Generating THz in Storage Rings. Part II

Fernando Sannibale

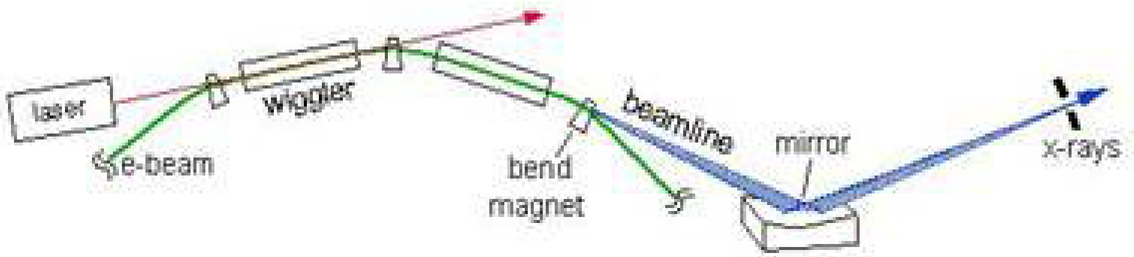
Contents

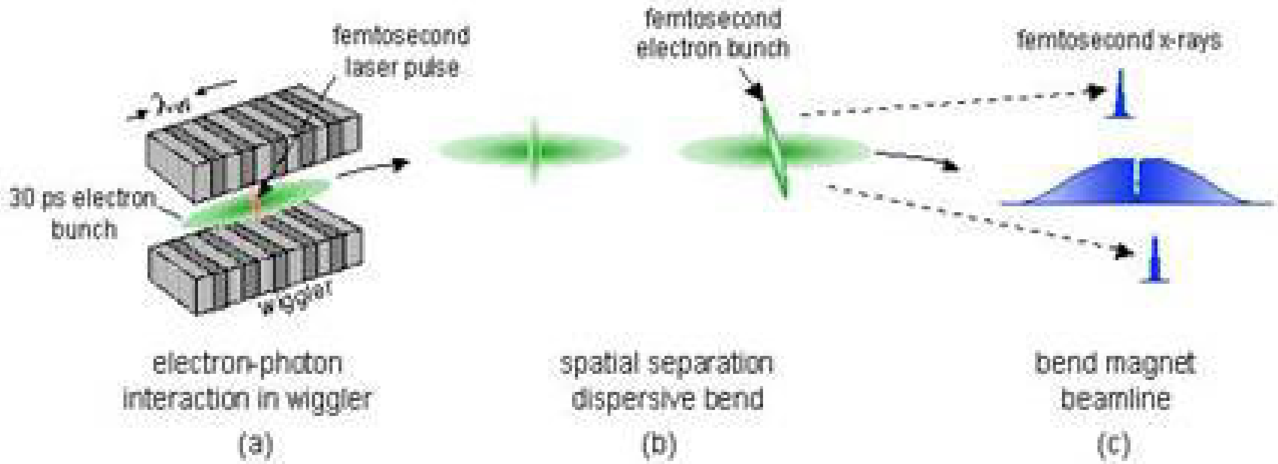
- CSR from "femtoslicing".

- CSR by seeding the microbunching instability.

The `Femtoslicing' Experiment

Beamline optimized for the generation of femtosecond x-ray pulses





- A. A. Zholents, M. S. Zolotorev, Phys. Rev. Lett. **76**, 912, (1996)

In operation at the ALS since 1999, and in the last few years also at BESSY II, SLS and UVSOR, …

Energy Modulation

$$f(z,\delta)=\frac{1}{2^{1/2}\pi^{3/2}\sigma_\delta}\int_{-\frac{E_M}{E_0}\exp\left[-(z-z_L)^2/2\sigma_L^2\right]}^{\frac{E_M}{E_0}\exp\left[-(z-z_L)^2/2\sigma_L^2\right]} d\tilde{\delta}\,\frac{\exp\left[-\left(\delta-\tilde{\delta}\right)^2/2\sigma_\delta^2\right]}{\sqrt{\left(\frac{E_M}{E_0}\right)^2\exp\left[-(z-z_L)^2/\sigma_L^2\right]-\tilde{\delta}^2}}$$

$$E_M^2\cong 4\pi\alpha E_L\hbar\omega_L\frac{K^2/2}{1+K^2/2}\frac{\Delta\omega_L}{\Delta\omega_W}$$

$E_0 \equiv beam\ energy$

$E_L \equiv laser\ pulse\ energy$

$E_M \equiv energy\ modulation$

$K \equiv wiggler\ parameter$

$\omega_L/2\pi \equiv laser\ frequency$

$\Delta\omega_L \equiv laser\ bandwidth$

$\Delta\omega_W \equiv wiggler\ radiation\ bandwidth$

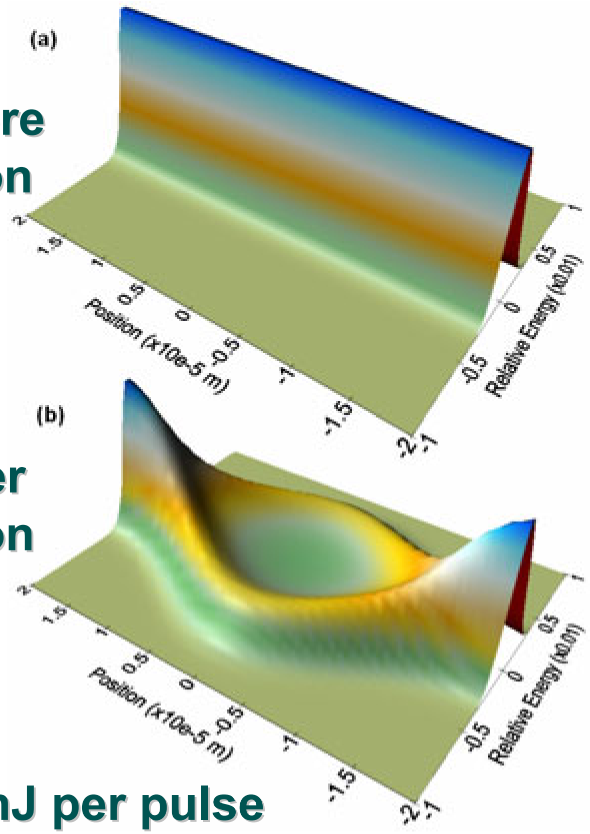
$\delta=(E-E_0)/E_0 \equiv beam\ energy$

$\alpha=e^2/4\pi\varepsilon_0\hbar c\cong 1/137$

$z \equiv particle\ longitudinal\ position$

Right before modulation

Right after modulation



For a few GeV electron beam, laser pulses with several mJ per pulse are required. This limits the max rep-rate to the order of few KHz

