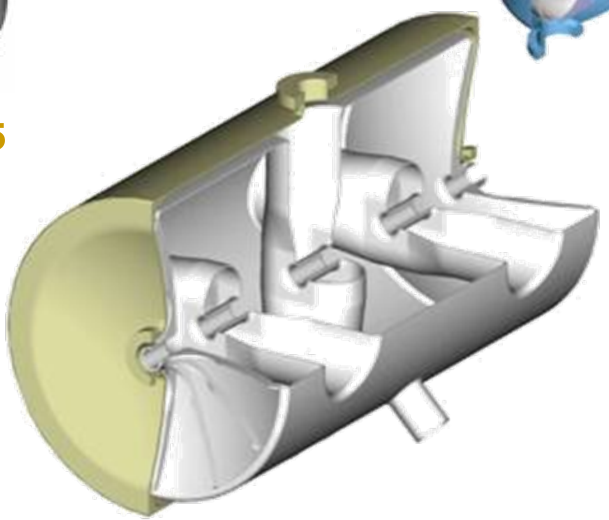


115 MHz =0.15
Steering-corrected QWR

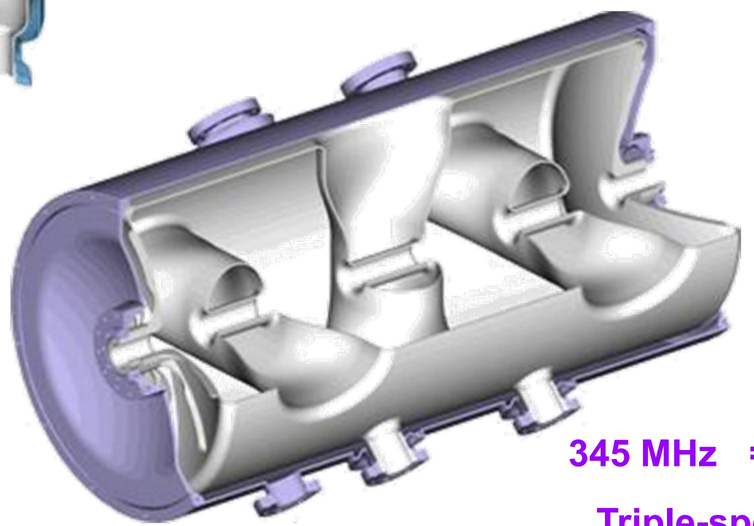
172.5 MHz =0.28 HWR



345 MHz =0.4 double-spoke



345 MHz =0.5
Triple-spoke

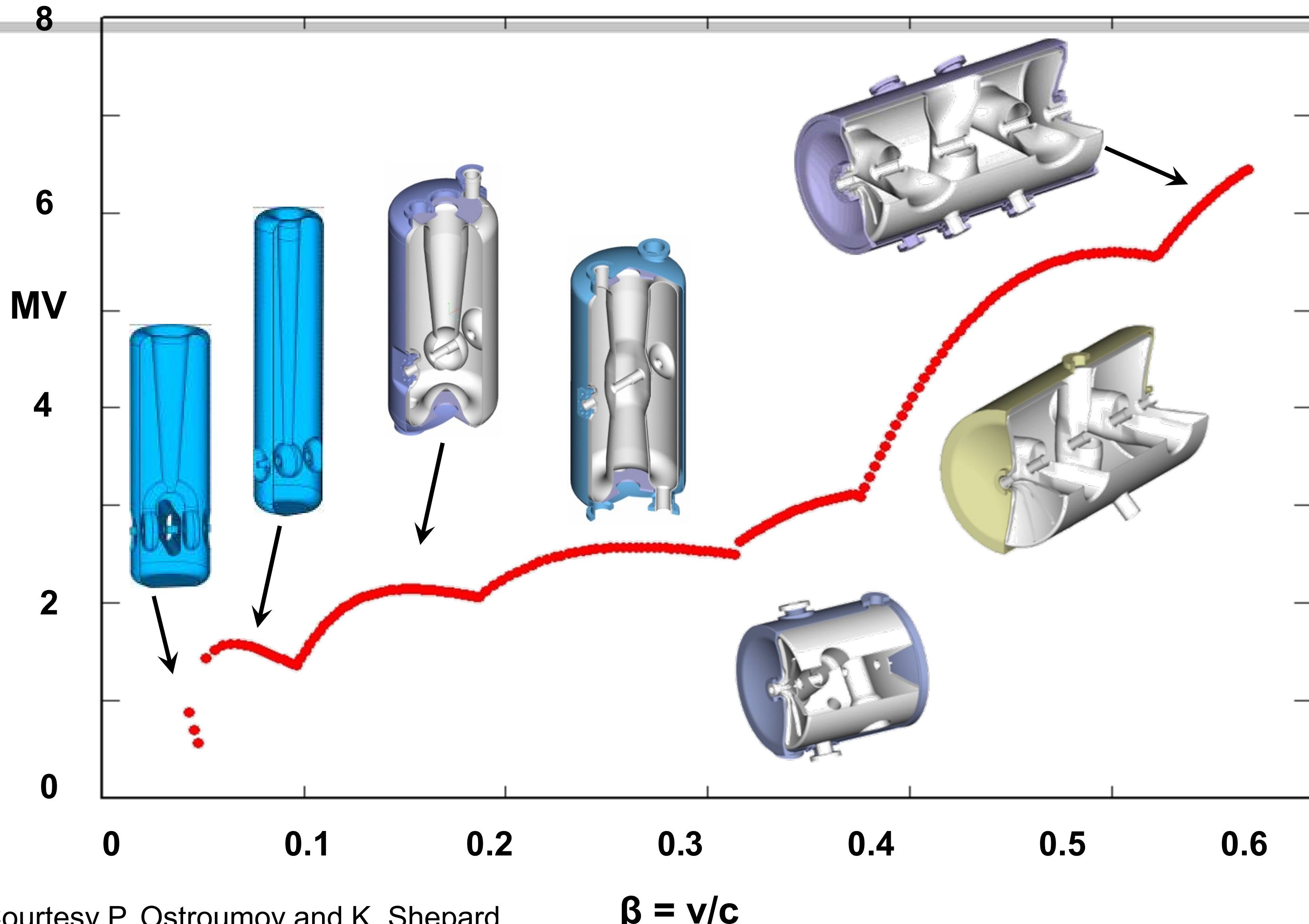


345 MHz =0.62
Triple-spoke

See publications by K.W. Shepard, et al.

Courtesy P. Ostroumov and K. Shepard

Cavity Walk – Voltage Gain per Cavity for Uranium Beam

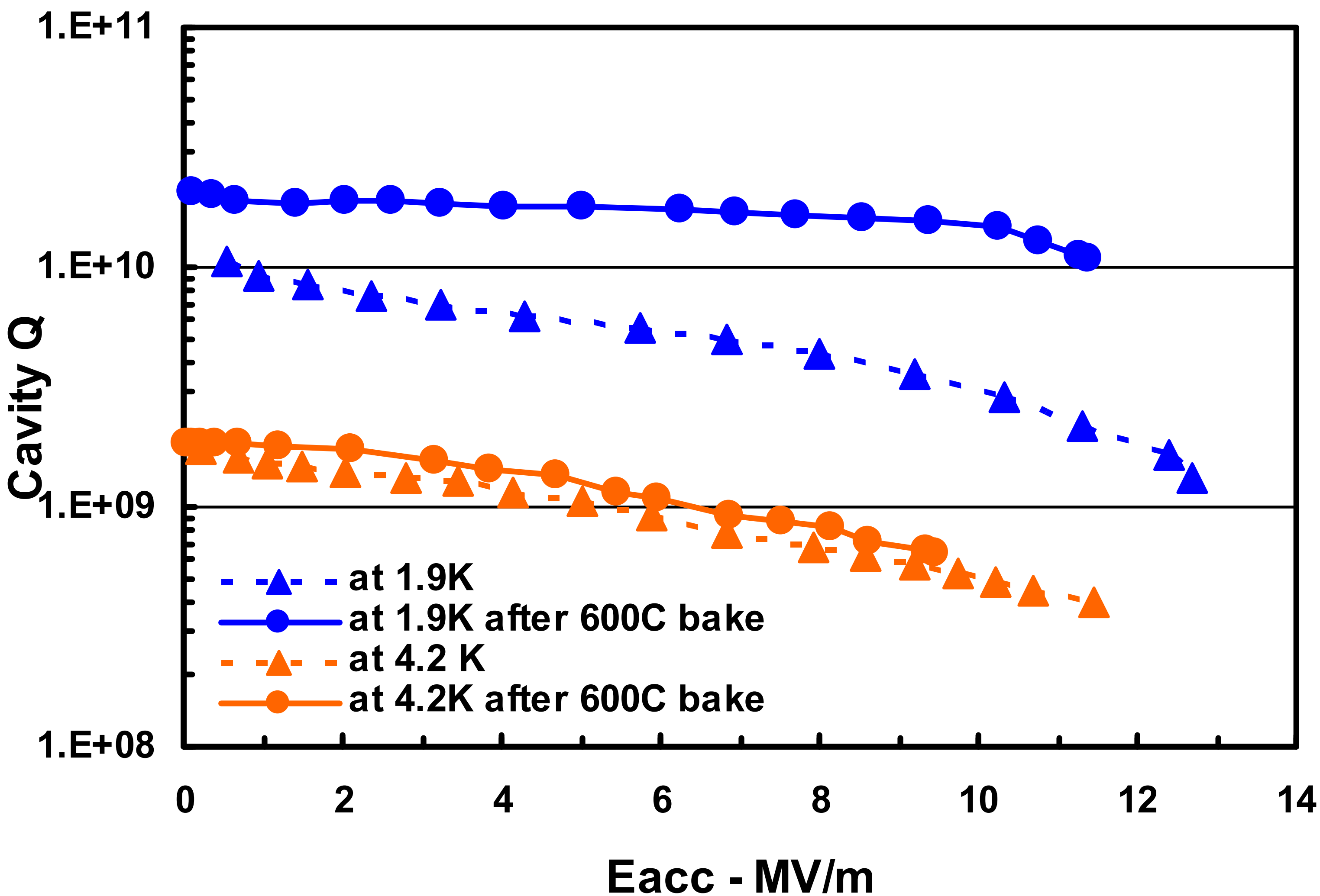


Courtesy P. Ostroumov and K. Shepard

ANL extended to TEM-class SC cavities the very high-performance techniques pioneered by TESLA

Courtesy P. Ostroumov and K. Shepard

Effects of interstitial hydrogen on triple-spoke cavity performance



Courtesy P. Ostroumov and K. Shepard

Parting Words

In the last 30+ years, the development of low and medium β superconducting cavities has been one of the richest and most imaginative area of srf

- The field has been in perpetual evolution and progress
- New geometries are constantly being developed
- The final word has not been said

The parameter, tradeoff, and option space available to the designer is large

- The design process is not, and probably will never be, reduced to a few simple rules or recipes
- There will always be ample opportunities for imagination, originality, and common sense