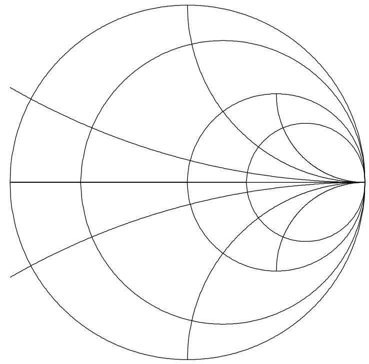


Microwave Measurements Laboratory for Accelerators

John Byrd, Stefano De Santis Derun Li,
Bob Rimmer
USPAS
June 2003
Santa Barbara

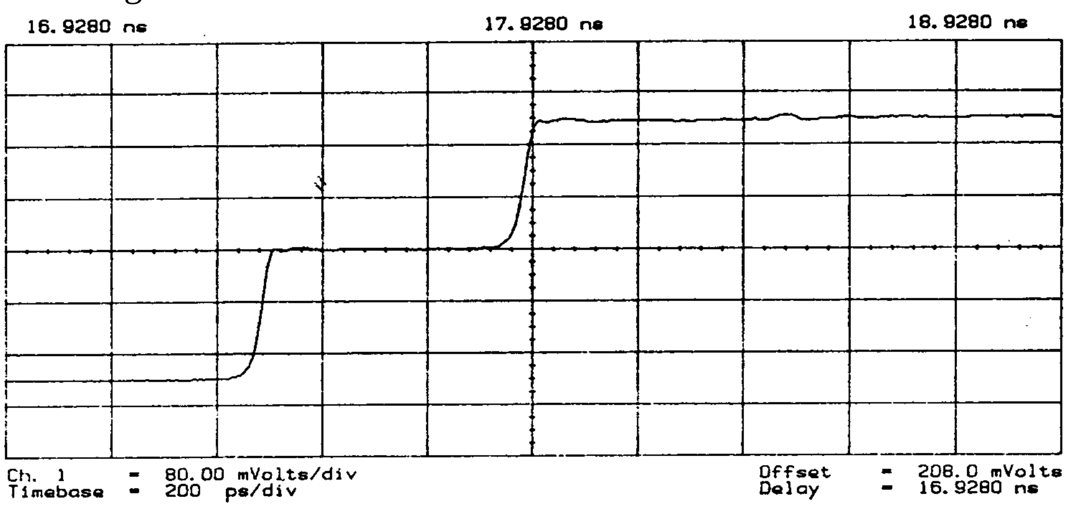
Table of Contents

Introduction to Time Domain Measurements

Time Domain Reflectometry (TDR)

Measuring lengths, reflection coefficient, and impedance of cables

Set the oscilloscope to TDR mode. Adjust the time base and sensitivity to see something like:



Note the position (in time) of the incident step and the reflected step coming from the open circuit at the output of the step generator. Connect a length of 50Ω cable (cable "A"), open circuited at one end. The step now travels along the length of the cable, is reflected by the mismatch at the (open circuit) end of the cable, and travels back into the sampling head where it is detected. The timebase should be about 1 ns/div to get the display on screen, then 500 ps/div for the measurements.

How much is the second step delayed?
What is the electrical length of the cable?
Measure the physical length of the cable. What is the cable velocity factor and dielectric constant?

Connect cable "B" onto the end of the first cable. Use a timebase of 2 ns/div, with the incident pulse to the left of the display. Plot the display (for a homework problem)

What is the reflection coefficient of the cables?

What is the characteristic impedance of cable "B"?
Does this agree with the calculated value of the reflection coefficient?
What can you say about the phase of the reflection coefficient?

Measuring discrete impedances

Set the oscilloscope to TDR mode. Connect cable "A" to the step generator, and note the position of the reflected step. Connect circuit "A" to the end of the cable. Adjust the timebase to 20 ns/div.

Examine the display qualitatively to determine the circuit, and sketch the circuit.
What are the values of the circuit components?

Determine the value of the resistive component by measuring the voltage at the end of the exponential rise, and the input step voltage (refer to homework problem, and figures on next page). Position a voltage cursor at the level of the maximum output from the circuit. Position the other cursor at the beginning of the exponential rise. Calculate the 1/e value (0.368) and subtract this from the first cursor position. Use the time cursors to measure the time between the exponential rise intersecting the two voltage markers - this is the e-folding time, the time-constant of the circuit. Use this time constant and the value of resistance obtained above in determining the capacitance.

Look at the detailed response around the end of the cable, with a timebase of about 500 ps/div.

What might be causing the peak, and the exponential decay?

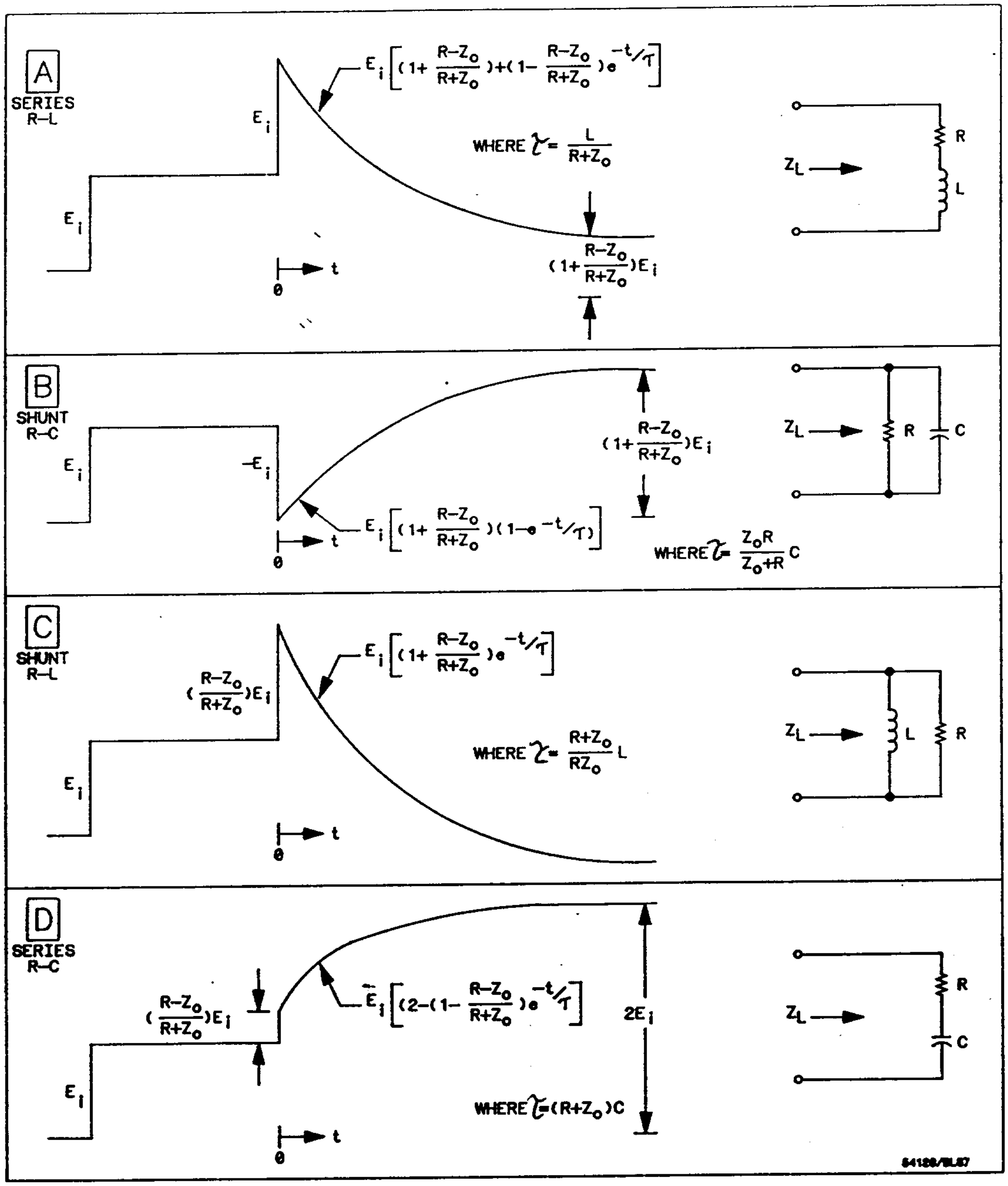
Disconnect circuit "A" and connect circuit "B".

What circuit element is in this circuit?
What is the value of the circuit element? {Use a technique similar to that described above. Note that any fraction of the exponential decay can be used, providing the appropriate time constant is employed. For example, using the time over which the voltage changes by a factor of two yields 0.693 times the exponential time constant. This may be useful in improving accuracy of the measurement}.

Look in detail at the region around the end of the cable.

What do you think may cause this perturbation to the simple exponential?

TDR waveforms for simple circuits

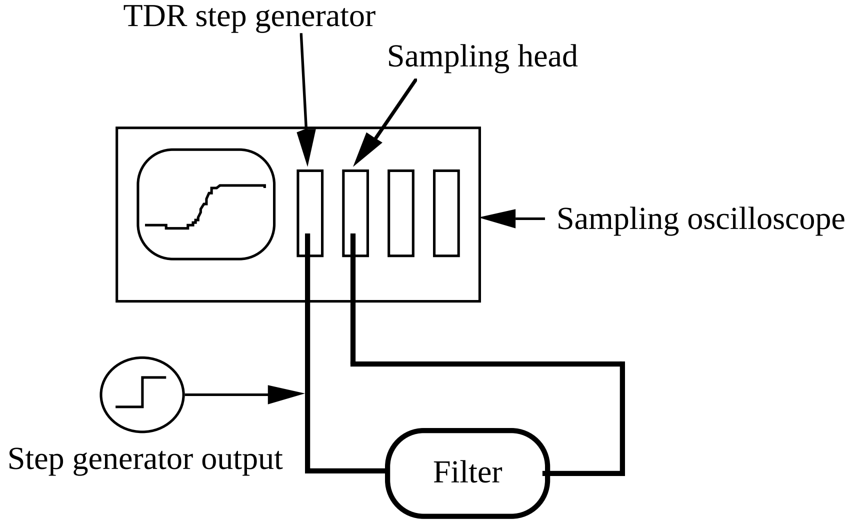


Time Domain Transmission (TDT)

Measuring step response of filters

Set the oscilloscope to TDT mode, where we will use the step generator as a signal source and measure the transmission through devices and into another channel of the oscilloscope.

Connect cable "A" to the step generator, and a similar cable to the input of another channel on the 'scope. Connect these cables together using a barrel (female/female connector), set the timebase to 500 ps/div and adjust the delay to position the step voltage toward the left side of the CRT. Connect the low-pass filter "A" as shown in the diagram.



Position a voltage cursor at the level of the base of the step, and another at the level of the step generator output voltage. Calculate 10% of the voltage difference, and position the lower cursor at the + 10% value and the higher cursor -10%. This gives the 10% and 90% (of maximum) voltage levels. Measure the time separation between these levels. This gives the 10 - 90 % risetime of the transmitted signal, a quantity often used in describing the time domain response of components. We have ignored the risetime of the step generator and sampling head, since these are typically approximately 25 ps, much less than the risetime of the filter).