From Energy to Density Modulation

$$\begin{pmatrix} x \\ x' \\ y \\ y' \\ c\tau \\ \delta \end{pmatrix} = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} \\ R_{21} & R_{22} & R_{23} & R_{24} & R_{25} & R_{26} \\ R_{31} & R_{32} & R_{33} & R_{34} & R_{35} & R_{36} \\ R_{41} & R_{42} & R_{43} & R_{44} & R_{45} & R_{46} \\ R_{51} & R_{52} & R_{53} & R_{54} & R_{55} & R_{56} \\ R_{61} & R_{62} & R_{63} & R_{64} & R_{65} & R_{66} \end{pmatrix} \begin{pmatrix} x_0 \\ x'_0 \\ y_0 \\ y'_0 \\ c\tau_0 \\ \delta_0 \end{pmatrix}$$

$$c(\tau - \tau_0) = R_{51}x_0 + R_{52}x'_0 + R_{53}y_0 + R_{54}y'_0 + R_{56}\delta \quad R_{55} = 1$$

For an off-energy particle initially on the reference orbit and for: $\eta_y = \eta'_y = 0$

$$c\Delta\tau = R_{51}\eta_{x0}\delta_0 + R_{52}\eta'_{x0}\delta_0 + R_{56}\delta_0 = \left(R_{51}\eta_{x0} + R_{52}\eta'_{x0} + R_{56}\right)\delta_0$$

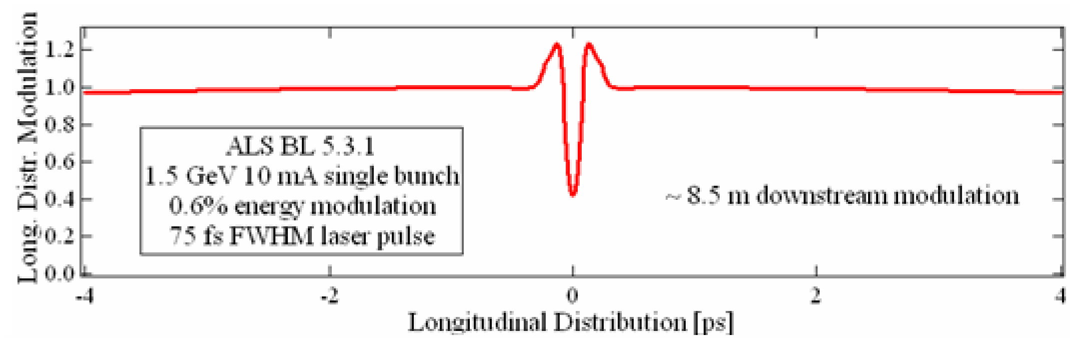
$$\Rightarrow \frac{c\Delta\tau}{L} = \frac{R_{51}\eta_{x0} + R_{52}\eta'_{x0} + R_{56}}{L}\delta_0$$

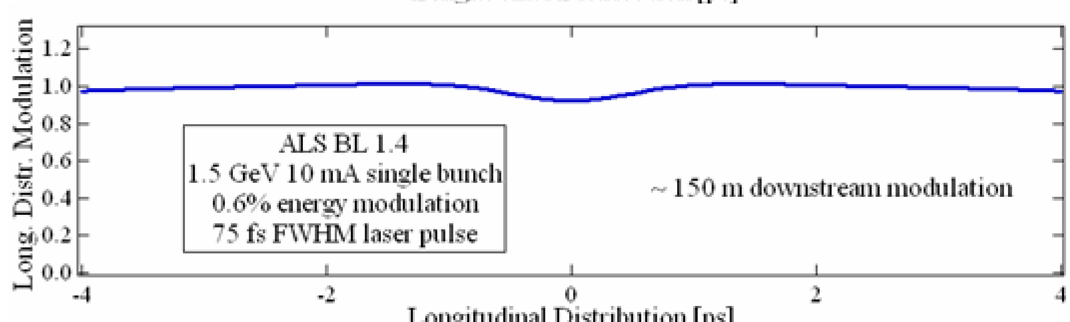
But by definition: $\dfrac{\Delta L}{L} = \alpha_C \delta$

$$\boxed{\alpha_C = \frac{R_{51}\eta_{x0} + R_{52}\eta'_{x0} + R_{56}}{L}}$$

Because of this dispersion term the laser induced energy modulation translates into a longitudinal density modulation.

The density modulation quickly smears out.
(In about one turn for the ALS case where $\alpha_C = 1.37 \times 10^{-3}$)





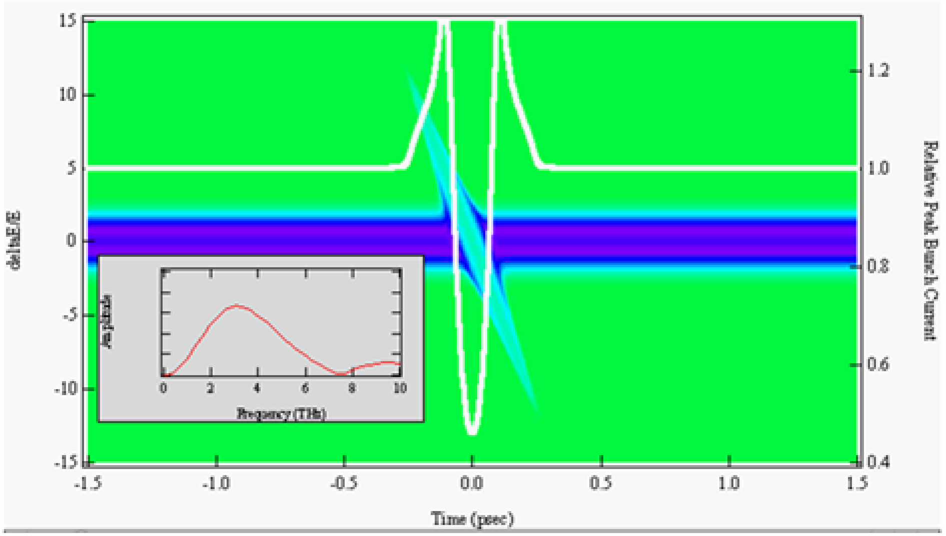
From Density Modulation to CSR Emission

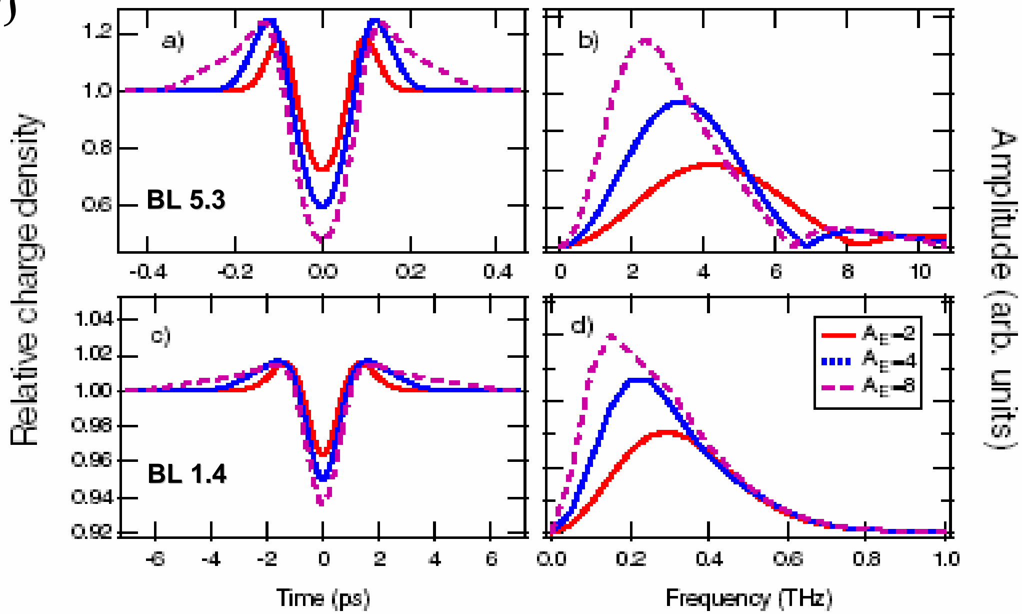
The characteristic length of these density modulations is about the laser pulse length (~100 fs) after the energy modulation, becomes of the order of the ps after ~ a turn and it is completely absorbed by the bunch in the next few turns.

