Beam Impedance, Coupling Impedance, and Measurements using the Wire Technique

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Beam, Coupling Impedance

When a particle beam of current *I* travels through a structure, it may give up some of its energy to that structure, reducing its energy. The energy loss may be expressed as a voltage *V*, and the **beam impedance** Z_b of the structure itself may be expressed as

$$Z_b = \frac{V_{loss}}{I_{beam}}$$

The structure may be a monitor device, where a signal *V* is generated by the beam and detected. In this case, we can define a **coupling impedance** Z_p for the structure

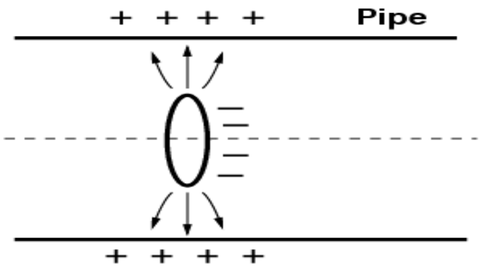
$$Z_p = \frac{V_{monitor}}{I_{beam}}$$

In general, these are not the same, and each depends on the details of the structure itself.

Stripline Monitor

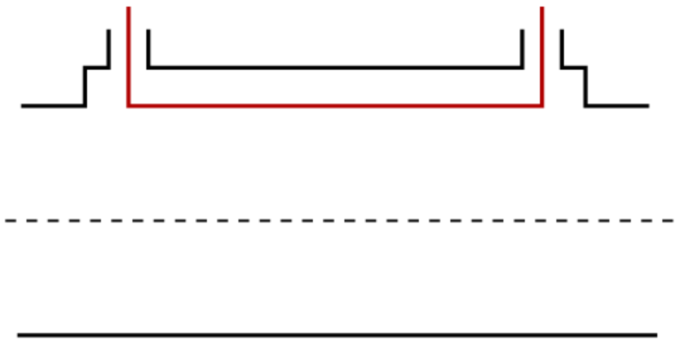
A beam bunch, as it travels down a pipe, induces images charges on the interior surface of the pipe. If the pipe is smooth and lossless, these charges do not affect the beam. An irregular beam pipe, however, will interact with the beam, as fields are induced in the beam pipe by the beam.

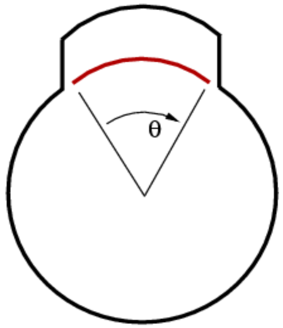
The fields of a relativistic beam are concentrated along the direction of motion.



One very useful type of monitor is the stripline monitor, which responds to the image charges. An electrode or electrodes are mounted in the beam pipe. Each end of the electrode is brought out and connected to a load. The electrode subtends and angle θ.

The geometry of the stripline resembles that of a *microstrip*, which has a characteristic surge impedance. The propagation velocity of an impulse along the stripline is assumed to be the speed of light c.

