# Linac Experiment
## — Coupled cell RF cavity structure

USPAS at University of California Santa Barbara, California

June 16 $\sim$ 27, 2003

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# 1 Introduction

Microwave linear accelerator (linac) structures can be considered as a series of single cell resonant cavities coupling together electro-magnetically through couplers (e.g. $\pi/2$ bi-periodic structures), beam iris (electric coupling) or coupling slots on adjacent cavity walls (magnetic coupling). A linac structure can be operated at either a traveling wave (TW) or a standing wave (SW) mode depending on the termination of the end cell cavity. We are now familiar with the single cell RF cavity, which is usually powered by a klystron through a coupler and operated in SW mode. Modes in a single cell cavity are categorized as either transverse magnetic (TM) or transverse electric (TE) modes. The two most important modes which are used in linear accelerators are $TM_{010}$ and $TE_{210}$ modes. The $TM_{010}$ mode has been used in many linacs such as the drift tube linac (DTL), coupled cavity linac (CCL) and side coupled linac (SCL), and the $TE_{210}$ mode is used in radio frequency quadrupole (RFQ) cavity. For beam stability, higher order modes (HOMs) in a single cell cavity may need to be damped by waveguide ports, antennas etc.

For a linac structure, in addition to HOMs which intrinsically exist in single cell cavity, coupled cavity modes (or structure modes) appear in the structure spectrum and are associated with these HOMs and the fundamental mode. This is due to the presence of coupling between single cell cavities. For instance, for a SW linac, the number of the coupled cavity modes depends on how many cells there are in the linac and where the linac is excited (or powered). For a structure consisting of a linear chain of $N$ cells, there will be $N$ modes associated with an excitation mode. Each coupled cavity mode has same field distribution pattern in each cell, but not necessarily the same field strength. A series of such cavity coupled mode is often referred as a band or *passband*. To characterize this property, we use *phase shift* between adjacent cavity cells to name the coupled cavity mode. For example, a 0 mode refers a multi-cell cavity structure (linac) with zero phase shift from cell to cell. Likewise a $\pi$ mode has 180° phase shift from cell to cell. Super-conducting cavities are typically operated in the $\pi$ mode, DTLs are in zero mode, SCLs are in $\frac{\pi}{2}$ mode which means that every other cavity is unexcited. The $\frac{\pi}{2}$ mode is preferred because of its good stability comparing with 0 and $\pi$ mode operation.

# 2 Dispersion

Dispersion is a general description of the passband of all coupled cavity modes, it is typically exhibited by a so called Brillouin diagram (see Figure 2 for example), a picture of $\omega$ vs $k$, where $\omega = 2\pi f$ with $f$ as the resonant frequency of coupled cavity mode, $k = \frac{\omega}{c}$ the propagation constant and $c$ the speed of light.

## 2.1 Phase velocity

Phase velocity is defined as the traveling speed of a wavefront, and can be easily calculated from dispersion diagram by,

$$v_p = \frac{\omega}{k}. \tag{1}$$

For a given waveguide, the phase velocity can be yielded by,

$$v_p = \frac{c}{\sqrt{1 - \left(\frac{f_c}{f}\right)^2}}, \tag{2}$$

where $f_c$ is the cutoff frequency of the waveguide determined by the geometry of the waveguide.

## 2.2 Group velocity

Group velocity is defined as the speed of energy propagation in a structure. It can also be calculated from dispersion diagram,

$$v_g = \frac{d\omega}{dk} \tag{3}$$

For a given waveguide, the group velocity is determined by,

$$v_g = c\sqrt{1 - \left(\frac{f_c}{f}\right)^2}. \tag{4}$$

An universal curve for phase and group velocity in a lossless waveguide is plotted as a function of $f/f_c$ in Figure 1. Phase velocity is infinite at the cutoff frequency and is always greater than the velocity of light; group velocity is zero at cutoff frequency and is always less than the velocity of light. As the frequency increases far beyond cutoff frequency, both phase and group velocities approach the velocity of light. For