$$\frac{dP}{d\omega} = \frac{dp}{d\omega}\left\{N\left[1 - g(\omega)\right] + N^2 g(\omega)\right\}$$

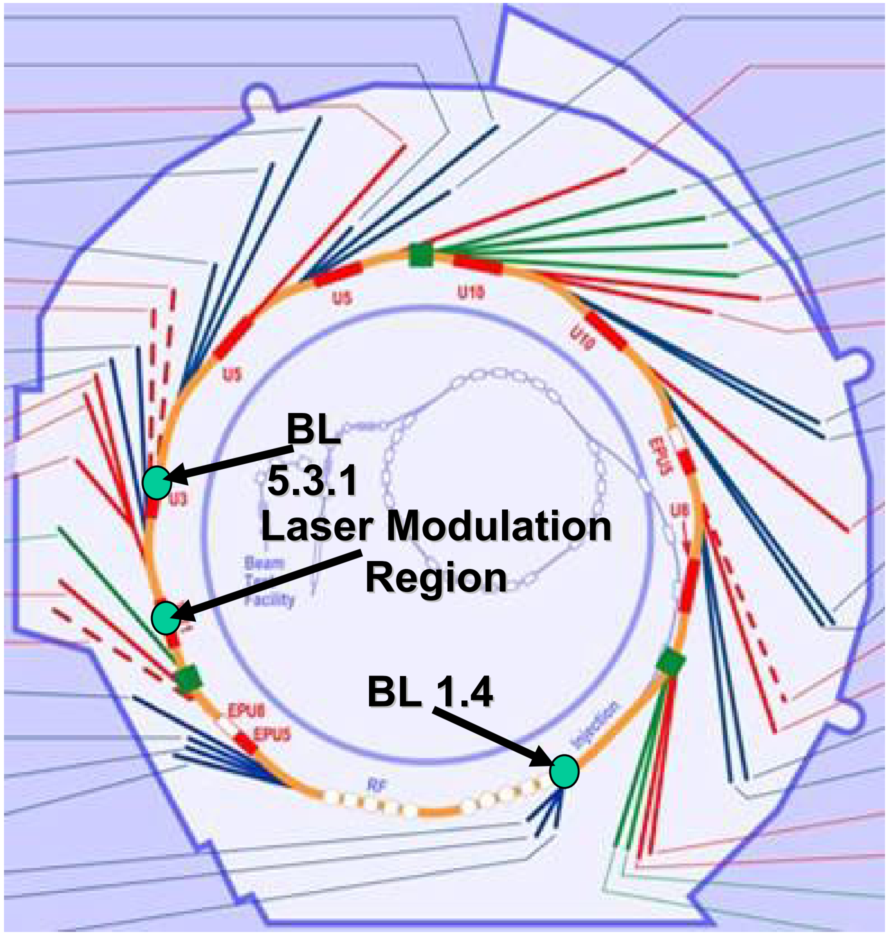
$$g(\omega) = \left|\int_{-\infty}^{\infty} dz\, S(z) e^{i\omega z/c}\right|^2$$

Such density modulation radiate intense CSR in the THz frequency range





"Femtoslicing CSR" Measurements at the ALS

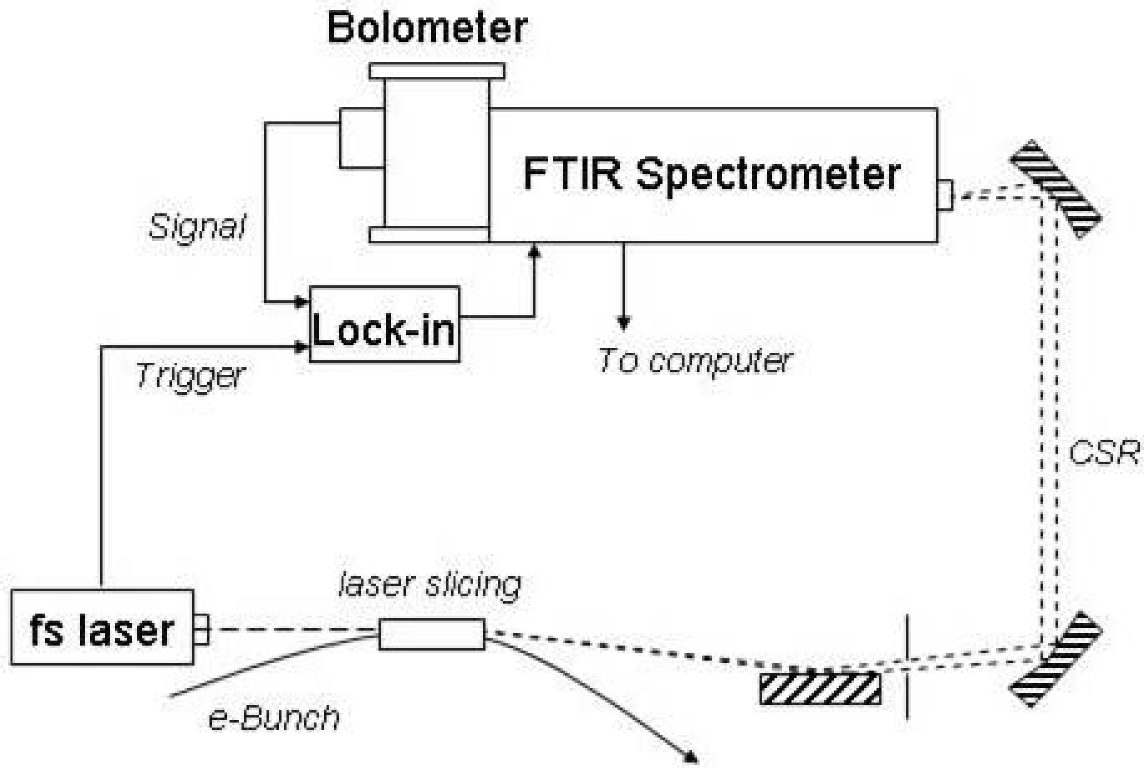


Parameter	BL 5.3.1	BL 1.4
Modulation-observation point distance [m]	8.4	149.5
Energy [GeV]	up to 1.9	
Current per bunch [mA]	up to 30	
Ring length [m]	196.7	
Dipole bending radius [m]	4.957	
Nominal momentum compaction	0.00137	
Relative energy spread	0.001	
Relative energy modulation	0.006	
Laser pulse duration FWHM [fs]	75	
Laser repetition rate [pps]	1000	