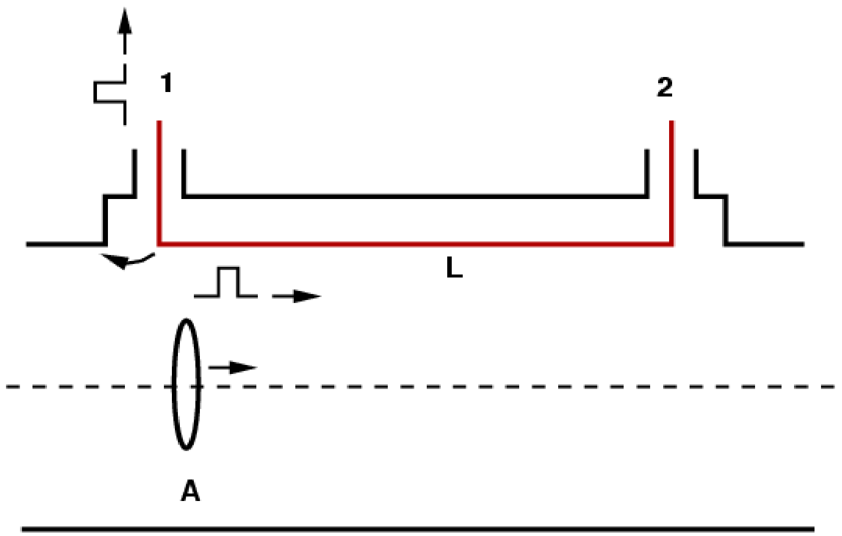




Brief Explanation of Stripline Response

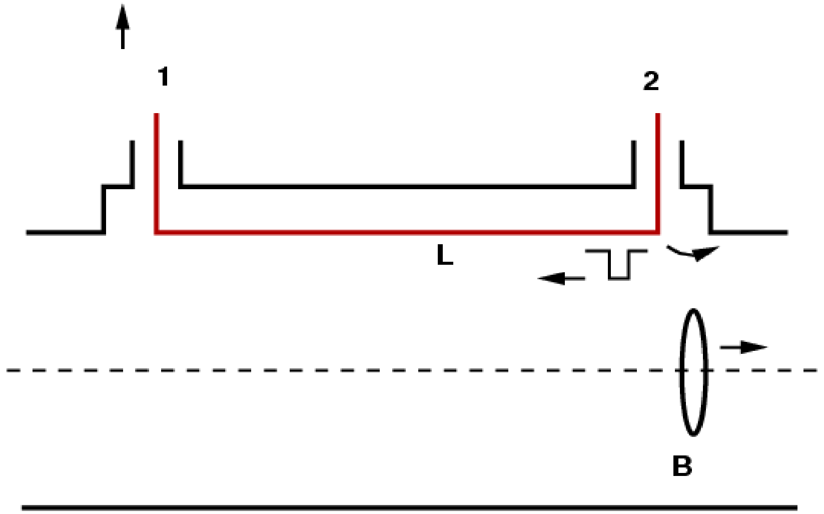
A beam pulse A passes by the front of the stripline and induces a field in the gap beween the pipe and the electrode. An induced charge q splits, one-half travels out port 1 and the other starts propagating down the stripline at velocity *c*, matching the beam velocity. If the stripline impedance is Z_L, then

$$V = -\frac{1}{2} g\, i(t)\, Z_L$$

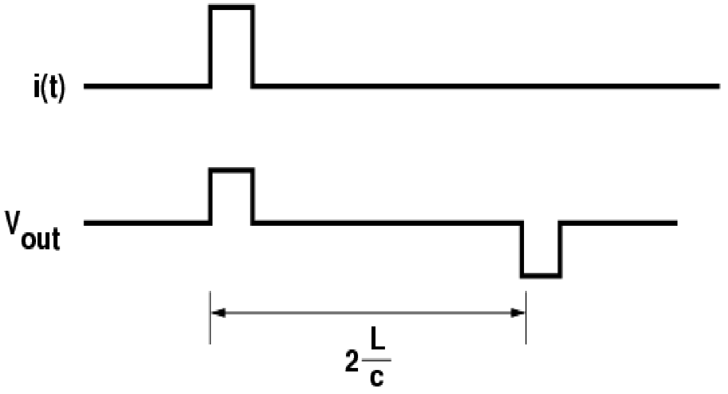


When the beam gets to port 2 it induces a field on the exit gap, opposite to that of the entrance gap. This cancels the pulse that would have emerged from port B, but the pulse of opposite polarity starts propagating back to port 1 and merges at a time t_d later from port 1.