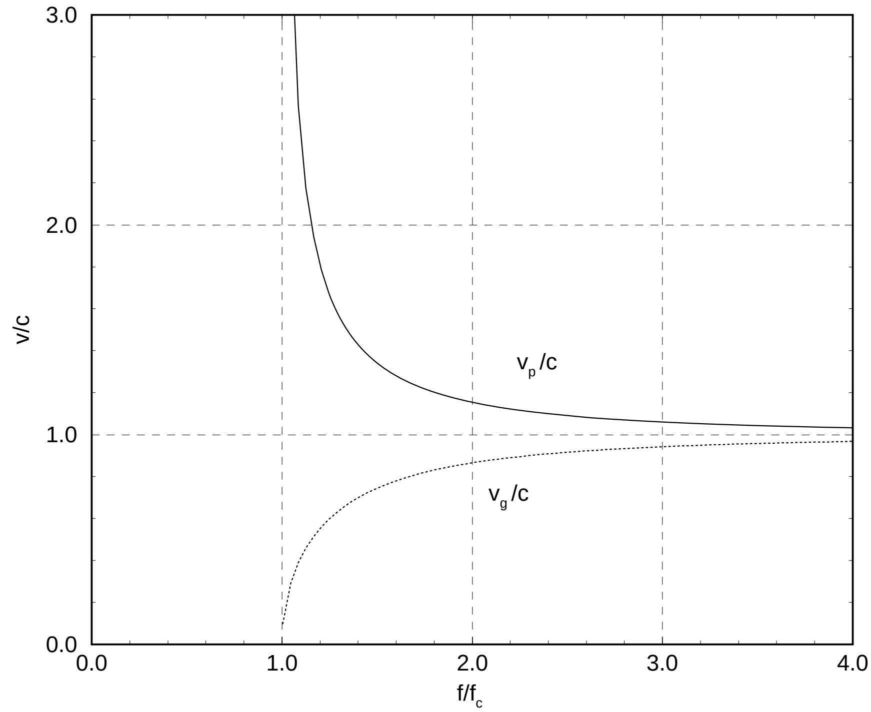


Figure 1: Frequency characteristics of phase and group velocity in waveguides

any linac structures, phase velocity $v_p$ has to be designed to synchronize with the speed of the particles being accelerated. As it has been implied in Figure 1 smooth waveguide can not be used directly as an accelerating structure. The phase velocity in a structure for a linac has to be *slowed down*! A slow wave structure is a necessity of any microwave linac.

For a given structure, if its phase velocity and group velocity are in the same direction (e.g. $v_p \cdot v_g > 0$) it is referred as a *forward wave* structure, otherwise (e.g. $v_p \cdot v_g < 0$) a *backward wave* structure.

## 2.3 Dispersion diagram

We have mentioned the dispersion diagram or relation above, how do we get the dispersion curve. As an example, Figure 2 shows a dispersion diagram calculated for a $\frac{2\pi}{3}$ traveling wave linac operated at 805 MHz. For this particular example, there is no beam iris on axis. Coupling between cells is realized by kidney shaped slots on adjacent cavity walls (*magnetic coupling*). In practice, a dispersion relation can be measured in a real structure. Information of individual cell frequencies and field distribution flatness is reflected in the dispersion. Equivalent circuit theory is often used to help understand the dispersion relation, field flatness and coupling between cells.