BL 5.3.1: provisional THz Port (~ 3 x 3 mrad^2 acceptance)

BL 1.4: ALS IR beamline (~ 40 x 10 mrad^2 acceptance)

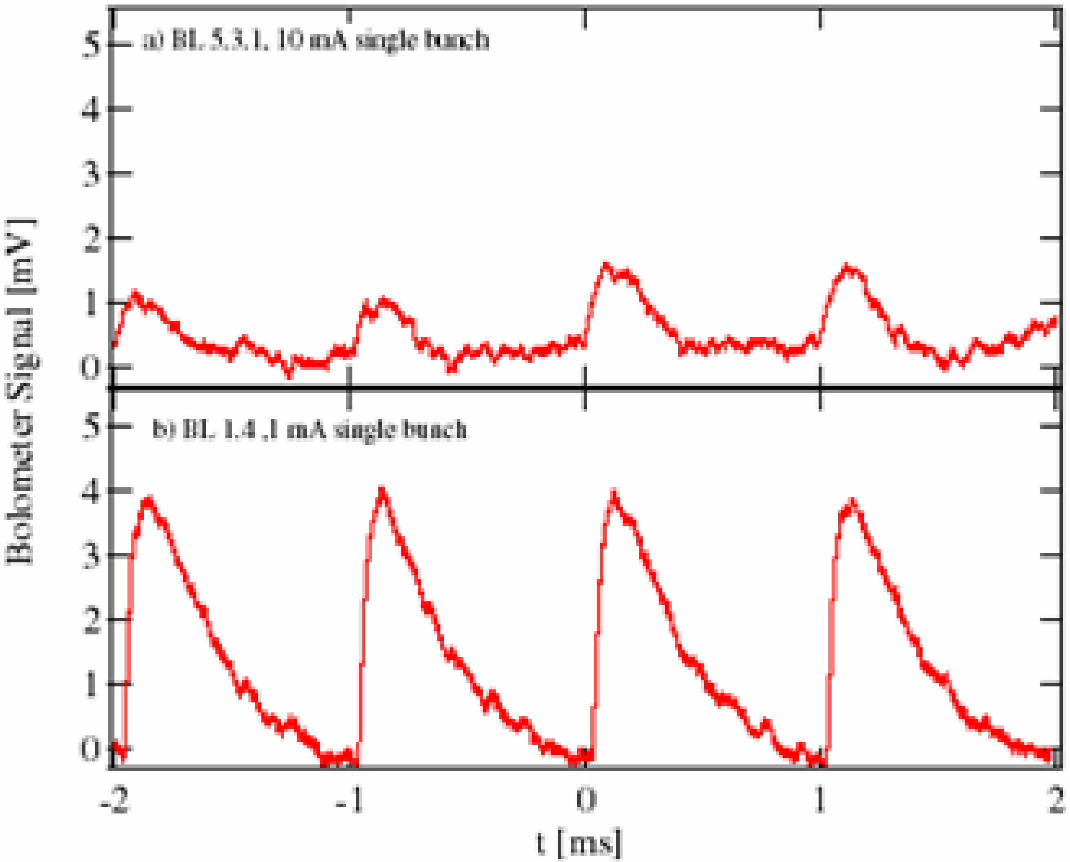
Experimental Setup



- **FTIR spectrometer: IFS 66V, *Bruker*.**
- **Bolometer: 4.3 K He, HD3, *Infrared Laboratories Inc*.**
  BW ~ 1 KHz - Frequency response from ~ 5 – 200 cm^{-1}
- **Lock-in amplifier: SR844, *SRS***

Time Domain Measurements

THz CSR pulses during slicing were indeed measured at the ALS. The figure shows the bolometer signal measured at the two different beamlines in the ring.



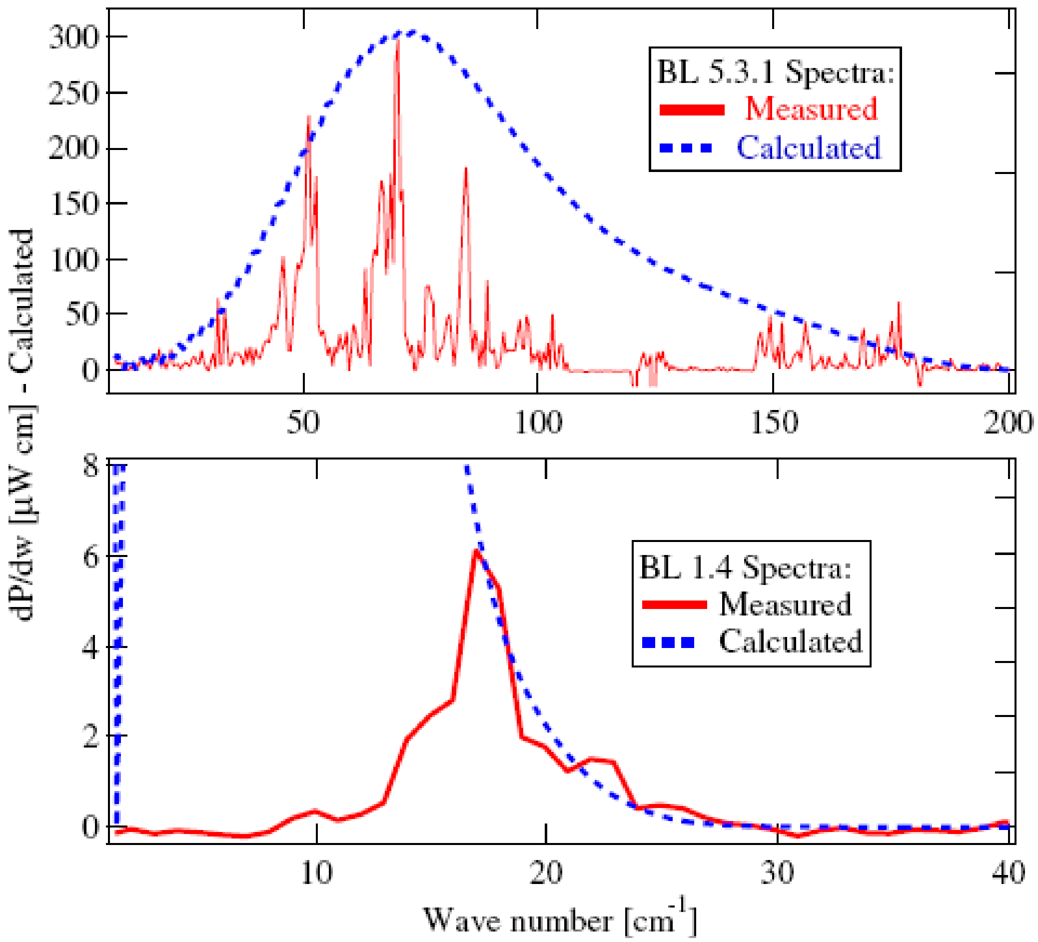
(J.M.Byrd *et al*., PRL **96**, 164801, 2006.)