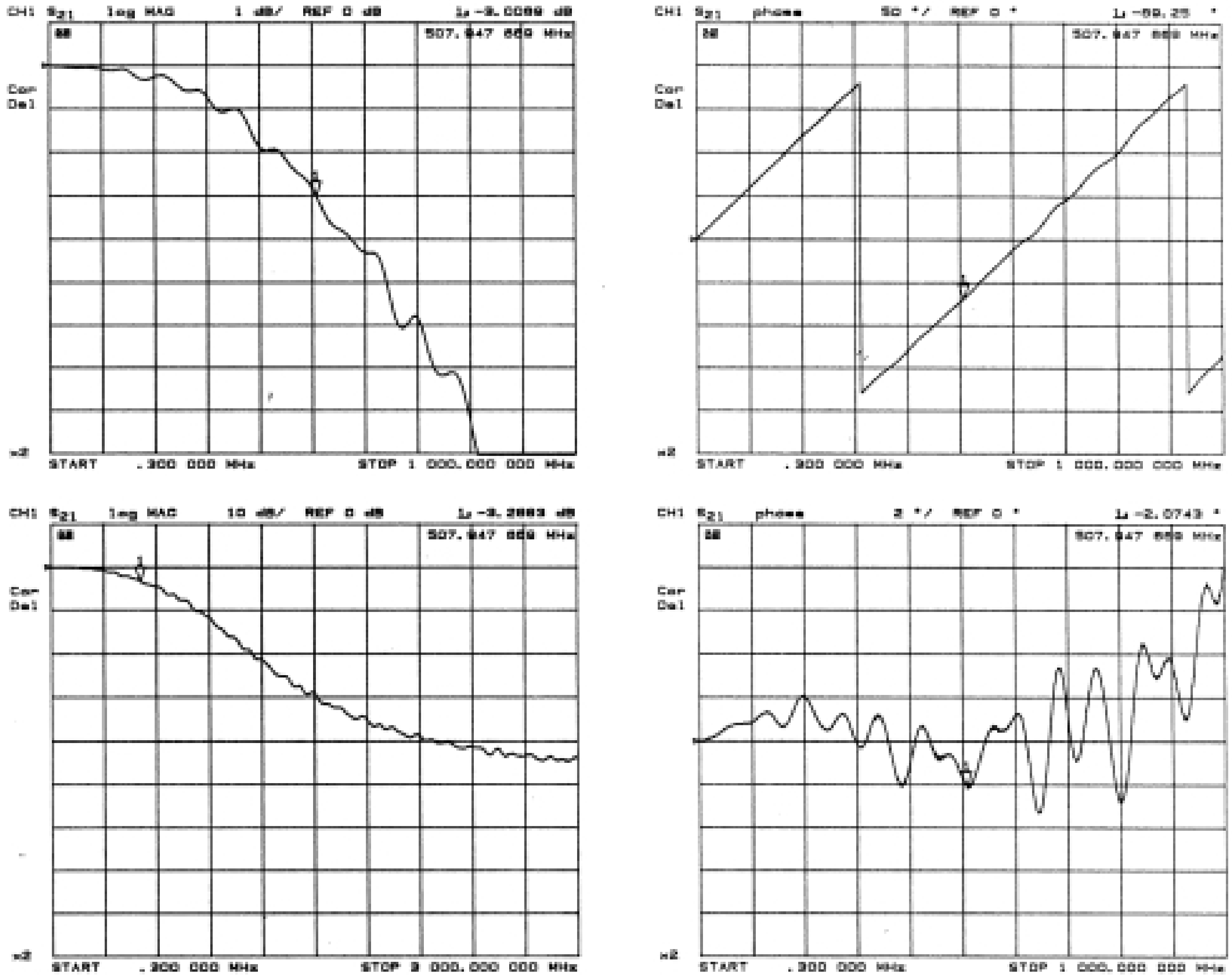
What is the 10 - 90% risetime of the filter?
What type of filter gives such a step response?
What is the bandwidth of the filter
{use the approximation $t_{rise} \approx 0.3/(3 \text{ dB bandwidth})$}

Compare your answers with the frequency domain response shown later. Note that this is a Thompson-Bessel filter with very linear phase response and smooth roll-off in amplitude. The step response is correspondingly very smooth, with little undershoot or overshoot.

Remove the filter, replace it with the barrel, and again position the display such that the input pulse is one division from the left of the screen and the timebase is 500 ps/div. Replace the barrel connector with bandpass filter "B" and increase the sensitivity to 5 mV/div.

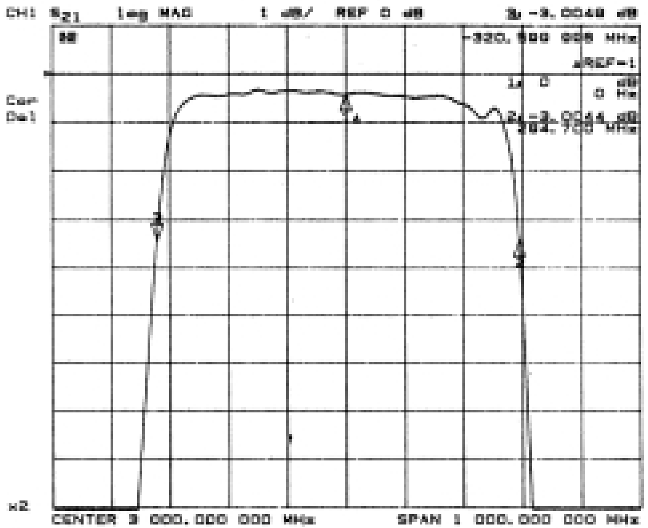
What is the frequency of the waveform you see?
What is the width of the tone burst (approximately, estimate the time for the tone burst amplitude to decay by 60%)?
What is the center frequency of the filter?
What is the filter bandwidth?

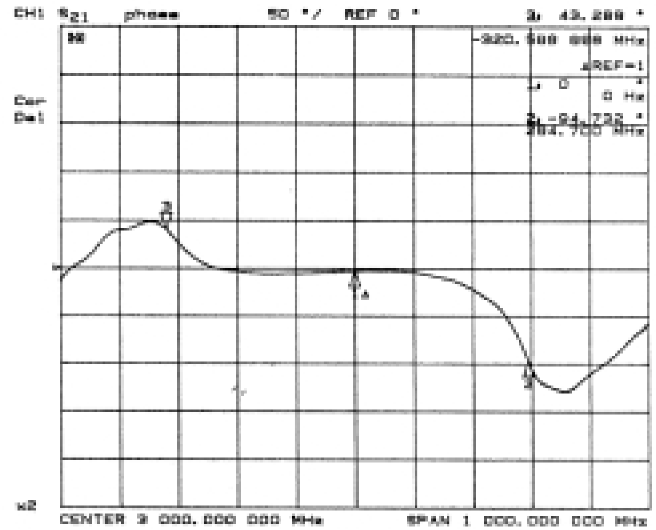
Compare your answers with the frequency domain response shown in the next pages.



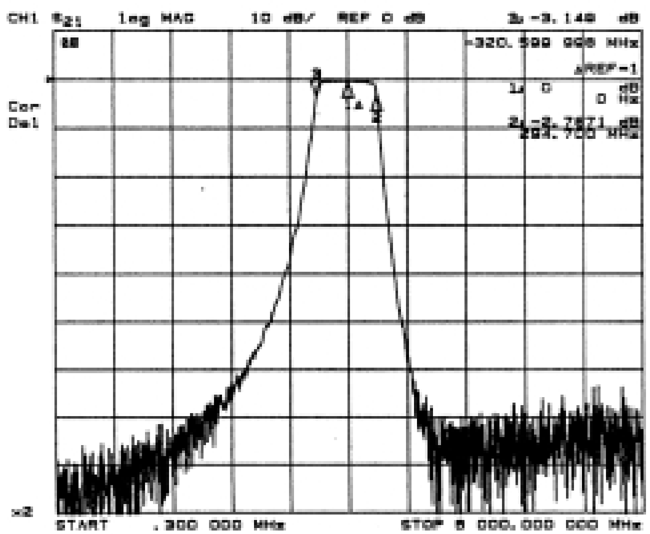
1.6ns
port
extension

FILTER "A"





65ps port extension



FILTER "B"

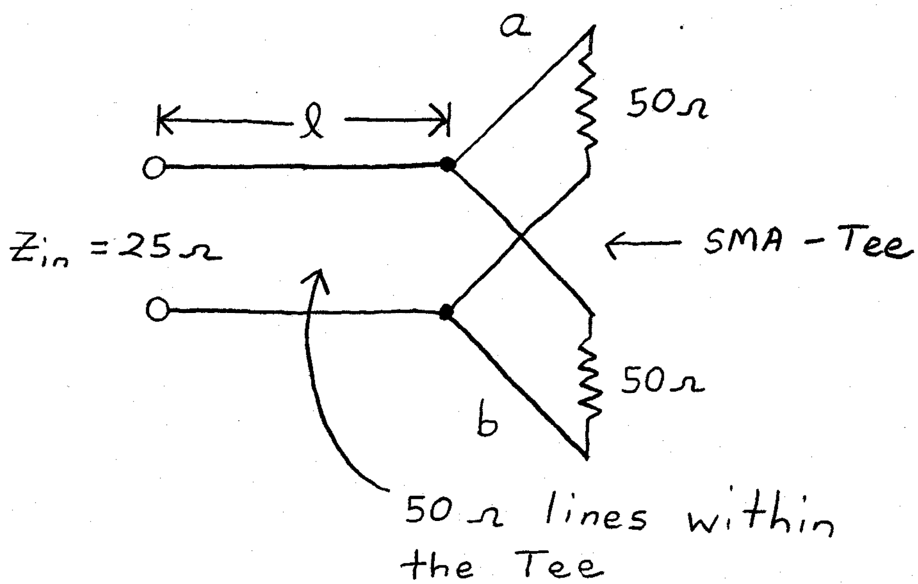
Introduction to Network Analyzers

Introduction

This lab introduces the basic functions of the network analyzer. This is accomplished through reflection and transmission measurements of a simple shunt element. The effects of transmission line on impedance, reflection, and transmission measurements is demonstrated. Calculation and measurement of cascaded networks is demonstrated.

Reflection of a 25 Ω Shunt

A 25 Ω shunt is easily made using two 50 Ω terminations in parallel using an SMA-Tee as shown below.



Experiment A

I. Set the NWA to measure S11 over the frequency range 300 kHz-3 GHz. Use 801 points and calibrate port 1 for S11 with and open short and load. Measurement the log magnitude of the reflection coefficient for the 25 Ω Tee.

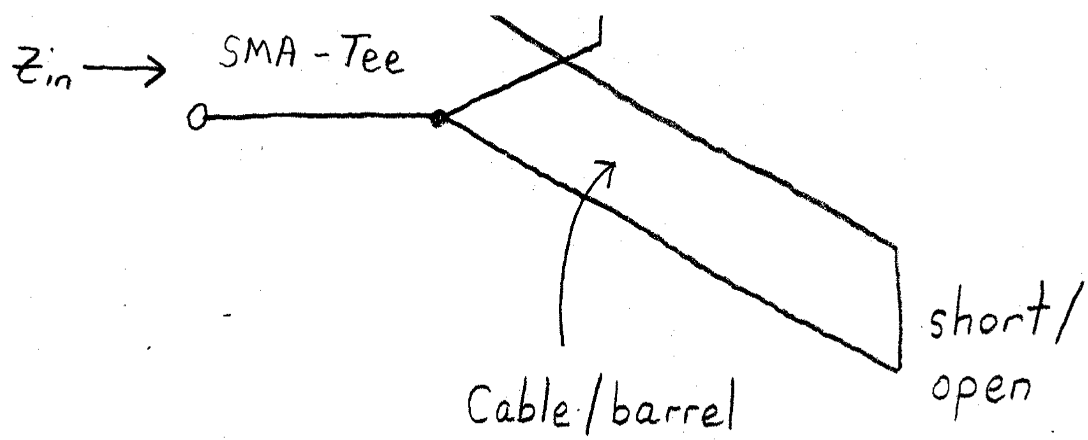
II. Go to phase format. Under the CAL MORE menu, turn on the port extensions and adjust port 1 to remove the slope of the phase response. This effectively takes out the effect of the SMA Tee input arm by extending the reference plane of port 1 to the junction of arms a and b. What is the electrical length of the input arm? Measure the input arm with a ruler and compare the electrical length.

III. Turn off port extension and go to delay format. With averaging on (16) and smoothing (aperture=1%), measure the group delay. How does this compare with the previous measurement?

IV. Go to Smith chart format (averaging, smoothing off). How does the complex impedance compare with what you expect? Using a short cable, investigate the effects of arm lengths a and b on the input impedance. Always terminate the cable with 50 Ω.