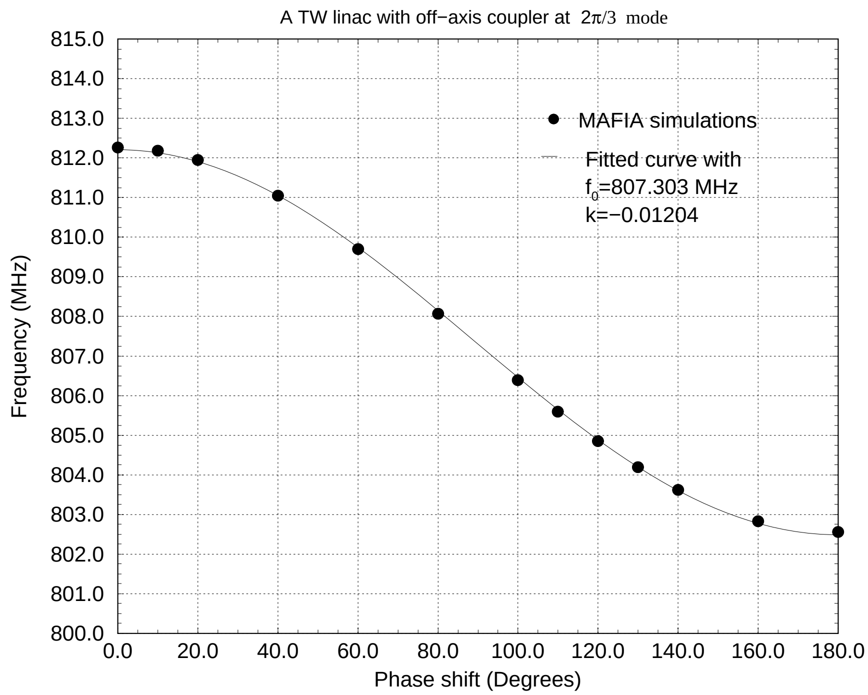


Figure 2: Dispersion relation of a $\frac{2\pi}{3}$ off axis coupled traveling wave structure.

# 3 A Disk Loaded Waveguide

We use disk loaded waveguide as an example to further explore the properties of an accelerating structure in detail. The disk loaded structure has been used for quite a long time as a TW accelerating structure for electron linacs. It consists of a chain of simple pillbox cavities coupled together through open beam irises on axis (see Figure 3). As one of our Microwave Measurement Labs, we have designed and fabricated a 3-cell disk loaded waveguide structure for the students to do measurements, let us call this structure the USPAS linac. Figure 3 is a 3D Model of the USPAS linac. Later we shall apply equivalent circuit theory to understand the structure where each cavity cell is represented by a RCL circuit at its resonant frequency, the coupling between the cavities is described by either mutual conductance (magnetic) or mutual capacitance (electric). Higher order couplings can also be treated readily by the theory.

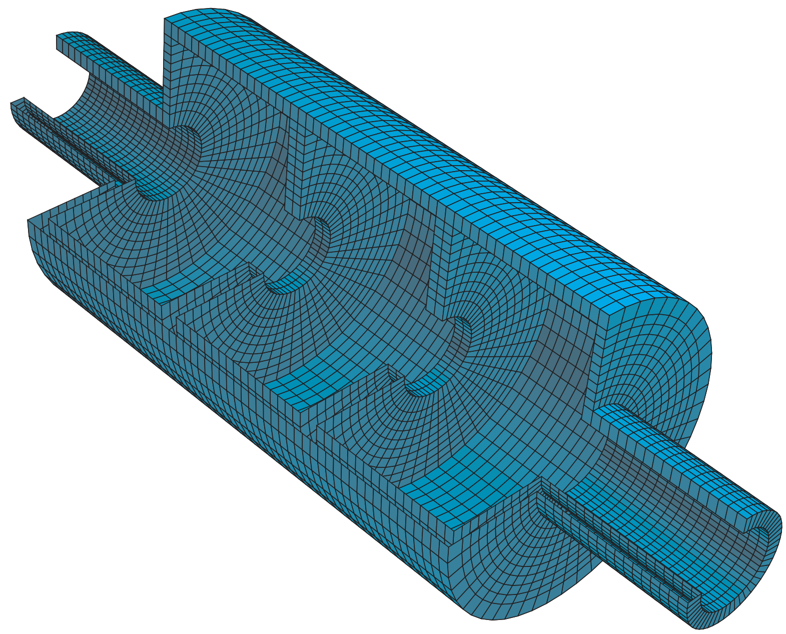

Figure 3: A 3D model (the USPAS linac) of a 3-cell disk loaded waveguide structure with beam pipe.

## 3.1 Coupled cavity modes

It is always hard to visualize what the field distribution in a RF structure may look like. Mathematical expressions help, but can only be obtained for very simple geometries, and are not always straight forward to interpret. With the development and help of available powerful computer codes, such as MAFIA, HFSS, SUPERFISH, URMEL etc., today we are much better equipped for understanding and simulating sophisticated problems which were impossible before, such as HOM damping, beam impedance, wakefields etc. For a demonstration and as well as to help us understand the coupled cavity modes in the USPAS linac, a 3D frequency domain MAFIA simulation has been carried out on the model shown in Figure 3. The simulation results show that three coupled cavity modes $(0, \frac{2\pi}{3}, \pi)$ of $TM_{010}$ exist in the structure. Electric boundary conditions are used at both ends of the beam pipe for the simulation. The simulation shows all the three modes are trapped well within the cavity cells.