The 1 kHz structure due to the slicing laser repetition rate is clearly visible.

By switching off the laser the signal disappears

The CSR signal is now one of the main diagnostics for the tune-up for the slicing experiment.

Frequency Domain Measurements



The agreement with calculations is reasonable, but the quality of the measured spectra is poor...

Similar results (but with better quality spectra!) have been obtained at BESSY II:

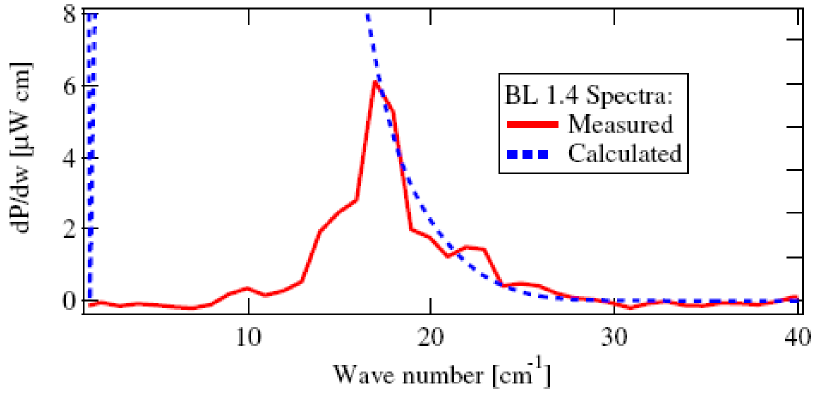
K. Holldack *et al.*, PRL **96**, 054801 (2006) and K. Holldack *et al.*, PRST-AB **8**, 040704 (2005).

Comments on the Measured Spectra

BL 1.4 Spectra:

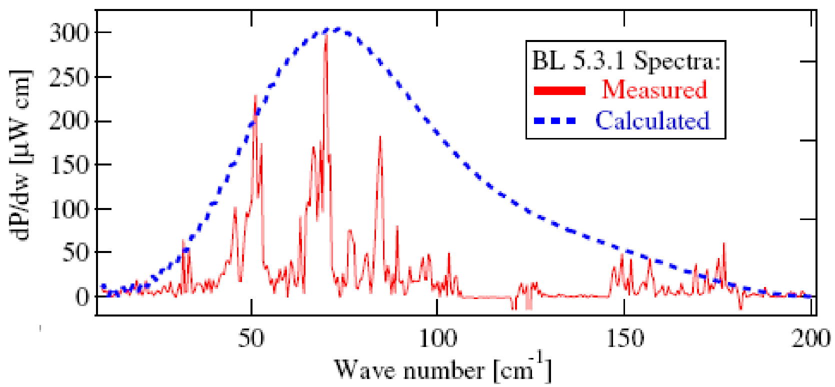
- Instrumentation bandwidth
- Vacuum chamber cutoff

Only the high frequency part of the spectrum can be measured

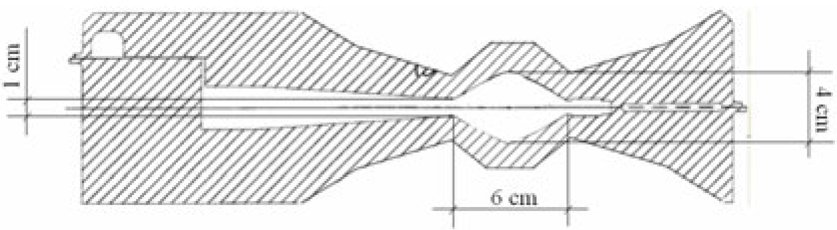


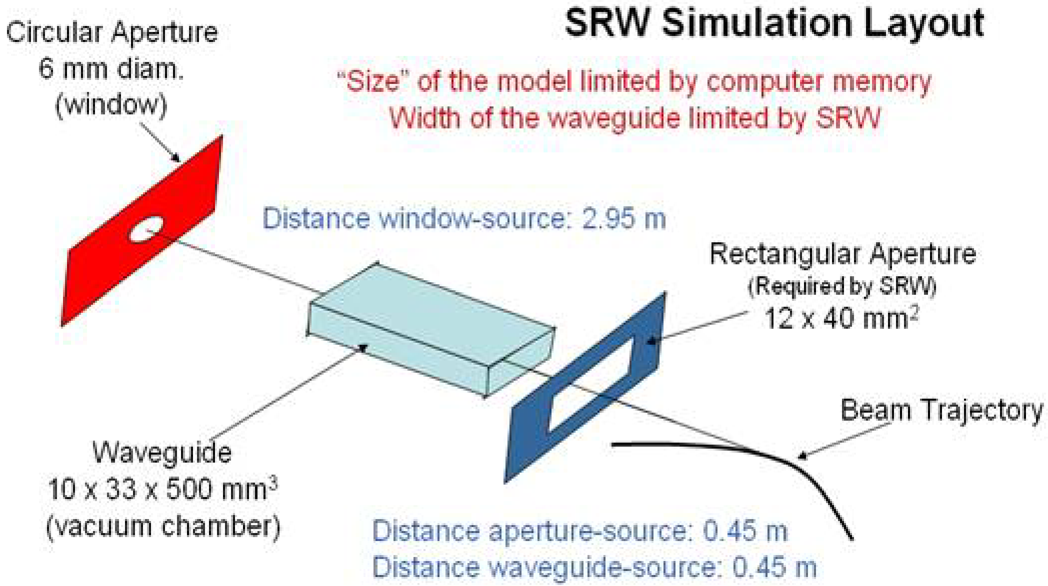
BL 5.3.1 Spectra:

- Fine structure due to water absorption.
- Larger structure due to interference with the vacuum chamber ('Waveguide effect').