$$t_d = 2\frac{L}{c}$$



Stripline Frequency Response

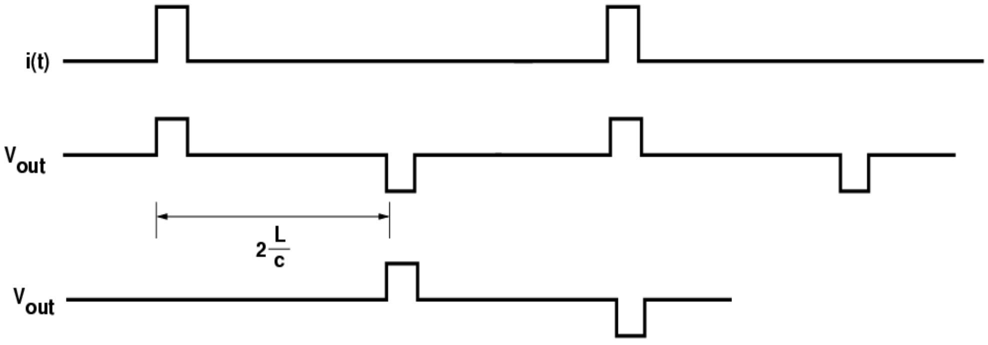


A short beam current pulse i(t) travels through a stripline pickup. The response function at port 1 would (ideally) show a bipolar signal, with the time difference between the direct and reflected signal of

$$t_d = 2\frac{L}{c}$$

Low-frequency limit: If the beam pulse is very long, the direct and reflected signals will tend to overlap and cancel each other out. Thus, at the low-frequency limit, we would expect the response to go to zero.

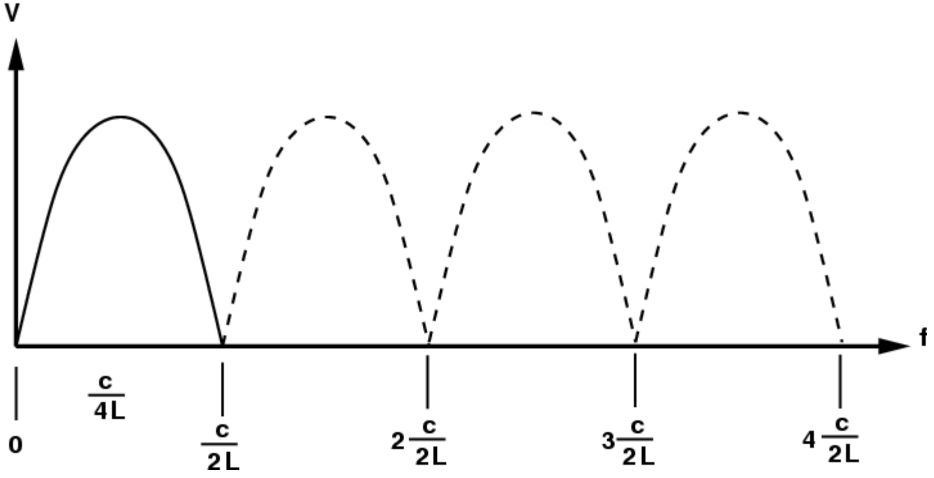
If the beam current contains pulses separated by t_d, or with a repetition frequency 1/td, then the next positive pulse will cancel with the negative reflected pulse from the last beam pulse, and the output will be zero.



If the beam pulses are separated by twice the above spacing, the pulses will add, rather than cancel, so we would expect a maximum in the frequency response function.

$$f_{first\ peak} = \frac{c}{4L} \qquad f_{next\ null} = \frac{c}{2L}$$

Stripline Pickup Response Function



The stripline pickup frequency response function consists of a series of peaks and nulls, separated by c/2L.

The Fourier spectrum of the beam signal will be convolved with the pickup frequency response function.

The actual response depends on the geometry of the pickup, how well it is matched to the load over its frequency response, and the angular fraction $\theta/2\pi$ of the pipe it subtends.

We will measure the frequency and also the amplitude response of the stripline pickup using the wire technique. The response is expressed as an impedance:

$$Z_p = \frac{V_{monitor}}{I_{beam}}$$

Beam Impedance Measurement

What is the beam impedance Z_b or the pickup transfer impedance Z_p?

With an actual beam, its energy loss may be measured. However, a beam is not always handy. We can simulate a beam by a wire that extends down the beam pipe and simulate the beam by a signal on the wire. We can detect on the stripline external electrode a signal induced by a signal on the wire to determine the pickup coupling impedance Z_p.