V. Add a small cable (~4-10 inches) between port 1 and the SMA-Tee (a female SMA barrel with be necessary.) Note the amplitude and phase response. Using the port extension, find the electrical length of the cable/barrel. What is the the dielectric constant of the cable/barrel.

VI. Put the SMA-Tee back on port 1 with extension off. Attach the cable/barrel to arm a or b leaving the other arm terminated in 50 Ω. Short the end of the cable as shown below and measure the log mag response. At what frequencies is the network matched. Where does total reflection occur? Explain this in terms of the cable/barrel length measured. Repeat this with the cable open (unterminated).



Analysis A

1. Assuming that the length of the input arm l, is negligible, calculate the reflection coefficient, Γ_0, for the 25 Ω load equivalent.
2. Write down an expression for the input reflection coefficient, $\Gamma(\omega)$, including the effects of the electrical length, l_e, of the input arm. What is the relationship between the electrical length of the arm and its physical length?

3. Assuming l_e is small compared to the wavelengths we are measuring, show that the input impedance to the SMA-Tee is given by

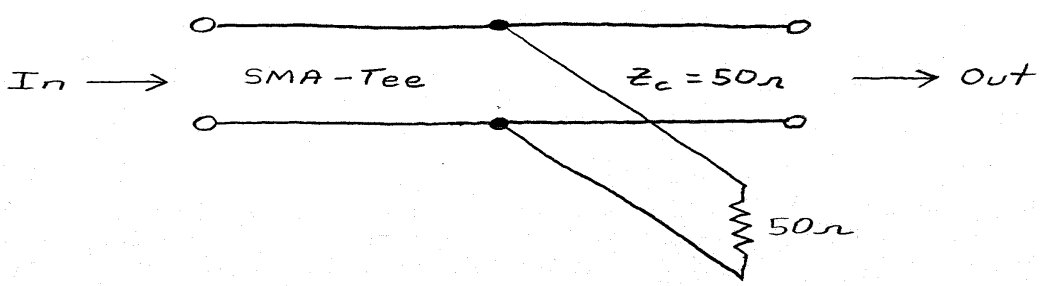
$$Z_{in} = Z_L + j\ \omega\left[\frac{l_e}{c} Z_c\left(1 - \frac{Z_L^2}{Z_c^2}\right)\right]$$

where Z_L=25 Ω and Z_c=50 Ω (characteristic impedance within the Tee)

4. Draw a low frequency equivalent circuit for the 25 Ω SMA-Tee. Write down an expression for the value of the inductor in this circuit in terms of the inductance per unit electrical length, L', of the input arm. Do the lengths of the 50 Ω arms, a and b, have an effect on the input impedance?

Transmission for shunt elements

A 50 Ω shunt can be easily made as shown below



Experiment B

1) Setup the NWA using a full two part calibration from 300kHz-3 GHz. Use 801 points. Calibrate at the ends of the cables where the shunt 50 Ω Tee will be inserted. Use a female-female SMA barrel for the thru standard and omit the isolation calibration.

2) Measure the log mag of the S21 for the 50 Ω shunt Tee. Measure S21 log mag of S11 and S22. Do these values agree with what you expect (see analysis section below)

3) Insert a small length of cable(~3-4 inches) between two 50 Ω shunt Tees and connect to the NWA.

4) Set channel 1 of the NWA up for S21 log mag. Set scale to 1 dB/div and the reference value to –7 dB. Set channel 2 to S21 phase. Under the CAL menu, set the port extension to take out the phase slope. Set the scale to 5 deg/div and the ref. value to zero. Fine tune the port 1 extension

Analysis B

1) Assuming that the input and output are matched at 50 Ω, show that the amplitude matrices for the 50 Ω shunt are given by

$$[S] = \begin{bmatrix} -1/3 & 2/3 \\ 2/3 & -1/3 \end{bmatrix}$$

$$[A] = \begin{bmatrix} 3/2 & 1/2 \\ -1/2 & 1/2 \end{bmatrix}$$

(Assume the lengths of the input/output arms are zero)

2) For a given input power to the 50 Ω shunt network, calculate the relative amounts of reflected and transmitted power. How much power is dissipated in the 50 Ω shunt? Is this a lossless 2-port network?

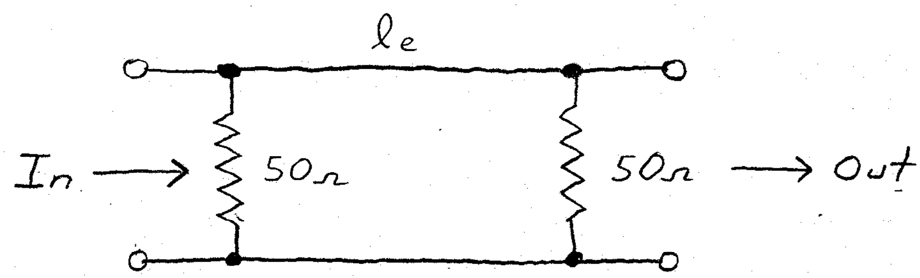
3) Show the wave amplitude transmission matrix for a transmission line of electrical length le is given by

$$[A] = \begin{bmatrix} e^{j\theta} & 0 \\ 0 & e^{-j\theta} \end{bmatrix}$$

where $\theta = \omega l_e / c$.

4) Show that the cascaded network consisting of two 50 Ω shunts separated by a transmission line of electrical length l_e is given by

$$[A] = \begin{bmatrix} 9e^{j\theta} - e^{-j\theta} & 3e^{j\theta} + e^{-j\theta} \\ -3e^{j\theta} - e^{-j\theta} & -e^{j\theta} + e^{-j\theta} \end{bmatrix}$$



5)Show that the magnitude of S21 for the cascaded network is given by

$$|S21| = \frac{4}{\sqrt{82 - 18\cos 2\theta}}$$

What are the maximum and minimum values of the nagnitude of S21 in dB?

6) Show that the deviation from linear phase of the cascaded network is given approximately by

$$\phi_\Delta = -\frac{\sin 2\theta}{9}$$

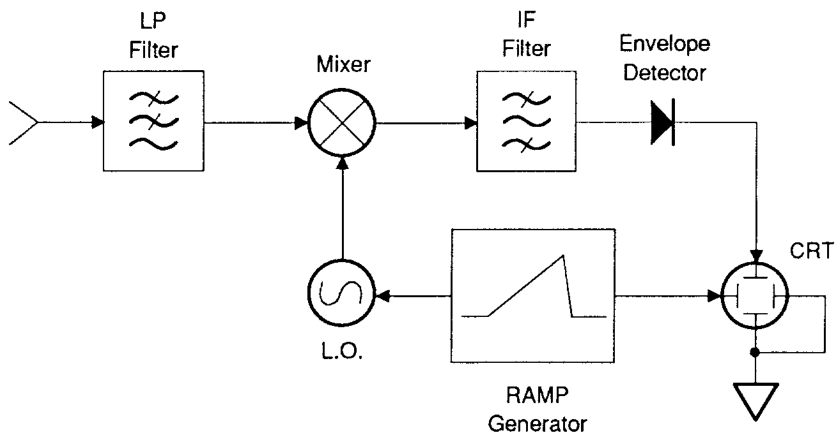
Introduction to Spectrum Analyzers

Introduction

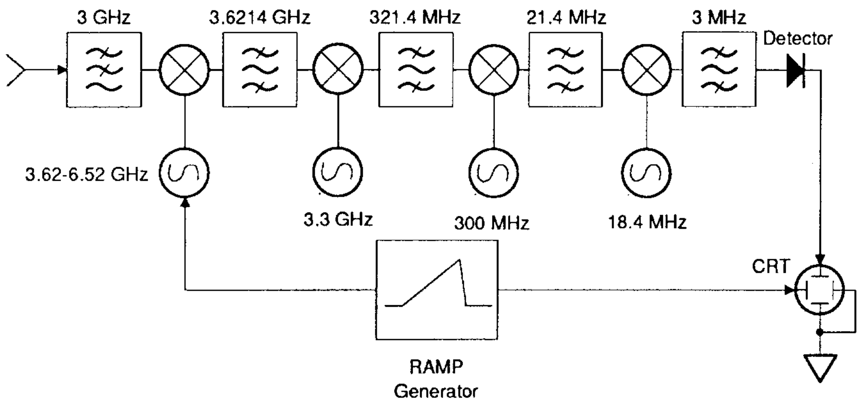
The importance of frequency domain analysis of signals to accelerators is attested by the presence of a spectrum analyzer in the control room of almost every accelerator, storage ring, etc. ever built. It is often one of the primary instruments for measuring many of the parameters of the beam. This laboratory introduces the spectrum analyzer and uses it to measure the frequency spectrum of several signals generated by an HP 33120A Arbitrary Waveform Generator.

Spectrum Analyzers

A schematic diagram of a spectrum analyzer is shown below. The input RF signal is first passed through a LP filter defining the maximum frequency range of the analyzer. The signal is then mixed with the local oscillator (LO) down to an intermediate frequency (IF) where it is passed through an analog filter of variable bandwidth. The envelope of the IF signal is then detected and that is used to control the vertical deflection on the CRT. The LO frequency is ramped through the desired frequency span. The ramp signal also determines the horizontal deflection on the CRT.



The analog spectrum analyzer works by demodulating a high frequency signal, similar to the operation of a simple car radio. However, the spectrum analyzer has much more control over the demodulation process. For example, a less simplified diagram of the spectrum analyzer shown below has multiple LO stages, each with its own IF filter of variable bandwidth.



Periodic Signals

Any time domain signal can be expressed as a sum of sinewaves. For periodic signals, this is called a Fourier series. This can be written as

$$V(t)=\frac{a_0}{2}+\sum_{k=1}^{\infty} a_n\cos(k\omega_0 t)$$

where the coefficients a_n are given by

$$a_n=\frac{2}{T_0}\int_{T_0} V(t)\cos(k\omega_0 t)$$

where the integral is over a single period of the signal.

Noise

We occasionally need to make sensitive microwave measurements, either of the beam or some other source. One of the fundamental limits on the smallest signal levels which can be measured is the thermal noise, either in the detector or the source to be measured. The noise arises from thermal fluctuations of the electrons in a conductor at a finite temperature (or somewhat equivalently, blackbody radiation of a source at a finite temperature T.)

$$\langle V_n^2\rangle=4kT\int_{f_1}^{f_2} R(f)df$$
$$=4kTR\Delta f$$

where

k=Boltzmann's constant=$1.38\text{x}10^{-23}$ Joules/deg-K
T=absolute temperature of source resistance R in degrees-K
_ f=noise bandwidth in Hz

R=source resistance in Ω

(note that this is an approximation valid up to ~$1x10^{13}$ Hz.)