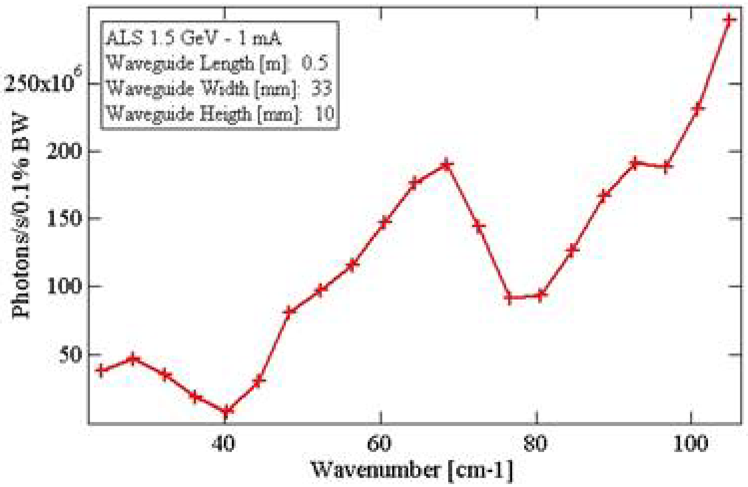
The ‘Waveguide’ Effect

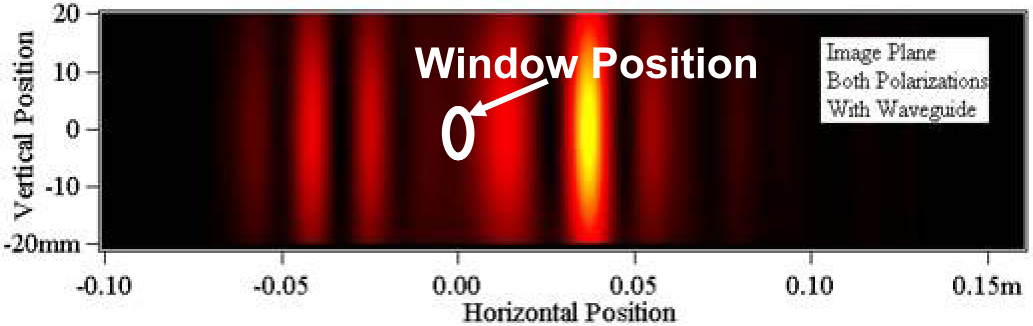




Qualitative agreement.

Actual geometry difficult to simulate.



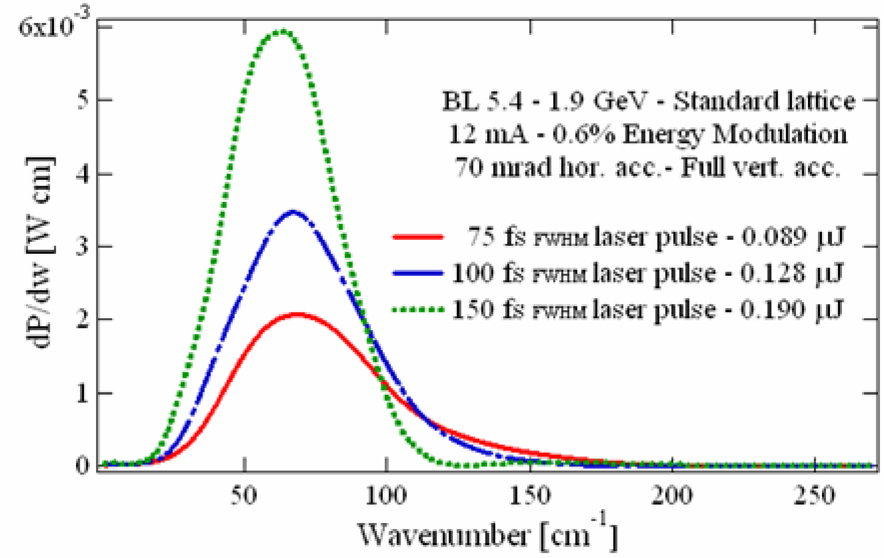


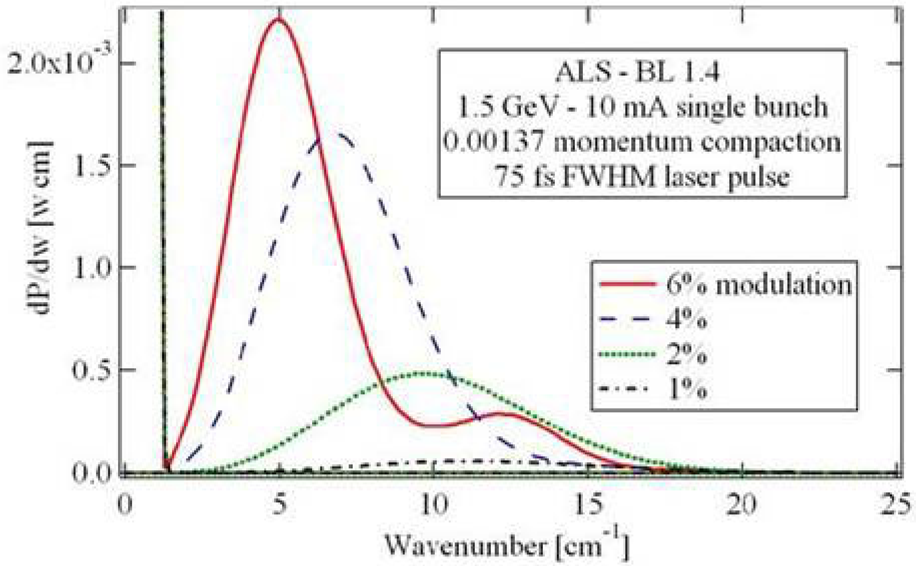
Wavefront for photons with wavenumber of 50 cm^{-1} at the window position

Controlling the CSR Spectrum

The CSR spectrum and intensity can be controlled by acting on slicing parameters such as:

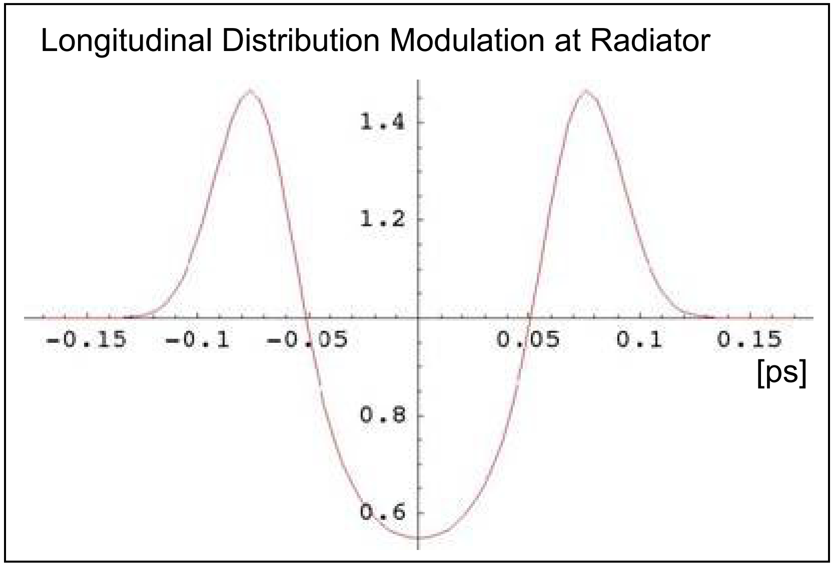
- **modulating laser pulse width**
- **modulating laser intensity**
- **bunch current**
- **storage ring momentum compaction**
- **distance modulator-radiator**