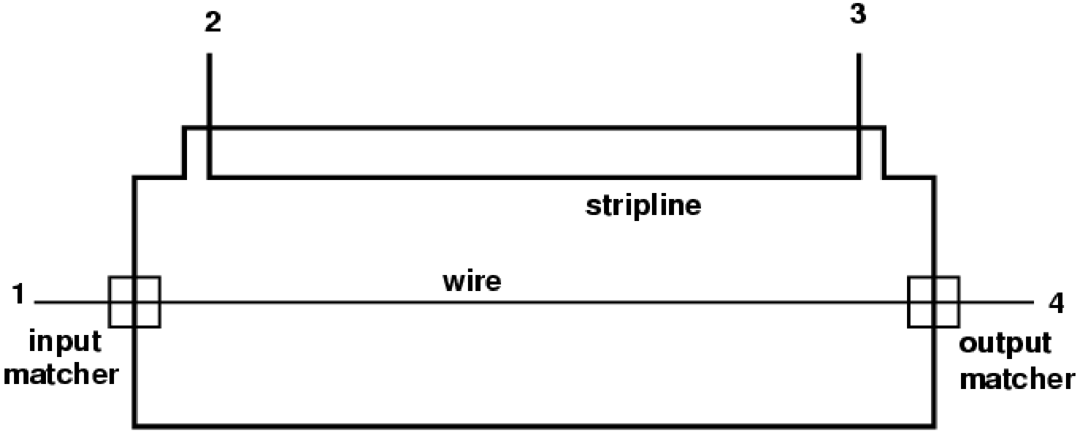
In the case of an irregular beam pipe itself, there is no external connection. Then, the only way to measure the beam impedance Z_b is by measuring the effect the pipe has on the signal on the wire after it has passed through a length of beam pipe.

This measurement uses the **substitution method**: a signal is propagated from one end of the *device under test* (DUT) and recorded. Then the DUT is replaced by a smooth beampipe, the *reference pipe*, and the signal again recorded. The beam impedance is calculated from the difference between the two signals.

Wire Measurement Setup

A beam pipe is closed at each end. A wire is threaded through the beam pipe, entering at node 1 and exiting at node 4.

The stripline is terminated on the upstream and downstream ends at nodes 2 and 3.



It is convenient to drive the stripline with a 50 ohm generator (the network analyzer). The impedance of an air-insulated coaxial cable is given by

$$Z_{surge} = 60\,ln\left(\frac{D_{outer}}{D_{inner}}\right), \qquad \frac{D_{outer}}{D_{inner}} = 2.3 \text{ for 50 ohms}$$

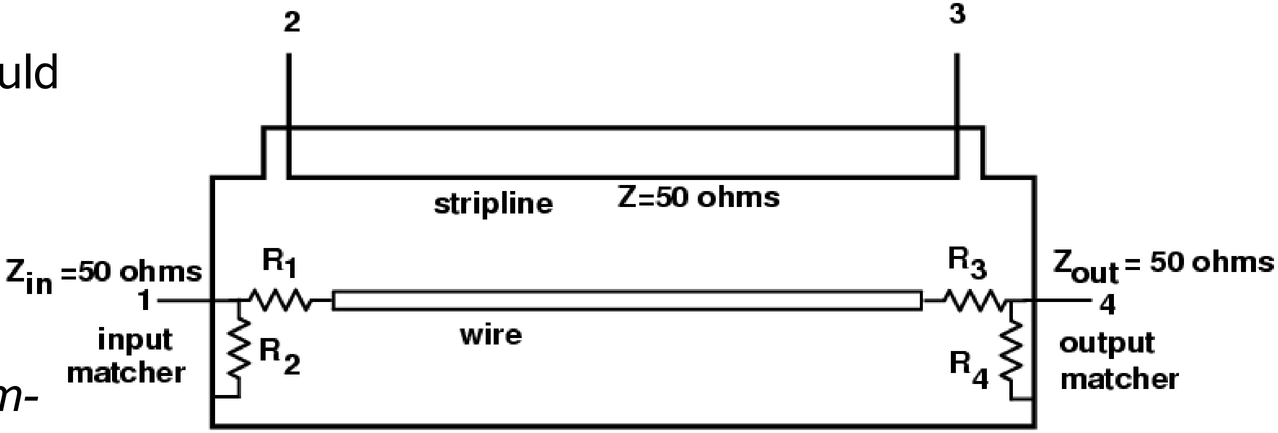
However, such a large inner conductor inside the beam pipe significantly alters the field configuration. In practice, a much smaller inner conductor is used. Here, D_{outer} = 3 inches and D_{inner} = 1/8 inches, for a characteristic impedance of 191 ohms.