If a matched load is connected to the noise source (i.e. load impedance=R), the maximum power transferred to the load is

$$P_n=kT\Delta f$$

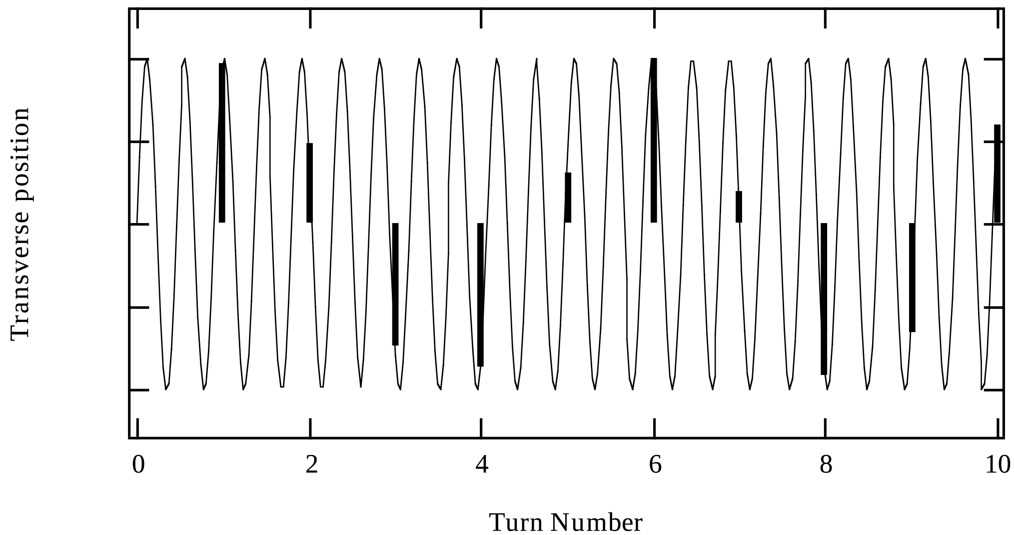
At room temperature (290 deg-K), the noise power density is -174 dBm/Hz

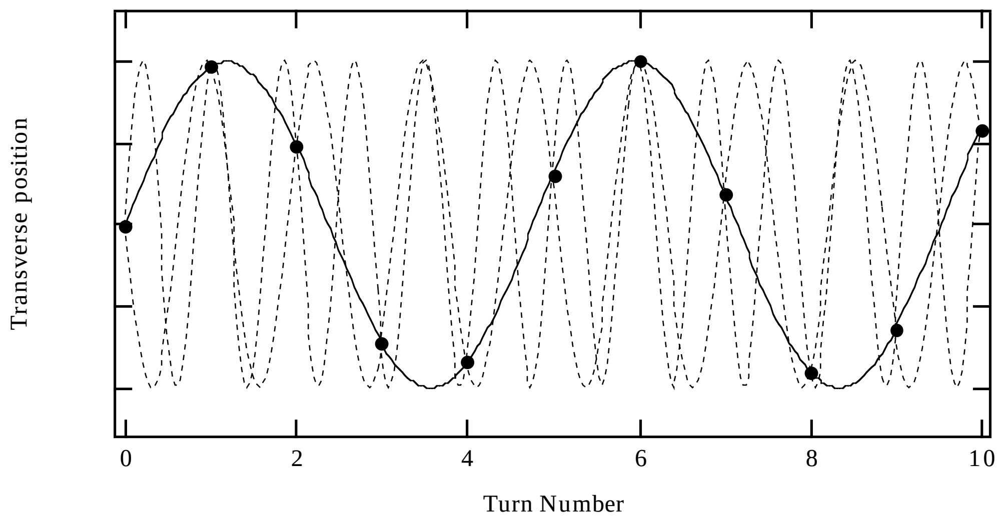
Signal Aliasing

In digitizing an sinusoidal signal, the sampling theorem tells us that we should sample the signal at least two times per oscillation for the minimum "true" representation of the actual signal. If we sample at a lower rate than 1/2 times the highest frequency we want to measure, the signal is undersampled and we see an effect called aliasing.

For example, a high frequency signal shown below is sampled at the times indicated on the signal. The resulting signal is shown below. One can say that the actual signal has been "aliased" into the sampling frequency band. Notice that an infinite set of sinusoids fits the sampled signal. If we looked at the sampled signal on a spectrum analyzer, we would see sets of AM sidebands spaced around harmonics of the sampling frequency.

In accelerators we usually observe the beam signal at a fixed single point in the ring. If the beam is making betatron oscillations, the difference signal at the detector is being amplitude modulated. However, since we only measure at a single point, we can never tell how many oscillations occurred through the rest of the ring. The betatron tune in most rings is greater than unity and so a single BPM signal is always undersampling the betatron oscillation.





The above shows a simplified betatron oscillation around a ring and the bar shows that sampled transverse position at a single point in the ring. The lower picture shows the sampled signal (as dots) and that an infinite series of frequencies fits the sampled data. This set of frequencies corresponds to the set of upper and lower sidebands of the rotation harmonics observed on a spectrum analyzer.

Experiment A

Characterize the frequency response of periodic signals in the frequency domain.

I. Set the frequency of the AWG to 100 kHz sinewave with a p-p amplitude of 100 mV. Setup the spectrum analyzer to measure the output of the AWG using a span of 20 kHz. Record the amplitude of the signal. Now vary the resolution bandwidth of and describe what you observe. Then return the resolution bandwidth to AUTO and decrease the sweep time and describe what you observe. Finally, reduce the resolution bandwidth of the spectrum analyzer until the noise floor of the doesn't decrease further (if possible). Record the signal/noise ratio at this point.

II. Set the AWG for a square wave output with a 50% duty cycle. Plot the frequency response from 0-2.5 MHz and record the amplitude of the first 10 peaks. Also plot the response for duty cycles of 20%, and 80%.

III. Plot the frequency response from 0-2.5 MHz of a) triangle wave b) ramp and c) sinc (the first choice in the arbitrary wave form list.) Record the values of the first 10 peaks for each waveform (do every other peak for the ramp and sinc waveforms.)

Analysis A

I. Verify that the amplitude of the sinusoidal signal is what you expect, knowing its amplitude in the time domain. What happens to the signal when you vary the resolution bandwidth and sweep time? Why? The AWG uses 12-bit digital synthesis to generate its signal. What signal/noise ratio (in dB) would you expect from this signal?

II. Compare the frequency spectrum of the square wave with what you expect for each value of the duty cycle. Explain the similarity between the 20% and 80% and between 40% and 60%.

III. Compare the frequency spectrum of the three additional waveforms with what you expect. What is the relationship between the triangle and square waveforms? Between the square and sinc waveforms? Why do the ramp and sinc waveforms have twice as many peaks as the square and triangle waveforms for the same frequency?

Experiment B

Noise measurement.

I. Preset the SA. Terminate the input into 50 Ω, set the center frequency to 10 MHz with a span of 1 MHz. Measure the signal with and without averaging on. Note the change in the fluctuations of the power level. Changing only the resolution bandwidth (averaging on), measure the noise power level for 3 kHz,... resolution bandwidths.

Analysis B

I. Plot the noise power level vs resolution bandwidth. What is the power law dependence of the noise power and RBW? Calculate the effective temperature of the input of the spectrum analyzer. Calculate the RMS voltage of this noise signal through a 1 MHz, 100 kHz, and 1 kHz bandwidth.

Experiment C: Signal Aliasing

Demonstrate signal aliasing using a the arbitrary waveform generator modulating a pulse train.

I. Setup the AWG to output a sinc waveform (push the ARB button) with a frequency of 2 kHz. Look at this waveform on the scope. This signal represents a series of beam pulses. Look at the signal on the spectrum analyzer. Tune to 6 kHz with a span of 5 kHz. Amplitude modulate the pulse train with a 20% modulation depth and a frequency of 400 Hz and record the spectrum. Raise the modulation frequency to 1.6 kHz and record what you see. Set the modulation to 16 kHz and then vary it ±2 kHz.

Analysis C

5. Explain what you observe in terms of signal aliasing. What is the equivalent "betatron tune" of the signal? When you observe a sideband, how can you tell if it's an upper sideband or lower sideband of the nearest harmonic?

Beam signals

Introduction

One of the simplest diagnostics on an accelerator is observation of the beam signal from a pickup (stripline, button, cavity, etc.) One can measure the orbit, betatron or synchrotron tune, chromaticity, bunch length, beam current, diagnose beam instabilities, and many other thing. Unfortunately, a real beam is not available for this laboratory experiment and thus an HP 33120A Arbitrary waveform generator is used to roughly simulate signals from a button BPM for a bunched beam.

Single Particle Current

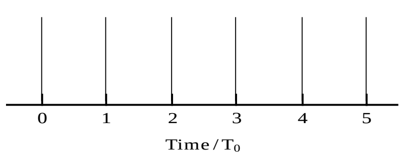
Consider a point particle going around a storage with revolution period T_0 and rotation frequency $f_0=1/T_0$. The current at a fixed point in the ring is given by

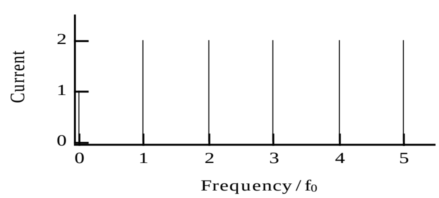
$$i(t)= e \sum_{n=-\infty}^{n=+\infty} \delta(t-nT_0) = e\ \omega_0 \sum_n e^{jn\omega_0 t}$$
$$= ef_0+2ef_0 \sum^{\infty} \cos(n\omega_0 t)$$

The FT of this given by

$$I(\omega)=e\ \omega_0 \sum_k \delta(\omega-k\omega_0)$$

The spectrum is a comb with signal only at the rotation harmonics





(Negative frequency components can be folded onto positive frequency. AC components are 2X DC component.)

Allow the particle to make synchrotron oscillations with angular frequency _ s of amplitude (in time) of _s

$$i(t)=e \sum_n \delta(t-nT_0+\tau_s\cos(\omega_s t))$$

The FT of this signal is given by

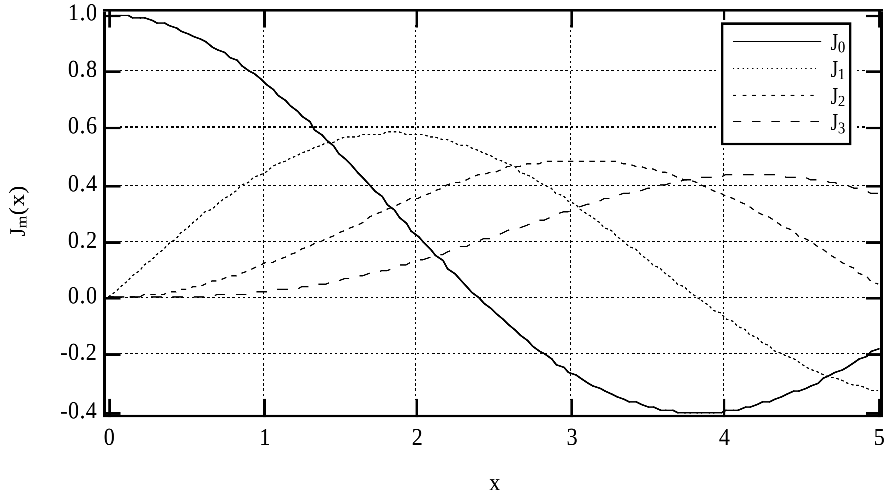
$$I(\omega)=e\,\omega_0\sum_n e^{-jn\omega_0(t+\tau_s\cos\omega_s t)}$$
$$=e\,\omega_0\sum_m j^{-m}J_m(\omega\tau_s)\sum_k \delta(\omega+m\omega_s-k\omega_0)$$

where the relation

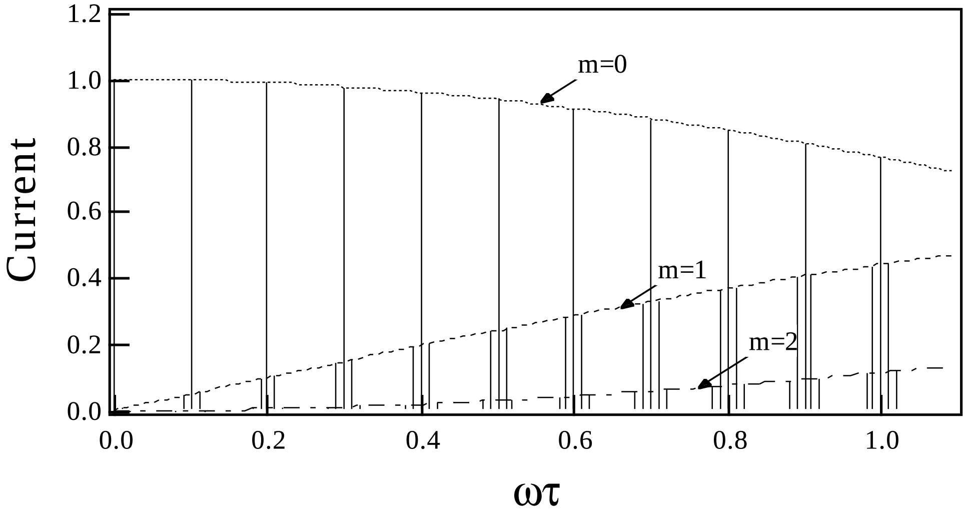
$$e^{jx\cos\theta}=\sum_m j^m J_m(x)e^{jm\theta}$$

has been used.