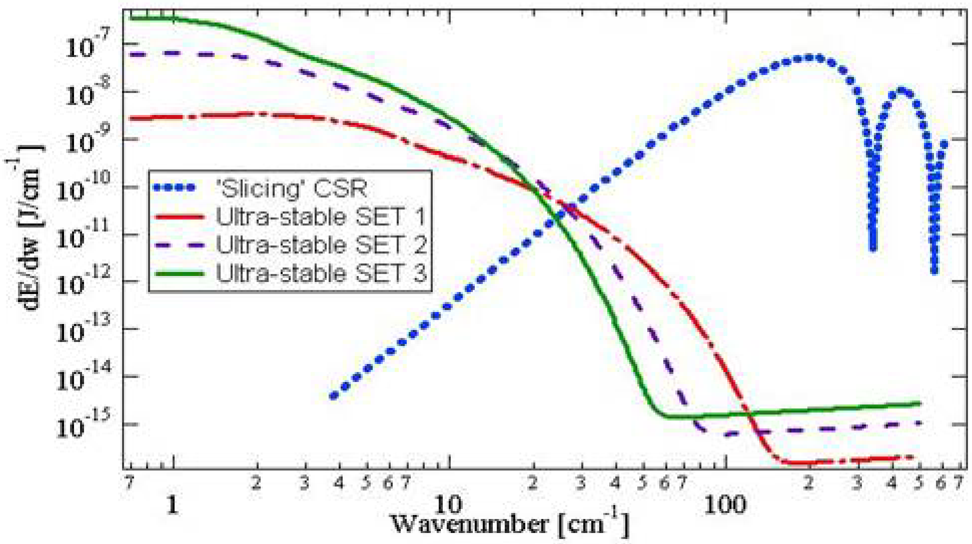




Optimized CSR by Slicing (CIRCE)

- Laser Modulation: 6 times the energy spread
- Laser pulse width: 50 fs FWHM
- Distance modulator-radiator: 2.5 m
- Current per bunch: 10 mA
- Horizontal Acceptance 100 mrad (single mode)





- Energy per pulse: 8.5 μJ
- Electric field ~ 1 MV/cm
- Rep. rate: 10 - 100 kHz
- Pulse shaping capability

"Multicolor" Experiments Capability

In the described femtoslicing experiment, several mutually synchronous photon beams with very different wavelengths are simultaneously available:

- x-ray pulses with ~ 100 fs length
- Near-IR or visible ~ 100 fs pulse from the slicing laser
- THz CSR synchrotron radiation pulse with transform limited length

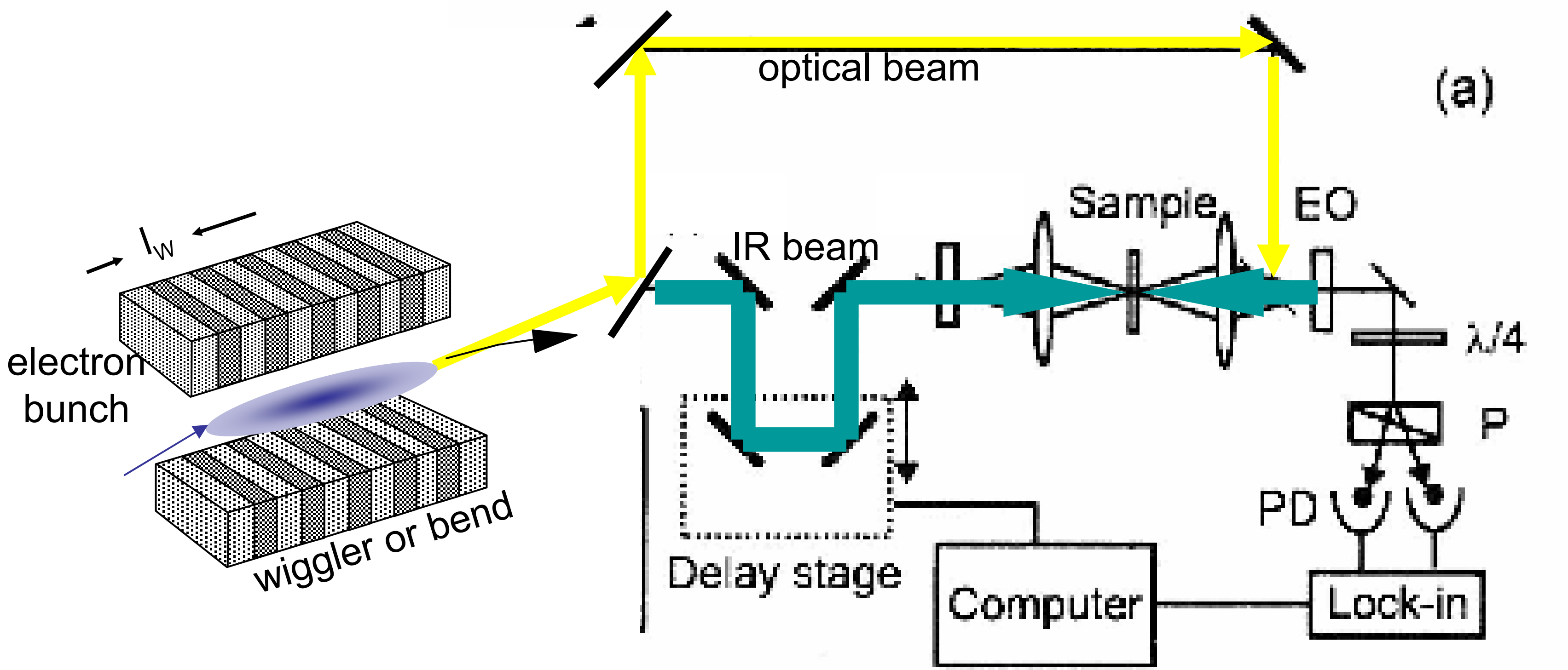
This opens the possibility for many interesting combinations of "pump and probe" experiments where one of the beams is used for exciting the sample while another is used for measuring its characteristics during the excitation transient.

By varying the delay between the pulses, one can reconstruct the whole sample response with resolution of the order of ~ 100 fs.

Example of Pump and Probe Experiment

Self-synchronized Electro-optic sampling

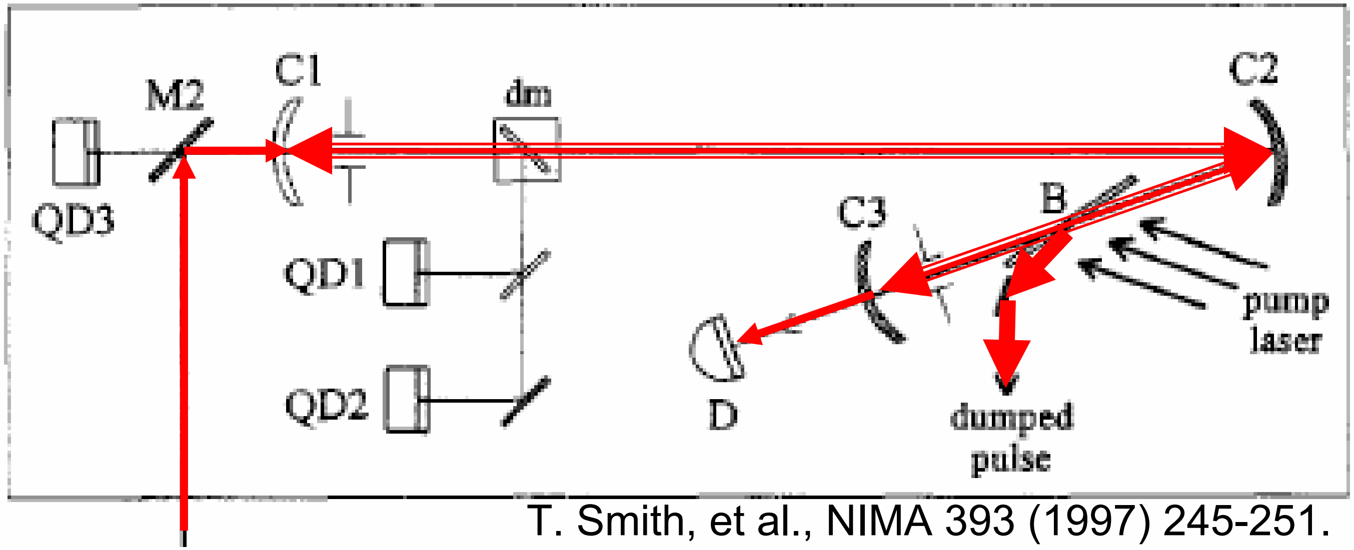
- provides functionality of benchtop setup w/1.5 GHz rep-rate
- use inherent synchronization of optical and THz beams
- optical source can be dipole (very weak) or undulator
- self-mixing techniques also possible.