### 3.1.1 Electric field distribution of 0 mode

The field distribution shown in Figure 4 is the electric field of the zero mode where phase shift from cell to cell is zero. As it should be and can be seen clearly in Figure 4, the electric field on the beam axis points in one direction. The field strength is indicated by the arrow size. This simulation is carried out in cylindrical coordinates, one quarter of the structure was cut out in order to show the field pattern inside the structure.

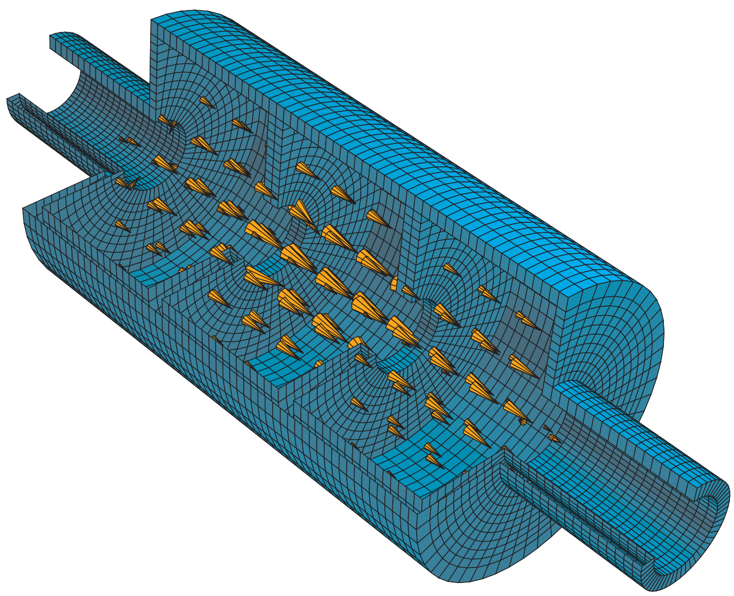

Figure 4: Electric field distribution of the 0 mode of the USPAS linac. The resonant frequency for this mode is calculated to be 2.599 GHz.

### 3.1.2 Electric field distribution of $\frac{\pi}{2}$ mode

Figure 5 gives the electric field distribution of the $\frac{\pi}{2}$ mode where phase shift from cell to cell is 90°. Unlike the 0 mode it is not so obvious to find out what exactly the phase shift is from the figure. However a closed look at the field on the beam axis, shows that the fields starts from zero at the beginning of an end cell and reaches to maximum at the center of the cell, and falls down to zero at the center cavity cell and reaches to another maximum (opposite sign) at the center of the other end cell and back to zero again. The phase shift from the beginning to the end cell seems to complete a $2\pi$ (in three cells), therefore the phase shift from cell to cell look like 120° ($\frac{2\pi}{3}$). In fact this is not true. A more careful look finds that the phase shift between two end cells is 180° (electric fields reach maximum with opposite signs at the center of the cells, and the center cell has no field). The fact that electric fields are forced to zero on the beam axis is due to the beam pipe of the end cells where the coupled fundamental frequencies are at cutoff.