This requires an impedance matching device at each end to avoid reflections of the transmitted signal.

End Impedance Matchers

A wide-band transformer would provide an ideal matching device, but one that covers near-DC to 2 GHz is difficult to obtain (for USPAS!).

Instead, we will use *minimum-loss resistive matching sections*.



The impedance looking into each end of the matcher presents the required impedance with the resistor values chosen for minimum loss.   These are called **resistor pads**.

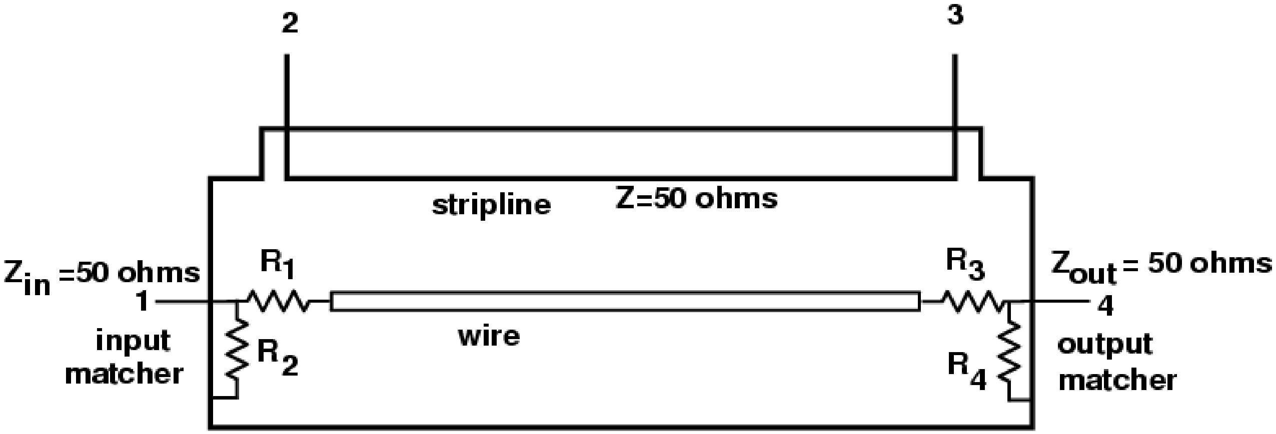
The R1-R2 network looks like 50 ohms looking from the left, and 191 ohms, looking from the right.   The values of the resistors is found by

$$R_1 = Z_w - R_2 \parallel R_g, \qquad R_2 = \sqrt{\frac{Z_w R_g^2}{Z_w - R_g}}$$

where R_g = 50 ohm generator impedance
Z_w = 191 ohm wire impedance

R_1 = 164 ohms, R_2 = 58 ohms.

Output Impedance Matcher



At the exit of the device under test, the 191 wire impedance must also be matched, but it is unnecessary to match the impedance going *into* port 4. (Why?)

In this case, R_3 = 191-50 = 141 ohms, and R_4 can be omitted.