The comb spectrum has added FM sidebands which are contained within Bessel function envelopes.



Rotation harmonics follow J_0, first order sidebands follow J_1, etc.



Single Particle Dipole Signal

The dipole signal at a fixed point is the d(t)=x(t)*i(t)

For an offset X_0 and betatron oscillation amplitude x_, the dipole signal is (no synch oscillations)

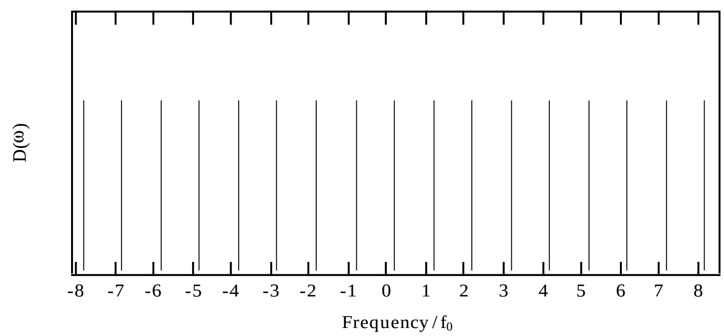
$$d(t)=e(X_0+x_\beta \cos(\omega_\beta t)) \sum_n \delta(t-nT_0)$$

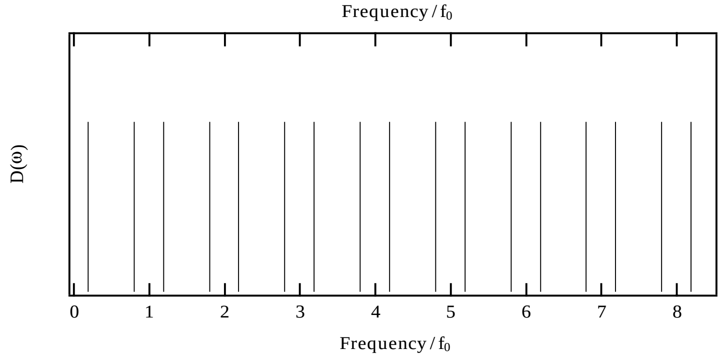
where _ _=Q_ _ 0 is the betatron angular frequency.

The frequency spectrum is given by

$$D(\omega)=e\ \omega_0 X_0 \sum_k \delta(\omega-k\omega_0) + e\ \omega_0 x_\beta \sum_k \delta(\omega-(\omega_\beta+k\omega_0))$$

The result is a comb spectrum at rotation harmonics with betatron sidebands. (Negative frequencies fold over to become lower sidebands in this form.) Note that the integer part of the betatron frequency cannot be measured because the detector "samples" the betatron oscillation at a single point in the ring.





Experiment A

The AWG is used to simulate beam signals. The frequency spectrum of the beam signals is then analyzed with amplitude and phase modulation.

I. Set the AWG frequency to 500 kHz and use the sinc waveform (press ARB). Observe this signal on an oscilloscope. This signal represents the pulses from a beam with spacing of 2 microseconds (except that real pulses are bipolar.) Look at this signal on the spectrum analyzer and understand what you are looking at. Turn on AM with a frequency of 20 kHz and a modulation depth of 20%. Zoom in on several of the "rotation harmonics." Are the sideband amplitudes constant for each harmonic?

II. Using the same set as part I, turn on FM with modulation frequency of 5 kHz and modulation depth of 1 kHz. Measure the amplitudes of the "rotation harmonics" and sidebands at 500 kHz, 3 MHz, and 6 MHz. Also observe this signal on an oscilloscope as the modulation depth is increased. Try to measure the amplitude of the time modulation of the signal.

Analysis A

I. If the modulation signal were that of a beam executing betatron oscillations, what would be the betatron frequency (or possible set of frequencies?) What would happen to the frequency spectrum if the beam were centered in the detector?

II. Why do the FM sidebands increase amplitude with frequency? Verify that the sidband amplitudes are what you expect for the "rotation harmonics you measured. Calculate the amplitude of time modulation from the frequency modulation and compare it to the direct measurement on the scope. If the signal is from a beam executing synchrotron oscillations, what is the corresponding amplitude of energy oscillations expressed as a fractional energy (momentum compaction is 1e-3). What is the smallest amplitude phase oscillation you can measure at 500 kHz? At 3 GHz?

Experiment B

Multibunch beam signals.

I. Set the HP AWG to sinc output at a 500 kHz rate. Observe the signal on the oscilloscope and the spectrum analyzer. This repetitive signal represents a ring with a symmetric fill pattern and an RF frequency of 500 kHz. Since all bunches are identical, we cannot determine the rotation frequency. Set the AWG to burst modulation with a burst frequency of 25 kHz and a burst count of 10 cycles. Burst frequency and count are found in the by turning on the option menu (shift-enter) under the A:Mod menu (down arrow). Change submenu selections with the left and right arrows. Look at the signal again on the scope. Plot the frequency spectrum over a 1 MHz span centered at 775 kHz. Do the same for a burst count of 15.

Analysis B

I. How much does the "bunch harmonic" decrease when you go to burst modulation and does the reduction agree with what you expect? For each burst count (i.e. beam gap or duty cycle,) what is the ratio of the bunch harmonic to the rotation harmonic between the bunch harmonics? What do you expect?

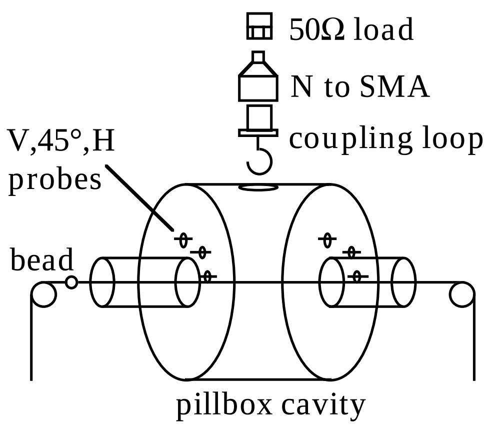
RF Cavity Characterization

Introduction

The experiment is designed to introduce the student to the concept of cavity modes by studying the mode spectrum of a simple model cavity using a network analyzer (NWA). The first few modes are identified by frequency and their Q's measured by a transmission method. For the TM_{010} mode, which would be the accelerating mode in a real cavity, the coupling through a drive loop is determined by means of a reflection measurement and the loaded and unloaded Q's are calculated. The loading of the higher-order modes (HOMs) is also observed and their Q reduction is measured. Finally The longitudinal field profile of the TM_{010} mode is measured using a perturbation method and the shunt impedance is calculated.

Equipment:

Aluminum model pillbox cavity
Network Analyzer + calibration kit
2 cables
2 N-type to SMA adapters
2 50Ω SMA loads
Coupling loop
E-Field probe
1 SMA F-F connector (for Thru calibration)
Bead-pull apparatus



Transmission measurement (unloaded cavity)

Connect the network analyzer to the probes on each end of the cavity to make a tranmission measurement (no coupling loop or damping antenna inserted). Observe Log Mag S_{21} on the NWA, set the frequency range from 700 MHz to 2.3 GHz and adjust the scale so that the resonant peaks in the spectrum are clearly visible (Note: up to 1601 points and an averaging factor of 64 may be necessary to resolve all of the modes with such a wide sweep). Plot the transmission response between the vertical pair of probes on the printer or plotter and save in the NWA memory. Connect the cables to the horizontal pair and compare the data with the memory. Are there any significant differences? Using the marker functions determine the frequencies of the peaks and compare with the mode frequencies in table 1, which are calculated by a cavity design program (URMEL). Make a table in order of measured frequency identifying each mode with it's URMEL counterpart and with additional columns for measurements of unloaded and loaded Q's. Zoom close in on a peak corresponding to a dipole mode and again compare vertical and horizontal spectra.

For each peak in turn set the scale, reference value and frequency span so that the -3.01 dB points can be accurately determined and record the unloaded Q (Qo) for each mode. Measure the half-power points manually for the first mode then try using the "widths" function on the NWA for the rest. (Do only for vertical probe pair)

Table 1: Mode frequencies calculated by URMEL

mode type	Freq (MHz)	R / Q* (Ω)
0E1	786	100.757
1E1	1245	9.761
1M1	1321	0.107
0M1	1417	14.298
2M1	1549	0.000
2E1	1675	0.227
1M2	1705	8.345
0E2	1808	8.004
2M2	2048	0.570
1M3	2104	0.092
0M2	2162	28.562
1E2	2272	0.045

(mode type = aE/Mb where a is the azimuthal order (0=monopole, 1=dipole, etc.) E/M indicates the longitudinal mode symmetry (E=electric boundary condition in midplane, M=magnetic), b= URMEL solution number. *R/Q is calculated on axis for monopole modes and at the beam-pipe radius for higher order modes)

Coupling measurement

Insert the coupling loop, remove the cables from the transmission probes and connect port 1 of the NWA to the coupler. Observe S_{11} for the TM_{010} (0E1) mode through the range of probe angles from zero to maximum coupling and choose a frequency span that is large enough to accomodate the shift in frequency but narrow enough to allow good resolution of the resonance curve. Calibrate port 1 of the NWA over this frequency range. Observe the response with calibration turned on for S_{11} or VSWR and on the smith chart as the coupler is rotated. (Note that when using the smith chart the electrical delay setting on the network analyzer should be adjusted so that the resonant frequency lies on the real axis and the marker mode should be set to read admittance (G+jB)).

Measure and plot β at resonance as a function of coupler angle. (What variation would one expect from a simple consideration of the loop area coupled to the azimuthal magnetic field?). Determine β_{max} and the angle for the best match (β=1). Set the coupler in the matched position and lock in place. Measure β accurately from the $|S_{11}|$ or VSWR curve, calculate the appropriate values for the half-power points for Qo and QL and measure the bandwidth at these values. Record the Qo and Ql. Check if Qo / QL=(1+β). Does Qo measured this way agree with the transmission measurement? Measure Qo again from the half-power points on the smith chart and compare?

Loaded Q measurement

Disconnect the cable from the coupler and replace with a 50Ω load. Go back to the transmission measurement set-up and re-plot the spectra for the three probe orientations. How are they different from before? Measure the loaded Q's in the Vertical and Horizontal orientations and add them to the table. Calculate the β for each mode. Does the loaded Q of the TM_{010} mode agree with the reflection measurement?

Field mapping

Remove the coupler and leave the NWA connected for transmission measurement. Assemble the bead-pull apparatus around the cavity with the thread running along the central axis and the bead just outside the beam-pipe. Set the frequency span so that the peak of the TM_{010} mode is on the right of the display and moves to the left of the display as the bead is pulled to the center of the cavity. This will give acceptable resolution of the frequency shift without having to adjust the display during the measurement (Note: some averaging may be required to get an accurate measurement). Measure the frequency and Qo with the bead outside the cavity and then proceed to pull the bead through the cavity and make frequency measurements at 0.5 inch intervals (use peak-search tracking on the NWA). Measure fo again once the bead is out of the cavity on the other side. If the frequency has drifted during the measurement a linear interpolation can be made between the end points to correct the data. Tabulate the frequency vs axial position and calculate RT^2/Q and the beam impedance $Z_{||}$ using the spreadsheet provided.

If time permits, repeat for the dipole TM_{110} mode with the bead offset close to the wall of the beam pipe at a known radius r. Calculate $Z_{||}(r)$ and Z

Signal Modulation

Introduction

We are all familiar at some level with modulation of signals. Every broadcast of intelligence involves modulation of the intelligence onto a carrier which is then demodulated at the receiver.

This laboratory exercise demonstrates simple signal modulation. It also demonstrates some applications of the spectrum analyzer as a receiver as well as FM/AM conversion.