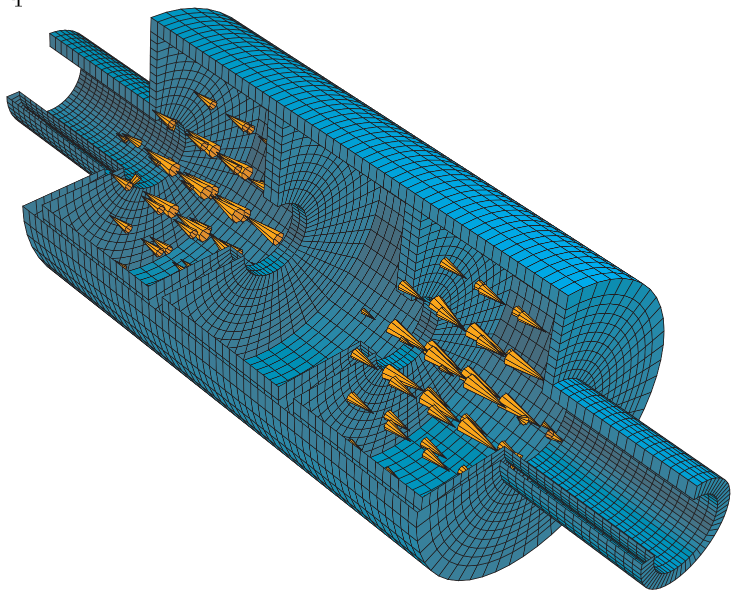

Figure 5: Electric field distribution of the $\frac{\pi}{2}$ mode of the USPAS linac. The calculated resonant frequency of this mode is 2.605 GHz.

### 3.1.3 Electric field distribution of $\pi$ mode

The $\pi$ mode is commonly used in super-conducting linacs. The length of each cavity cell is designed to be $\frac{\lambda}{2}\beta$, where $\lambda$ is free space wavelength at $\pi$ mode frequency and $\beta = \frac{v}{c}$ with $v$ the speed of particles being accelerated. Therefore the phase shift that the particles experience in each cavity cell is exactly $\pi$. As shown in Figure 6, apparently electric field on the beam axis alternates its sign from cell to cell, a clear indication of $\pi$ phase shift. The total phase shift for the USPAS linac is $3\pi$.

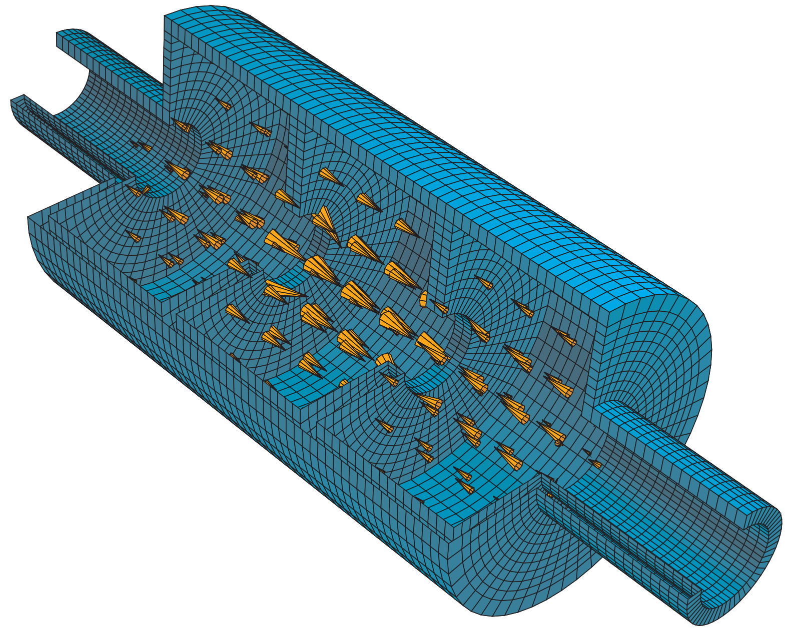

Figure 6: Electric field distribution of the $\pi$ mode of the USPAS linac. The resonant frequency of this mode is 2.611 GHz.

# 4 Equivalent Circuit For Linacs

Resonant cavity cells in a linac structure can be represented equivalently by a RCL circuit, coupling between cells and dispersion relations can be treated using circuit theory. The theory has played an important role in microwave technology. It is also very useful in microwave measurements. Figure 7 is an equivalent circuit for an infinite long magnetically coupled linac structure. Each cell is represented by a RCL circuit, coupling between cells is described by mutual conductance $M$ (or mutual capacitance for electrical coupling). Higher order couplings (defined as coupling other than adjacent cells, for example $M_2$ is a second order coupling) can be treated readily with the theory.