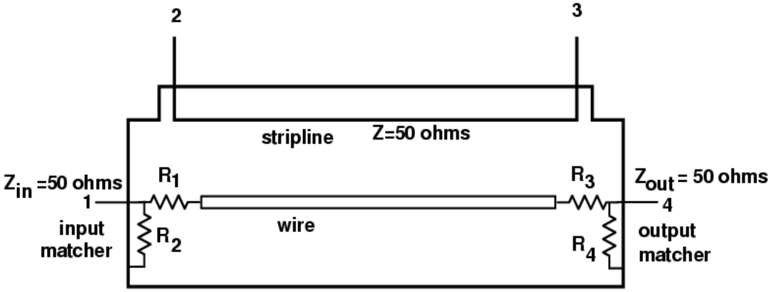
This results in a larger signal at port 4 that is returned to the network analyzer, as the current need not divide between two branches at the exit.

Accounting for the Matcher Loss

The excitation function is a *current* on the wire, producing a *voltage* at the stripline output port. For beam impedance measurements, it is the loss of beam energy in the device under test.

Therefore, we must determine the ratio of input current to the current on the wire.

At the input at port 1, the current divides between R_2 and R_1. The impedance looking into R_1, omitting R_2 for the moment, is the resistance of R_1, 164 ohms, in series with the impedance of the wire itself, 191. The total resistance looking down the R1 to the wire is 355 ohms.



The current divides between the two branches by the ratio of these two resistances:

50/58 = 0.86 will go to R_2, and 50/355 = 0.14 will go down the wire. The attenuation A_1 at the entrance is then $\boldsymbol{A_1}$ **= 0.14**.

At the exit, all the current in the wire goes to the output (R_4 is absent). Therefore the attenuation $\boldsymbol{A_2}$ **= 1.0** at the exit.

Note that these networks are to be treated as *current dividers*, not *voltage dividers*.

Measurement of the Stripline Beam Impedance Z_b

The network analyzer measures the quantity S_{21}, which is just the ratio of input to output voltage (or current in a constant-impedance configuration) of the device under test (DUT).

We will use the **substitution method** to determine the beam impedance of the stripline. This is a general method, useful for a beam pipe without any external signal ports.

The substitution method comprises two steps: measure the DUT, and then measure S_{21} for a device with a smooth beam pipe. The difference is then used to calculate the beam impedance of the DUT. In measuring the DUT, some energy is extracted from the beam (wire signal), and this is compared to the wire signal on a **reference** pipe.

This is a measure of the *longitudinal beam impedance*. It is also possible to measure the transverse beam impedance with a different setup, that characterizes the effect of transverse beam offsets on the back-induced beam voltage or the coupling impedance of a beam position monitor.

$$Z_b = 2\,R_w\left(\frac{S_{21\,REF}}{S_{21\,THRU}} - 1\right)$$

Z_b is the beam impedance

R_w is the impedance of the wire in the tube

Measuring the Beam Impedance

We will skip the theory and go directly to the technique.

The experimental apparatus comprises three sections: The wire section and two top sections: the stripline and the reference section.

An SMA connector is located at each end of the wire section, along with the resistor matcher. One end is labeled as the input.

The stripline section has two SMA connectors on it, tied to each end of the stripline.

The reference section completes the round 1.5 inch radius cross-section of the wire section.

Screws at each end secure the top piece to the wire section.

Calibrate the Network Analyzer

The signal-to-noise of this measurement is poor, as the effect is subtle, so care must be taken to securely screw each top section to the wire section.

The network analyzer (NWA) must be set up to measure the throughput parameter, S_{21}.

First, run the analyzer through its throughput calibrate sequence by connecting the cable from port A of the NWA through a barrel connector to the cable to port B of the NWA. Set the NWA to scan from 0.3 MHz to 1.7 GHz, using 801 points.

Choose from the calibrate menu the through option and calibrate the NWA, along with the two cables to give a flat S_{21} scan. You should find that $S_{21} = 1.00$ after the calibration over the entire frequency scan.

Then connect port A of the NWA to the input SMA connector on the wire section and the output SMA connector of the wire section to port B of the NWA.

Measure the Reference Line

Place the smooth reference section on top of the wire section. Using the small hardware, screw the ends of the wire section to the reference section.