Amplitude Modulation

A simple form of amplitude modulation occurs when a sinusoidal signal of frequency ω_m (i.e. modulation frequency) is used to determine the amplitude of another sinusoid of frequency ω_c (i.e. carrier frequency). This can be written mathematically as

$$V_s(t)=V_m\cos(\omega_m t+\phi_m)V_c\cos(\omega_c t+\phi_c)$$
$$=\frac{V_m V_c}{2}\left(\cos((\omega_m-\omega_c)t+(\phi_m-\phi_c))+\cos((\omega_m+\omega_c)t+(\phi_m+\phi_c))\right)$$

The result is two sinusoids, one with a frequency equal to the difference between the carrier and modulation frequencies and another frequency equal to the sum. The process of multiplying the two signals is commonly referred to as mixing. The two resultant mixing products are also called sidebands of the carrier. Note that this process does not leave any signal at the original carrier frequency, sometimes called DBSC (double sideband suppressed carrier.)

AM modulation including the carrier is usually written as

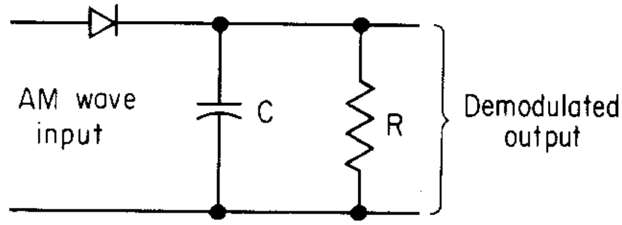
$$V_s(t)=V_c\cos(\omega_c t+\phi_c)\left(1+m\cos(\omega_m t+\phi_m)\right)$$

This is essentially the same as the above except that only some fraction *m* of the original carrier is mixed. *m* is commonly referred to as the modulation depth (e.g. *m*= 0.1 = 10% modulation.) In this scheme, the carrier is not suppressed as in the above case. The ratio of the sideband to the carrier is simply *m*/2.

Demodulation

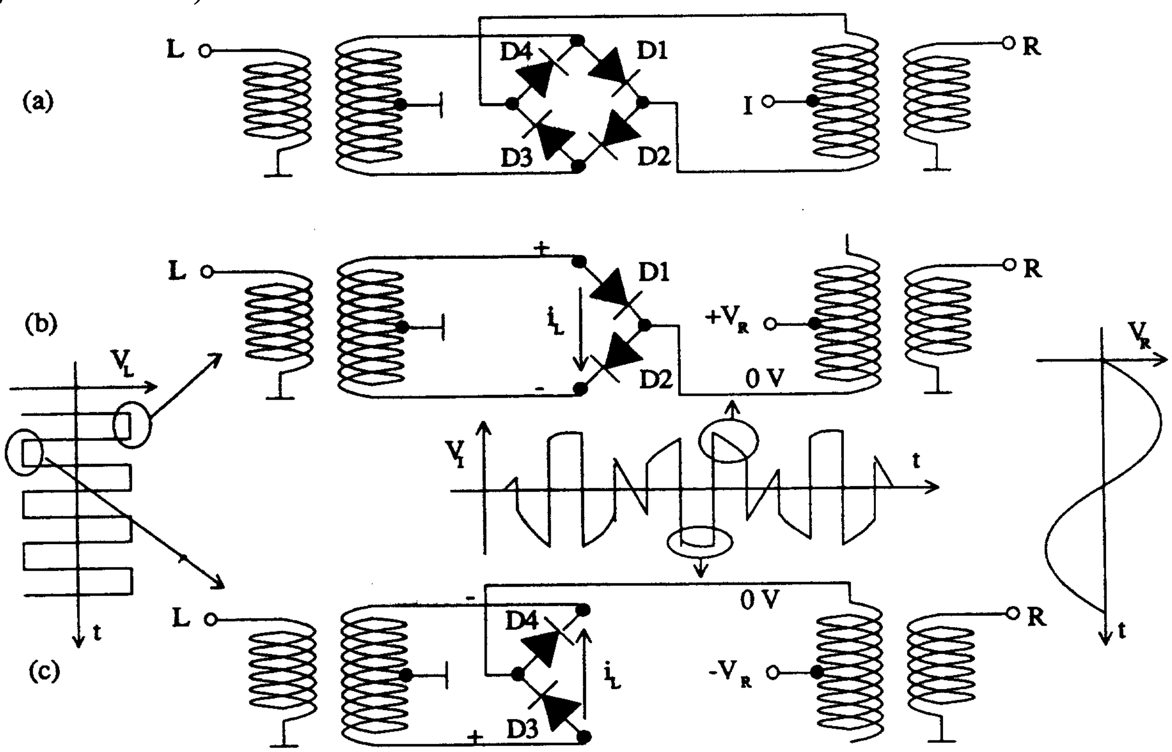
Consider the above case where a signal has been modulated onto a carrier. How do we demodulate the the result in order to recover the original modulation signal? One technique is to multiply the signal by the carrier resulting in two sets of frequencies, ω_m and $2*\omega_c \pm\omega_m$. If the signal is low-pass filtered, we can recover the original modulation.

This is sometimes referred to a demodulating or mixing a signal down to baseband. Another common technique illustrated in Figure 1 is a diode or envelope detector.



Methods of Mixing

A common technique for mixing uses a double balanced mixer, shown in Figure x. The nonlinear response of the diodes multiplies signals at the RF and LO ports (and also gives higher harmonics.)



Phase and Frequency Modulation

The phase of a signal can also be modulated

$$V_s(t)=V_c\cos(\phi(t))$$

where

$$\phi(t)=\omega_c t+\hat{\phi}\sin(\omega_m t)$$

Frequency modulation is just a special case of phase modulation. The instantaneous angular frequency is given by

$$\omega(t)=\frac{d\phi}{dt}=\omega_c+\hat{\phi}\omega_m\cos(\omega_m t)=\omega_c+2\pi\Delta f\cos(\omega_m t)$$

where Df is the peak frequency deviation from the carrier. The peak phase deviation and frequency deviation are related by

$$\hat{\phi}=\frac{\Delta f}{f_m}$$

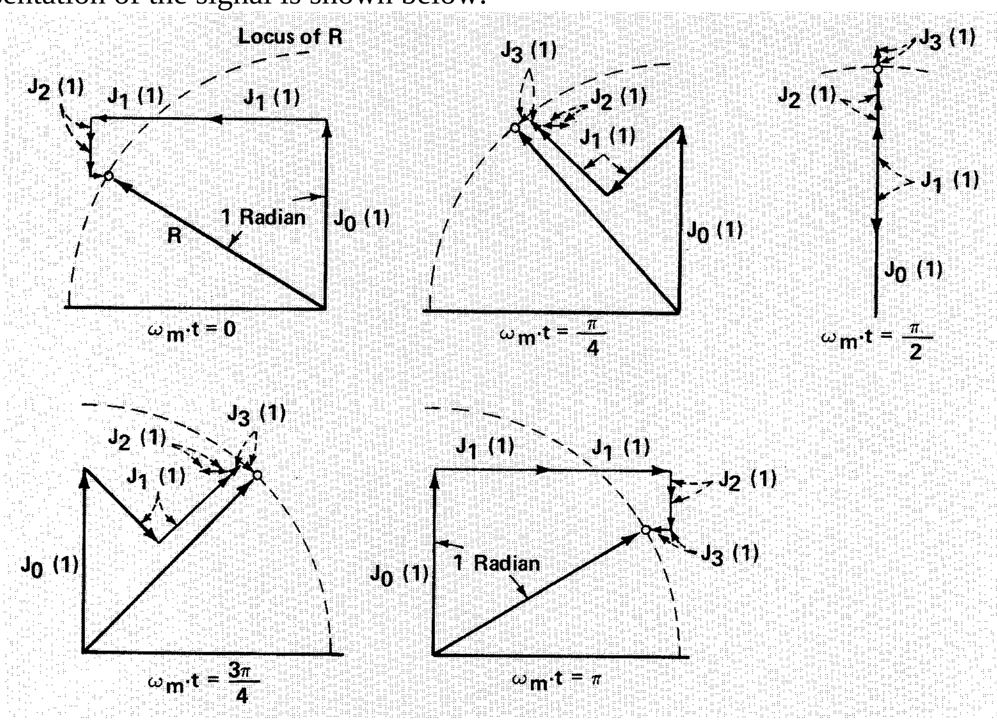
Using the Fourier expansions

$$\cos(x\sin\theta)=J_0(x)+2J_2(x)\cos 2\theta+2J_4(x)\cos 4\theta+...$$
$$\sin(x\sin\theta)=2J_1(x)\cos\theta+2J_3(x)\cos 3\theta+...$$

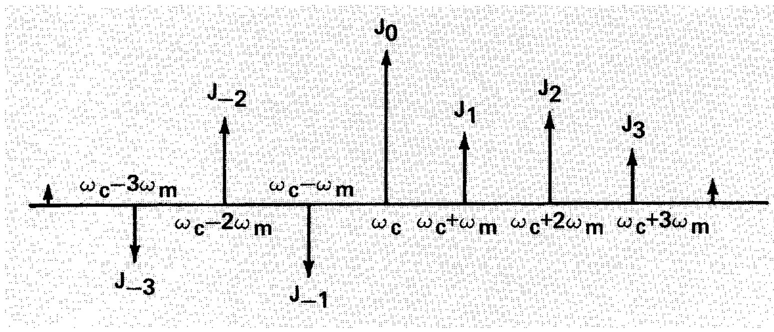
where J_n are the Bessel functions, the PM signal can be written as

$$\begin{aligned} V_s(t)=J_0(\hat{\phi})\cos(\omega_c t)&-J_1(\hat{\phi})(\cos(\omega_c-\omega_m)t-\cos(\omega_c+\omega_m)t) \\ &+J_2(\hat{\phi})(\cos(\omega_c-2\omega_m)t+\cos(\omega_c+2\omega_m)t) \\ &-J_3(\hat{\phi})(\cos(\omega_c-3\omega_m)t-\cos(\omega_c+3\omega_m)t) \\ &+...... \end{aligned}$$

The signal becomes an infinite sum of harmonics of the modulation frequency, each proportional to the Bessel function corresponding to that harmonic. A phasor representation of the signal is shown below.



The frequency spectrum is shown below. Note that the relative phase of the upper and lower sidebands is different. The does not appear on the spectrum analyzer because phase information is lost.



Signal Modulation

A generically modulated signal can be written as

$$V(t)=\mathrm{Re}\left\{V_c(1+a(t))e^{j(\omega_c t+\phi(t))}\right\}$$

where a(t) and f(t) are any arbitrary amplitude and angle modulation.

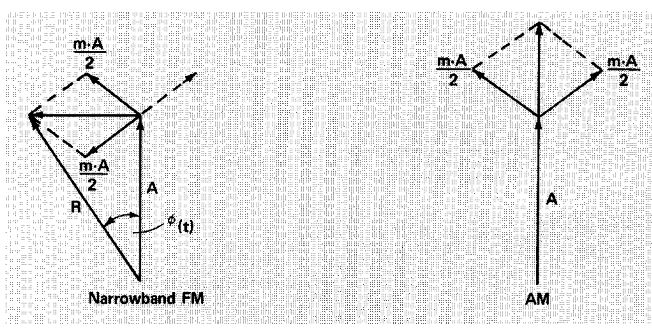
For purely sinusoidal modulations, this can written as

$$V(t)=\mathrm{Re}\left\{V\ (1+\hat{a}\cos(\omega\quad t))e^{j(\omega_c t+\hat{\phi}\sin(\omega_{pm}t))}\right\}$$

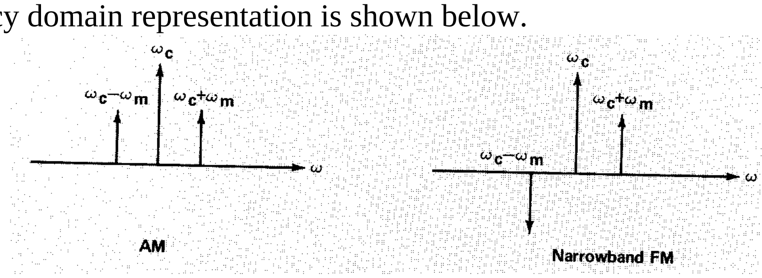
Assuming that the amplitude of amplitude and phase modulation is small, this can be written as

$$V(t)\approx V_c\mathrm{Re}\left\{e^{j\omega_c t}\left[1+\frac{\hat{a}}{2}(e^{j\omega_{am}t}+e^{-j\omega_{am}t})+\frac{\hat{\phi}}{2}(e^{j\omega_{am}t}-e^{-j\omega_{am}t})\right]\right\}$$

We can compare amplitude and frequency modulation using a phasor representation shown below. The carrier is a long phasor with two counter-rotating small phasors attached to its end.