For simplicity, we have assumed this linear chain consists of an infinite number of resonators (cells), and there are no losses in the structure (e.g. $R = 0$). From Kirchoff's law, we have:

$$\left(\frac{M_2}{M}\right) i_{n-2} + i_{n-1} + i_n \left\{\omega^2 L - \frac{1}{C}\right\} \frac{1}{\omega^2 M} + i_{n+1} + \left(\frac{M_2}{M}\right) i_{n-1} = 0, \tag{5}$$

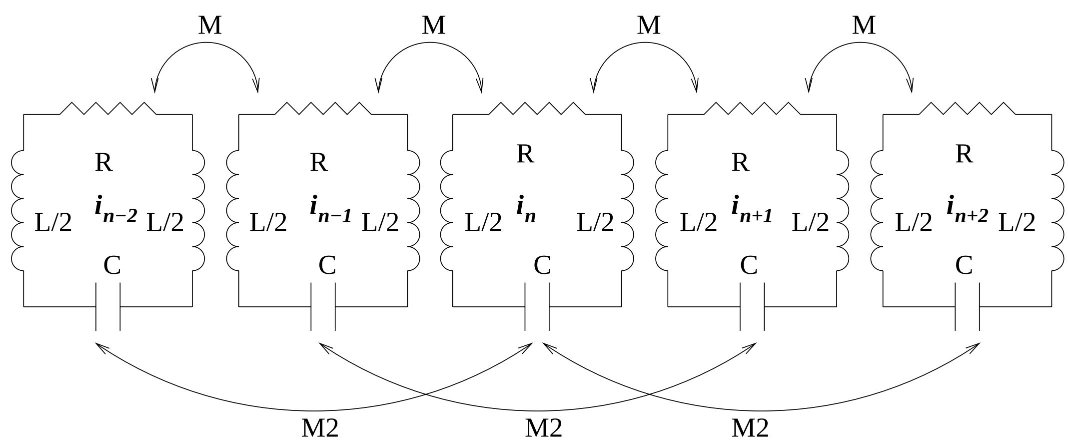


Figure 7: Equivalent circuit for an infinitely long magnetically coupled linac structure. This linac is a single periodic structure meaning that every cavity cell is identical.

The first and second order coupling constants $k_1$, $k_2$ are defined as,

$$k_1 = \frac{2M}{L}, \quad k_2 = \frac{2M_2}{L} \tag{6}$$

respectively. To solve Equation (5), we assume a solution with the following form,

$$i_n = A_n e^{j(nkd+\chi_0)} e^{j\omega t} \tag{7}$$

with $A_n$ the amplitude of the field in cell $n$ and $\phi = kd$ the phase advance per cell and $d$ the length of each cell. After some algebra, we obtain the dispersion relation,

$$\omega^2 = \frac{\omega_0^2}{1 + k_1 \cos\phi + k_2 \cos(2\phi)} \tag{8}$$

with $\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$, $f_0$ as resonant frequency of each cell. Using Equation (8) group velocity is deduced,

$$v_g = \frac{k_1 \sin\phi}{2(1 + k_1 \cos\phi)^{3/2}}(\omega_0 d), \quad \text{assuming } k_2 = 0 \tag{9}$$

The solid line in Figure 2 is fitted using Equation (9) for obtaining $f_0$ and coupling constant $k$.

# 5 Coupling between a cavity and RF source

This section gives one more example of using equivalent circuit theory to treat a coupling problem between a resonant system and a RF source, its equivalent circuit is shown in Figure 10. We first assume that there is no beam loading. The normalized input admittance, $y_{in}$ can be expressed as,

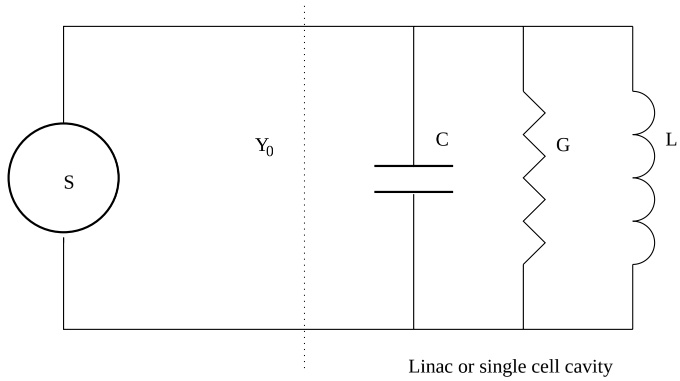


Figure 8: Equivalent circuit for a linac or a single cell cavity coupling to a RF source.