Set up the NWA to measure the S_{21} parameter from port A to port B. The most convenient display is the linear magnitude (lin mag) display. Adjust the scale factor so that the trace fills most of the screen.

You should see that S_{21} is on the order of 0.14, the overall current transfer function found in analyzing the resistive matchers at the end of the wire section. This quantity, which does show a frequency dependence is S_{21REF}, the transmission of the reference beam pipe.

Use averaging to improve the signal-to-noise value of the measurement. Average over at least 25 scans.

Save this measurement in the memory of the NWA.

Remove the smooth upper pipe section.

Measure the Stripline Section

Put the stripline section on to the wire section and screw it to the ends with the small hardware.

The stripline section has two ports. Terminate both with 50 ohm SMA terminators.

As with the reference section, measure S_{21} of the stripline. This is $S_{21\text{THRU}}$.
(Don't save this to memory, or the reference value would be lost.) Use signal averaging.

The NWA has a math division function **data/mem** This will display $\frac{S_{21\,THRU}}{S_{21\,REF}}$

(If you do things in reverse order, you will get the inverse, of course.)

Set the NWA reference position to somewhere in the middle of the scale, and the reference value to 1.0, establishing a line at 1.0, from which you can then measure the deviation of the ratio from unity. The amplitude is proportional to Z_b.

Knowing the wire and tube diameters, calculate the wire impedance R_w and then the beam impedance of the terminated stripline.

$$Z_b = 2\,R_w\left(\frac{S_{21\,REF}}{S_{21\,THRU}} - 1\right)$$

Calculate the Beam Impedance Z_b

If all goes right, you should get a display like the picture.

As $\dfrac{S_{21\,THRU}}{S_{21\,REF}}$ is displayed, the plot is inverted, and the peaks extend downward.

Measure the frequencies and amplitudes of the peaks and nulls by using the marker function of the NWA.

Calculate Z_b at the peaks.

$$Z_b = 2\,R_w\left(\frac{S_{21\,REF}}{S_{21\,THRU}} - 1\right)$$

Calculate the spacing of the peaks and the spacing of the nulls. Are they equal?

From that, calculate the effective length of the stripline itself.

Why is there a factor of 2 in the equation for Z_b?

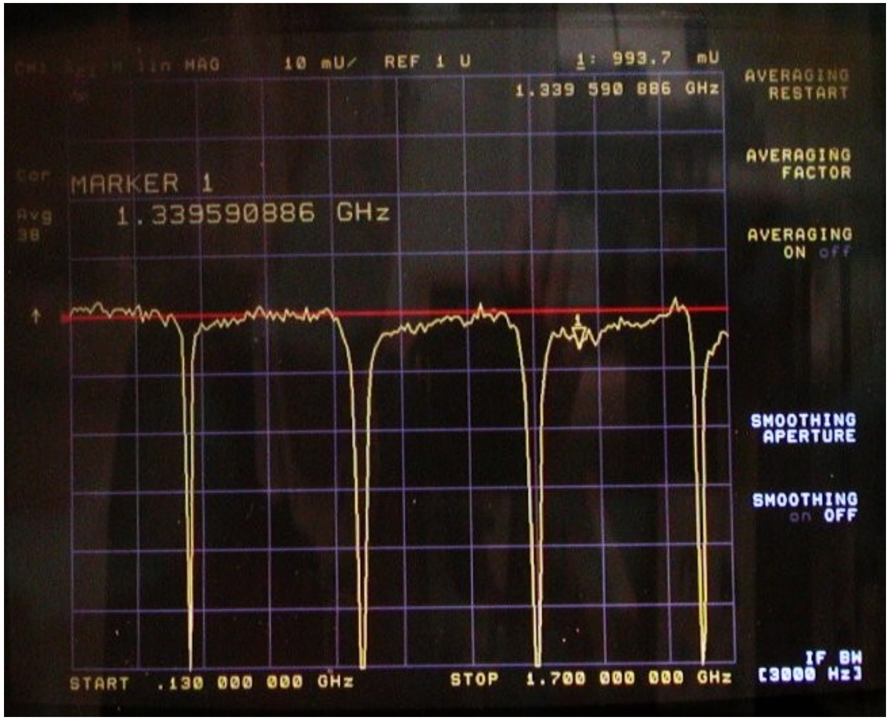
Stripline Termination

The signal generated in the stripline from the passage of the beam signal is dissipated in the terminators at each end of the stripline. This power is lost to the beam signal and is responsible for the finite beam coupling impedance of the stripline.