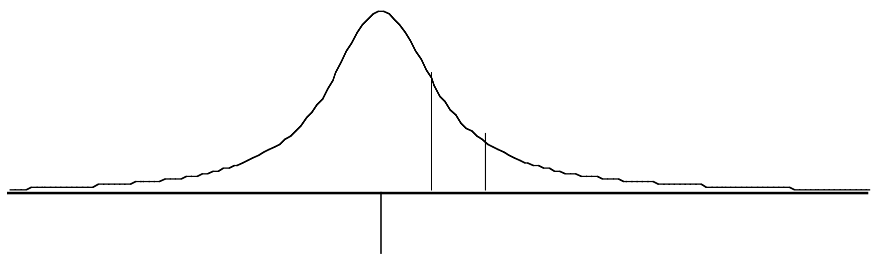


In the frequency domain representation is shown below.



The relative phase of the upper and lower sidebands for the PM results from the choice of the phase of the modulation (i.e. sin rather than cos).

When the modulated signal passes through a filter (e.g. an RF cavity), AM can become FM and vice versa. Shown below in an FM signal. When the cavity is tuned to the carrier, the upper and lower sidebands have the same response and the signal induced by the two sidebands cancel since they are out of phase by 180 degrees. When the filter is tuned asymmetrically about the sidebands, the transmitted response the two sidebands is unequal. The resulting signal has an amplitude modulation.



Experiment A: Amplitude Modulation

Characterize AM using the HP 33120A Arbitrary Waveform Generator (AWG) and the spectrum analyzer in swept frequency and tuned receiver mode.

I. Set the HP AWG to AM with a center frequency of 1 MHz. Set the modulation frequency to 5 kHz. Setup the spectrum analyzer to observe the carrier and sidebands. Measure the ratio of the sidebands to the carrier at 3 values of the modulation depth (10%, 50%, 120%). Vary the resolution bandwidth (RBW) from the default setting up to 100 kHz.

II. Using the same AM modulation as in part A (50% modulation depth), observe the amplitude modulation of the carrier with the spectrum analyzer tuned to the carrier in receiver mode (i.e. zero span.) It is easiest to record the signal level with the SA displaying in linear mode. Remember to adjust the RBW of the analyzer accordingly. Use the video trigger to help stabilize the signal. Vary the center frequency of the SA over ±100 kHz and record what you see. Vary the RBW back down to 3 kHz and record the amplitude of the signal.

Analysis A

I. Why do you observe some second harmonic component of the modulation in the spectrum?

II. Verify that the p-p modulation and its frequency agree with what you expect for 50% modulation depth. Explain qualitatively what you observe as you vary the center frequency. Explain qualitatively what happens to the signal as you decrease the RBW. Extra credit: Try to do it quantitatively by measuring the width of the analog filters used on the IF signal or by using the information in the HP application note on spectrum analysis (pp. 9-11.)

Experiment B: Phase and Frequency Modulation

Characterize FM using the AWG and the spectrum analyzer in swept frequency and tuned receiver mode. The second part uses the SA as a receiver and is somewhat more advanced.

I. Set the AWG to FM with a carrier frequency of 1 MHz. Set the modulation frequency to 5 kHz and the modulation depth to 100 Hz. Setup the spectrum analyzer to observe the carrier and sidebands. Use a span of 50 kHz and set the resolution bandwidth to auto. Measure the first three sidebands (as they appear above the noise floor) as a function of modulation depth (0.1, 0.5, 5 kHz.) Record the noise floor of the measurement for use in the analysis.

II. a) Set the AWG to a carrier of 1 MHz and modulation depth of 500 Hz. Set the SA tuned to the carrier in receiver mode (i.e. zero span.) Set the RBW to 10 kHz and the sweep time to 1 msec. Also have the SA display in linear mode. Vary the center frequency ±5 kHz and record what you observe. Also try a RBW of 3 kHz.
b) Now set the AWG to modulation depth of 5 kHz. With the center frequency at 1 MHz, vary the RBW from 10 kHz down to 300 Hz and record what you observe.

Analysis B

I. Verify that the amplitudes of the FM sidebands you measured are what you expect. You can use the provided table of Bessel functions if you need it.

II. a) Draw pictures of the overlap of the response of the SA IF filter with the FM spectrum for a center frequency of the 1 MHz and 1 MHz ± 5 kHz. Use these to explain qualitatively what you observe as you vary the center frequency of the SA. Why do you begin to see amplitude modulation? Why is there no amplitude modulation when the SA is tuned to the carrier frequency?
b) Draw a picture of the overlap of the response of the SA IF filter with the FM spectrum. Use this to explain qualitatively what you observed as the RBW is increased.

Network Analyzers: Two simple networks commonly used in particle accelerators

Introduction

In this set of exercises, the measurement skills acquired in LAB-1 are used to investigate two simple microwave networks commonly used in particle accelerators, the long coaxial cable and the two-tap notch filter. These devices will be measured in the time domain as well as the frequency domain using a sampling oscilloscope with time domain transmission (TDT) capability.

The long coaxial cable

Long cable runs are common in all particle accelerators. For broadband RF/Microwave signals, the cable quality significantly affects transmission quality. As discussed in the lecture/homework, the skin effect in conductors results in complex series surface impedance per unit length in the cable. This impedance is proportional to $\omega^{1/2}$ and therefore causes dispersion as well as loss in the signals being transmitted through the cable. The transfer function for a cable of physical length z is given by:

$$H(\omega) = \exp\left(-\left[\Psi\sqrt{\omega} + j\ (\Psi\sqrt{\omega} + \beta)\right] z\right)$$

where

$$\Psi = \frac{1}{4\pi Z_c}\sqrt{\frac{\mu_0}{2\sigma}}\,\frac{a+b}{ab}$$

Note that:
$Z_c = 50\ \Omega$
a,b=inner, outer conductor radius
σ=conductor conductivity
$\beta=\omega/v$ (normal propagation constant)

Also note that, in general, a real term which is linearly proportional to frequency must be included in the exponent of the above to account for shunt-conductive losses of the dielectric in the cable. This effect dominates at high frequencies for cables with lossy dielectrics. However, for our experiments we neglect this factor because the small diameters of the conductors in our cables cause conductive effects to dominate over our frequency range.

Experiment A

1) Calibrate the network analyzer for S21 over the frequency range 300 kHz - 3 GHz. Use 1601 points and a simple thru calibration. Use a small length of BNC cable as your thru standard. Appropriate adapters are provided.

2) Connect the long BNC RG 233/U cable to the network analyzer and measure it's electrical length by going to phase format and using the port 1 extension. Note that the electrical length will be different for different frequencies because of the dispersion. Therefore, measure the asymptotic low frequency electrical length by extending port 1 to take out the linear phase slope due to the $e^{-j\beta z}$ term in (1). When you are close to the correct extension, the phase vs. frequency curve should have a - $\omega^{1/2}$ dependence as given by (1). Make fine adjustments by measuring phase at two frequencies and adding or subtracting port extension until the - $\omega^{1/2}$ dependence is obtained.

3) What is the low frequency electrical length of the cable (neglect the length of your calibration standard)? Note that this is greater than the physical length, z, in (1) by $\varepsilon_r^{\ 1/2}$ of the cable dielectric. What is the low frequency delay of the cable, τ?

4) Determine a value for ψ_z using the phase curve.

5) Go to log magnitude format. Using the value for ψ_z obtained above, show that the log magnitude plot on the network analyzer obeys the amplitude/frequency dependence given in (1). Remember, the amplitude dependence given in (1) will not be exact because dielectric losses are not included, but it will be close in our case. What is the 3 dB bandwidth of the cable?

6) Go back to phase format and find an approximate value for the maximum deviation from linear phase of the cable (use the port 1 extension again). This parameter is important for feedback systems utilizing long cables.

The long coaxial cable in time domain

The step response of the long cable is easily measured with a sampling scope in time domain transmission (TDT) mode. The scopes used in this part of the lab were not available at the time when this was written so only general operational guidance is given here. Detailed instruction concerning the operation of the sampling scopes can be obtained from the instructors during the lab.

Using Laplace transform techniques, one can derive the step response of the long cable from (1):

$$h(t) = \mathrm{erfc}\left(\frac{\Psi z}{\sqrt{2(t-\tau)}}\right) \qquad for\ t > \tau$$

Where τ=low frequency electrical delay of cable and the complimentary error function

$$\text{erfc}\,(x) = \frac{2}{\pi}\int_{x}^{\infty} e^{-\lambda^2} d\lambda$$

Experiment B

1)Set the sampling scope up for TDT mode using the TDR step output as your step signal source and one of the other channels as the receiving port. Calibrate with the short BNC reference cable used in step one of the network analyzer measurements.

2)Using a time marker, determine the arrival time at the receiving port of the leading edge of the step for the reference cable.

3)Replace the short reference cable with the long cable and turn in electrical delay under the time base menu until the leading edge of the step is found. Note that the edge of the step will now be rounded according to (2) due to the cable dispersion. Set another time marker at the beginning of the rounded step and determine the low frequency electrical length of the cable (neglecting the reference length). Does this value agree with step 3 of the network analyzer measurements?

4)Measure the 10% - 90% rise time of the cable.

A commonly used relationship between rise time and 3 dB bandwidth is the inverse of three times the 3 dB bandwidth. This relationship is derived by considering the response of a simple, single pole low pass filter. How accurate is this relationship for the long cable? What is the exact rise time/bandwidth relationship for the long cable based on your measurements?

Analysis B

Derive the theoretical relationship between rise time and the 3 dB bandwidth for the long cable using (1) and (2). Note that erfc(x) = 1 – erf(x). Tables attached, compare with your measurement results.

Derive (2) from (1). The following Laplace transform relation may be useful:

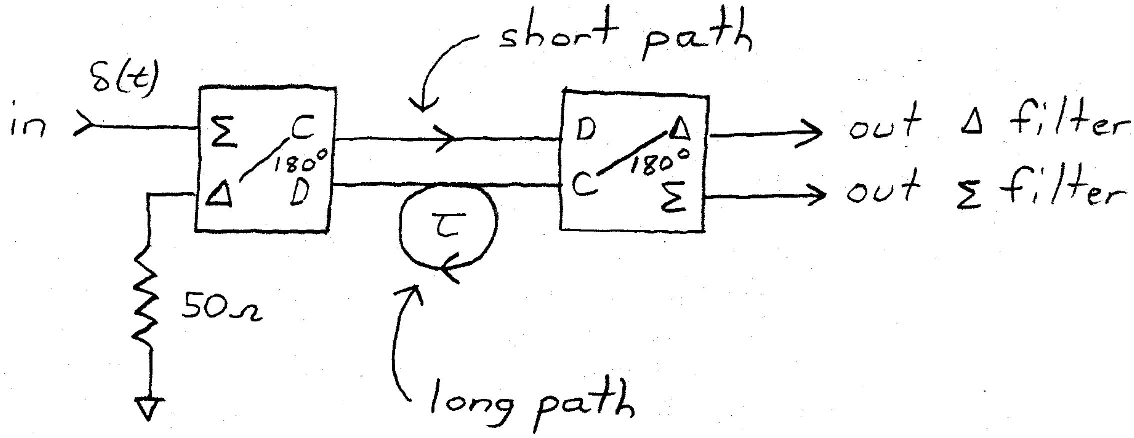
$$\frac{1}{s}e^{-k\sqrt{s}} \Leftrightarrow \text{erfc}\left(\frac{k}{2\sqrt{t}}\right)$$

The two-tap notch filter

Notch filters are commonly used in feedback systems for controlling coupled-bunch instabilities in storage rings. These filters reject frequencies which are integer multiples of the revolution frequency of the ring (orbit harmonics). These signals represent static orbit or phase offsets for the bunches and must be rejected by the feedback system in order to save the full dynamic range of the system for coupled-bunch oscillations.