$$
\begin{aligned}
y_{in} &= \frac{G + j\left(\omega C - \frac{1}{\omega L}\right)}{Y_0} && (10)\\
&= \frac{Q_e}{Q_0} + jQ_e\left(\frac{\omega}{\omega_0} - \frac{\omega_0}{\omega}\right), && (11)
\end{aligned}
$$

The reflection coefficient is,

$$
\Gamma = \frac{1 - y_{in}}{1 + y_{in}} \tag{12}
$$

Therefore the input power to the cavity $P$ vs the power $P_0$ from RF source is given by,

$$
\frac{P}{P_0} = 1 - |\Gamma|^2
$$

Now let us consider RF energy buildup process in an RF cavity. According to the energy conservation law,

$$
\begin{aligned}
P &= P_w + \frac{dU}{dt}\\
&= \frac{\omega_0 U}{Q_0} + \frac{dU}{dt}, && (13)
\end{aligned}
$$

where $P_w$: RF power losses on cavity wall; $U$: stored energy in the cavity. During the energy buildup process, we may define a transient quality factor,

$$
Q_0' = \frac{\omega_0 U}{P} = \frac{\omega_0 U}{P_w + \frac{dU}{dt}} \tag{14}
$$

After some algebra, we have an equation for $U(t)$,

$$\frac{dU}{dt} + \frac{\omega_0 U}{Q_L} - 2\sqrt{\frac{\omega_0 U P_0}{Q_e}} = 0 \tag{15}$$

with $Q_L = \frac{Q_0}{1+\beta_c}$, $\beta_c$ the coupling constant. Solving Equation (15),

$$U(t) = \left(\frac{Q_0}{\omega}\right) \frac{4\beta_c}{(1+\beta_c)^2} P_0 \left\{1 - e^{-\frac{t}{\tau_L}}\right\}^2, \tag{16}$$

with $\tau_L = \frac{2Q_0}{(1+\beta_c)\omega}$

$$P_w = \frac{4\beta_c}{(1+\beta_c)^2} P_0 \left\{1 - e^{-\frac{t}{\tau_L}}\right\}^2 \tag{17}$$

# 6 Linac Experiment

The USPAS linac is a disk loaded waveguide consisting of three pillbox cavities. Coupling between cavities is realized through open irises on beam axis. The linac is made from aluminum and resonant frequency is around 2.6 GHz. Each cavity has its own tunner and probe (a screw and a SMA connector on the cavity side wall).

Two rigid SMA coaxial cables are provided for exciting and measuring cavity frequency through beam iris, they can also be used to detune the cavities.

## 6.1 Frequency of individual cells

Measure cavity frequency through $S_{11}$ measurement. Set up NWA's center frequency around 2.6 GHz with frequency span of 50 MHz, connect a SMA cable to the probe on cavity being measured. Please note that adjacent cavities need to be detuned in order to measure the cavity frequency correctly. Think why? and how to detune other cavities?

## 6.2 Dispersion measurement

- Measure dispersion through $S_{11}$ measurement. Using NWA and setting the frequency span to 50 MHz and center frequency to 2.61 GHz.
  - connect the SMA cable to probes on end cell cavity, how many peaks do you see on the screen? and measure the frequency of each peak.
  - connect the SMA cable to the probe on the center cell, how many peaks do you see now? and measure the frequency of each peak.
  - Compare frequency measurement between above two measurements, what do you find? Explain your findings.
  - Identify the mode of each peak.
  - Plot measured frequencies against modes.
- Repeat the dispersion measurement through $S_{21}$ and measure $Q$ value of each mode.
- Use tuners in each cell and tune them to have the same resonant frequency.

- Repeat the dispersion measurement and compare to the previously measured dispersion, do you see any difference?

## 6.3 Field distribution measurement

Set up bead pull measurement, measure electric field amplitude $E$ at center of each cell ($E \propto \sqrt{\Delta f}$) (hint: find maximum frequency shift of each cell) for each mode.

## 6.4 Field flatness*

From above field distribution measurement, do you get the same amplitude in each cell? If not, why? Can you tune each cell's frequency and flatten the field distribution (do your best)? Record cell frequencies when you flatten the field.