What if the terminators are open circuits or shorts? Would no power then be dissipated in the stripline and the coupling impedance be zero?

Not necessarily.   What might be the consequence of an open or shorted stripline?

This measurement might give a clue.



Measurement of the Stripline Coupling Impedance Z_p

The coupling impedance measures the voltage output V(t) on the upstream stripline electrode generated by a beam current I(t) passing down the center of the pipe.

$$Z_p = \frac{V_{monitor}}{I_{beam}}$$

This value is related to, but distinct from, the beam impedance Z_b of the stripline.

It is also measured with the NWA, this time measuring S_{21} from the wire input to the stripline output terminals.

Since there is an attenuation of the current at the input matcher, this loss must be taken into account.

Two measurements are made:

S_{21} from the input to the output of the wire.   This includes the matcher loss.

S_{21} from the input to the stripline upstream port.

$$Z_p = R_0 \frac{S_{21\,stripline}}{S_{21\,THRU}}$$

where R_0 is the 50 ohm input or stripline impedance

Signals at Wire Output and Stripline Upstream Port

You will find that S21 THRU is not a constant 0.14, but slightly frequency-dependent.

This is why the response function from the stripline port must be normalized by the response function.

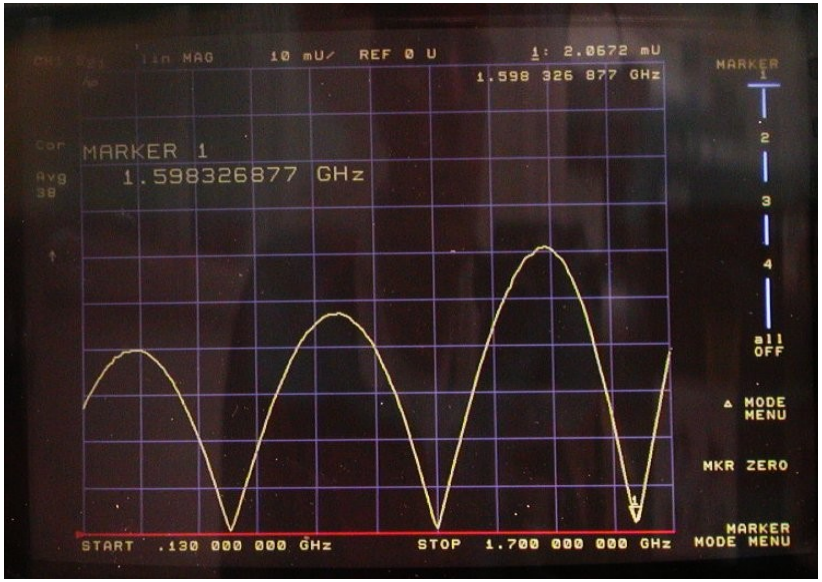
What errors might still be present in this procedure?

Measure the frequencies and amplitudes of the peaks and nulls by using the marker function of the NWA.

Calculate Z_p at the peaks.

Calculate the spacing of the peaks and the spacing of the nulls. Are they equal?

$S_{21\ \mathrm{THRU}}$



$S_{21\ \mathrm{stripline}}$

Other Stripline Signals

Is there a signal on the downstream end of the stripline when the upstream port is terminated? Would you expect one?

What happens if the upstream port is shorted or open?

If the stripline subtended a full 360 degees of the beampipe, what would expect the coupling impedance to be? Can you calculate the actual fraction of the circumference occupied by the stripline from the measured coupling impedance?