Background/problems

Two versions of simple two-tap notch filters are easily realized with 180 degree hybrids:



If τ is the difference in time delays between the two cables connecting the hybrids, the impulse responses of the Σ and Δ filters, apart from an overall time delay, are given by:

$$h_{\Sigma}(t) = \frac{\delta(t)}{2} + \frac{\delta(t-\tau)}{2}$$
$$h_{\Delta}(t) = \frac{\delta(t)}{2} - \frac{\delta(t-\tau)}{2}$$

Show that the frequency responses of the Σ and Δ filters are given by:

$$H_{\Sigma}(\omega) = e^{-j\ \omega\tau/2}\cos\left(\omega\tau/2\right)$$
$$H_{\Delta}(\omega) = e^{-j\ \omega\tau/2}\sin\left(\omega\tau/2\right)$$

Derive and plot the steps responses of the two filters.

Experiment C

1)Using the hybrids provided, assemble the filters as shown in the background figure. For the short path, use a male-male SMA barrel. For the long path, use a 3' – 6' SMA cable. Also, make sure that the output port that is not being tested is connected to a 50 Ω termination.

2)Do a simple thru (S_{21}) calibration on the network analyzer from 300 kHz – 500 MHz. Use 1601 points and any SMA cable for the thru reference. Set up channel 1 for linear magnitude format and channel 2 for phase and go to dual channel display. Connect the notch filter to the analyzer. Adjust scales and references as needed. Use the port 1 extension to take out the overall phase slope due to the short path and hybrid delays to obtain the transfer functions in background problem 1. Verify these transfer functions.

Note: the response of the hybrids starts to deteriorate beyond 400 MHz. This is why the notch depth decreases at the higher frequencies.

3)Take out the long SMA cable path and replace it with the very long BNC cable use previously. The resulting notch filter could conceivably be used in a feedback system for a storage ring with circumference equal to the electrical length of the BNC cable.

4)Change the format of channel 1 to log magnitude and turn off the dual display. Adjust scale and reference as necessary. In either the Σ or Δ configuration, note the envelope of the notches. Why doe the notch depths decrease with frequency? Will the notches be evenly spaced in frequency in this case? Explain. What modifications to the filter would you make to improve the performance over the whole band? Assuming insertion lost is not a problem, what would you do to make the filter performance more symmetric about the center of the frequency band?

<u>The two-tap notch filter in time domain</u>

Set up the sampling scope for time domain transmission as in the case for the long cable measurements. Take the long BNC cable path off the notch filter and replace it with the 3' –6' SMA cable again. Measure the step responses of the Σ and Δ filter configurations and compare with your answers to background question 2. What causes the step edges to be rounded in this case?

Beam impedance of a stripline kicker

Introduction

This experiment introduces the wire method of impedance measurement. This is the primary method measuring the impedance of vacuum chamber components. The characteristics of a simple stripline are measured.

Equipment:

Aluminum model stripline shell
wire impedance rig with reference shell
Network Analyzer + calibration kit
2 cables
2 50Ω SMA loads
1 SMA F-F connector (for Thru calibration)

Experiment:

•Set the NWA frequency range from 300 kHz to 2.3 GHz with 801 points. Do a full 2-port calibration (omit isolation calibration.)

•Set up the impedance rig with the stripline. Terminate the stripline terminals. Measure S21 and store the result in memory.

•Replace the stripline with the reference piece are measure S21. Using the math functions on the plot the ratio $S21_{stripline}/S21_{ref}$. Calculate Z_B at the two maxima where

$$Z_B = 2R_W\left(\frac{S_{21,ref}}{S_{21,object}} - 1\right)$$

•Move terminal two to the upstream pickup. Measure S21 and plot the ratio of the $S21_{upstream}/S21_{ref}$.

-Calculate $Z_p=A_2R_0(S21_{upstream}/S21_{ref}.)$

-Calculate the effective width of the strip. ($Z_p=R_{W_w}/2$__

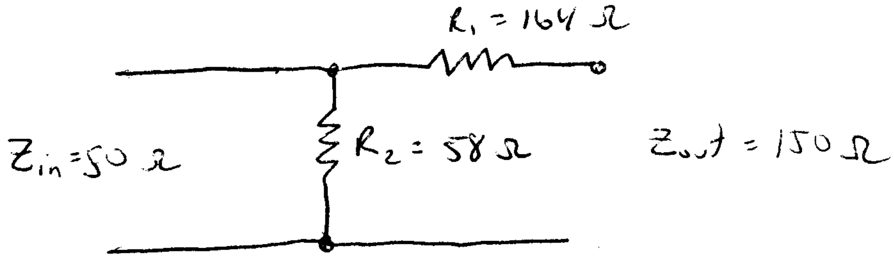
-From the frequency of the zeroes in calculate the effective length of the stripline.

-What happens when the downstream port is unterminated? Explain.

•Measure the pickup impedance at the downstream pickup port. Why is this not zero? What is the fraction of upstream pickup impedance do you observe?

Resistive Matching

In order to make a good measurement of the beam impedance, it is important to match the characteristic impedance of the test setup (usually 50 Ω) to the impedance of the coaxial line formed by the wire and device under test. This is usually done by transforming the impedance using smooth tapers. However, tapers are more elaborate mechanically and so for this we are using resistive matching techniques for simplicity. This experiment uses a T-network to match from the 50 Ω input impedance to the 150 Ω of the wire/pipe coaxial line (=60*ln(a/b)) as shown below. This matches the impedance in both the forward and reverse directions.

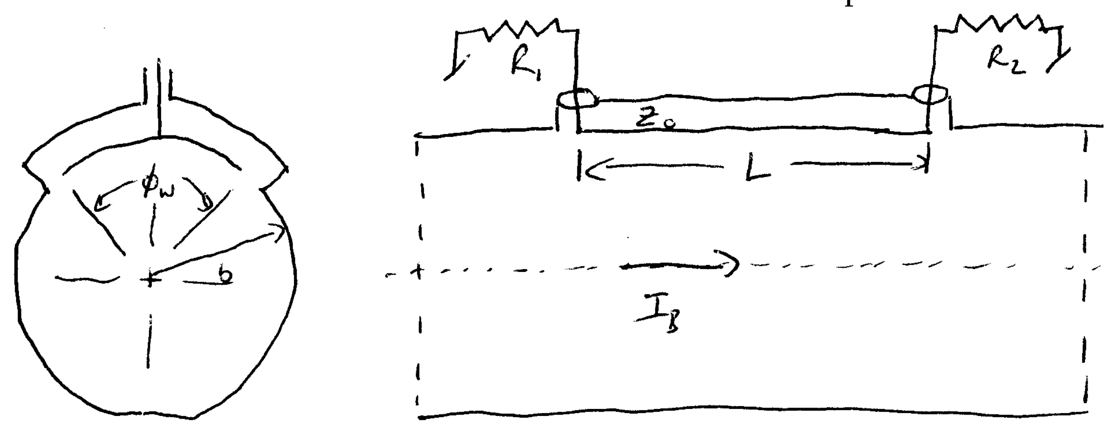


One of the disadvantages of resistive matching is that the signal levels can be considerably attenuated by the resistors, complicating the analysis. However, impedance transformers are somewhat impractical at lower frequencies where resistive matching is ideal. The attenuation for the resistive matching at ports 1 and 2 has been found to be A1=0.14 and A2=0.54.

N.B. It is not necessary to match the impedance in both directions at port 2 since we only need to measure S21 (i.e. port 1 to port 2) to determine the impedance. A single series resistor is sufficient to match at port 2 and provides lower attenuation than the series/shunt arrangement shown above.

Striplines

Consider a stripline as shown below. A relativistic charged particle (v=c) moves along the center axis with current I_b. As the beam passes the upstream end of the stripline, an image current $-gI_b$ is induced on the inner side of the stripline plate, where g is the fraction of a circle subtended by the plate, and travels along with the beam. An equal opposite current is drawn from the load resistance and is distributed on the outer side of the stripline.



Assuming that the upstream terminal impedance R_1 is matched to the stripline impedance Z_0, half of the current exits the upstream ($gI_b/2$) terminal and half travels down the stripline with velocity c. As the beam passes the downstream port, the current on the inner electrode plate, $-gI_b$, combines with the current of the outer side of the plate. A current of $-gI_b/2$ travels back towards the upstream port and exits.

The voltage at the upstream port is given by

$$V_1(t) = \frac{1}{2} gR_0 I_b \left(\delta(t) - \delta(t - 2L/c) \right)$$

where L is the stripline length, and $R_0=R_1=R_2=Z_0$ is the characteristic impedance of the stripline. The voltage as function of frequency is

$$V_1(\omega) = \frac{1}{2} gR_0 I_b \left(1 - e^{-2jkL} \right)$$

where k=_/c. This is shown below.

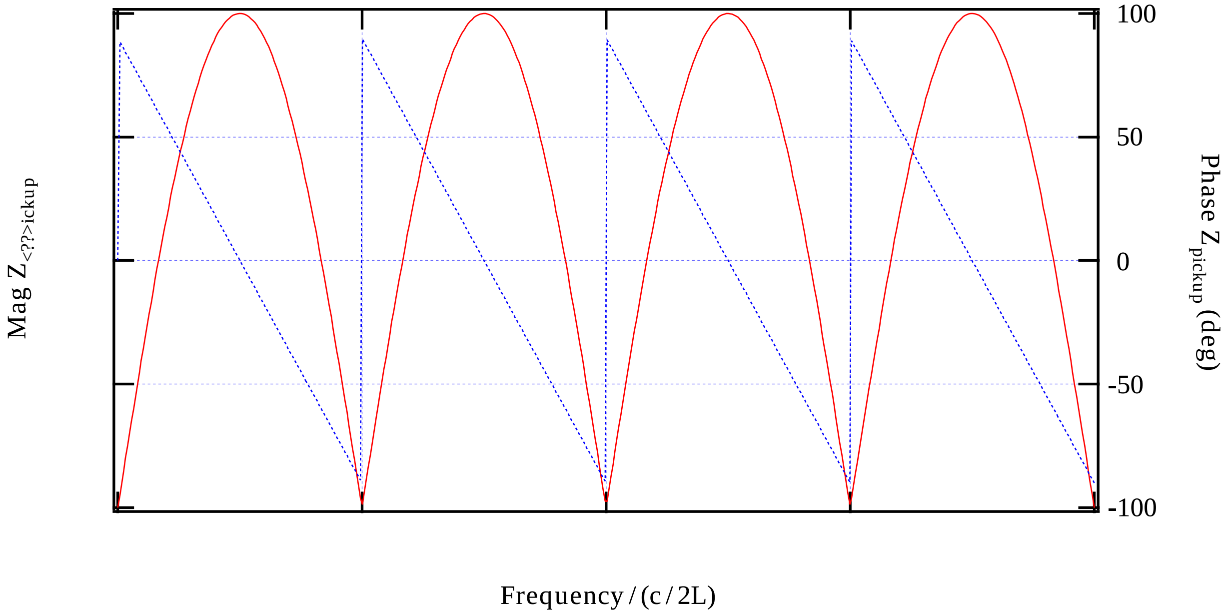


Figure 2. Pickup impedance of a matched stripline. Voltage at upstream port only.

The voltage at the downstream port is zero for ideal matching of Z0 and the output terminal. In practice it is difficult to get perfect matching and thus signals can also be observed at the downstream port.