Pulse Stacking

Experiments interested in exciting samples in the nonlinear regime require electric fields on the sample >~ 1 MV/cm. Such fields can be obtained by focusing photon pulses of energy >~ 10 μJ down to their diffraction limited beam size.

In our case, because the input pulses are coherent, it is possible in principle to resonate the signals to gain high pulse power levels at a reduced repetition rate.



T. Smith, et al., NIMA 393 (1997) 245-251.

Input CSR pulses

Peak power limited by cavity Q and phase stability of pulses

Tailoring the Shape of the THz Pulse

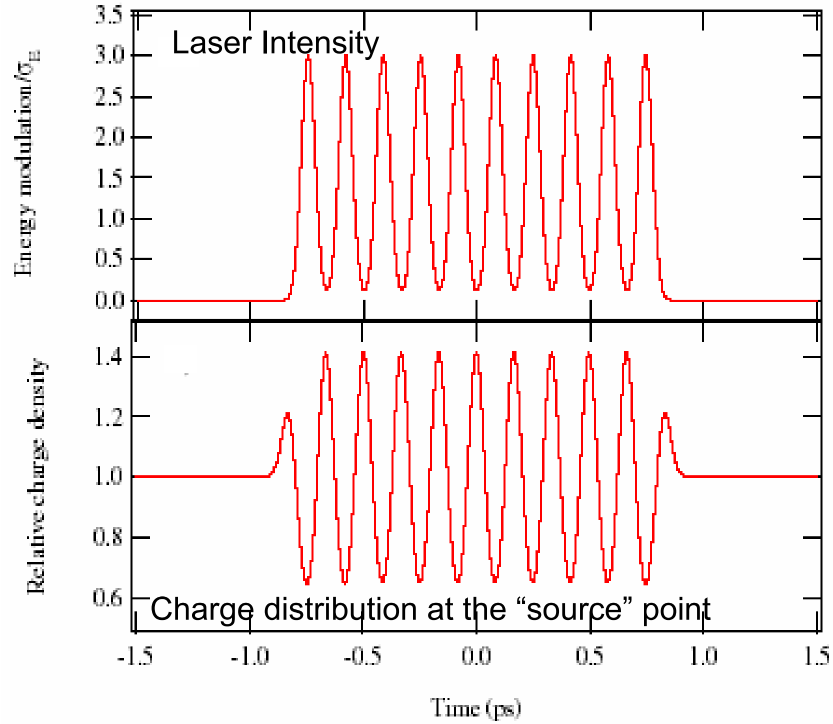
One additional interesting possibility of this scheme is the ability of tailoring the electric field of a terahertz pulse by an appropriate shaping of the slicing laser pulse.

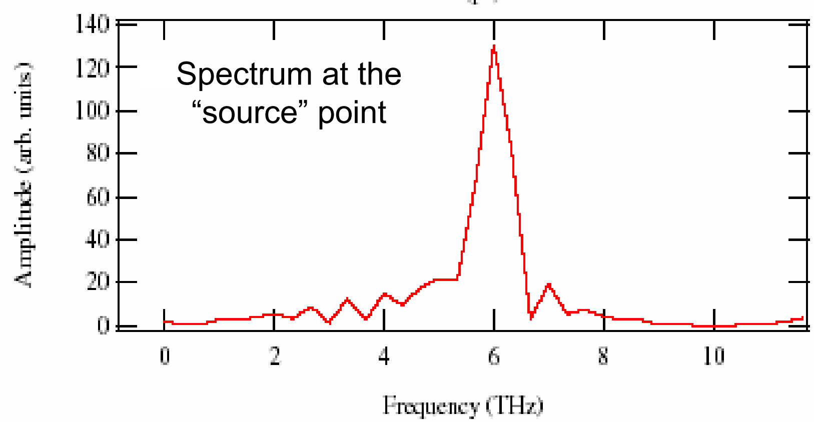
The example shows how by using a train of laser pulses instead of one single pulse one can concentrate the CSR power within a narrow bandwidth.

The number of pulses defines the bandwidth while the distance between pulses defines the central frequency of the peak.

In principle by this technique, arbitrary spectrum shapes can be obtained

An example of application that could benefit from this capability is the control of complex chemical reactions where the shape of the exciting radiation is dynamically adjusted for optimizing the reaction.





(J.M.Byrd *et al*., PRL **96**, 164801, 2006.)

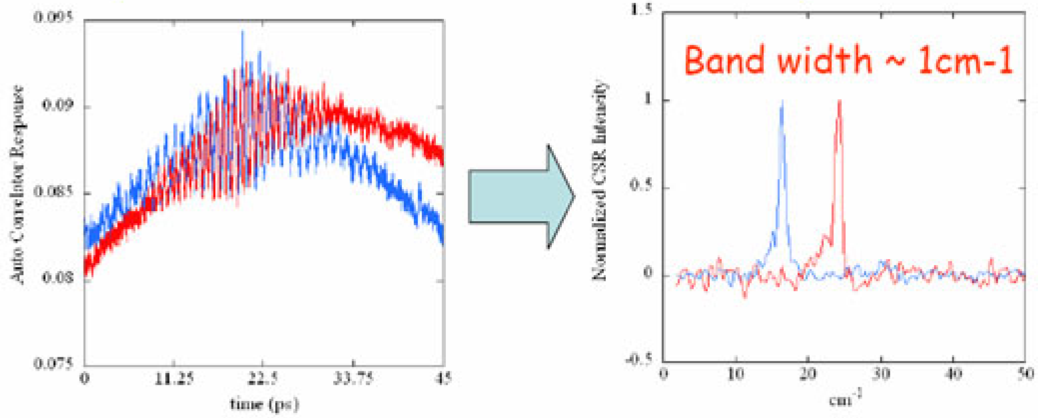
Tailoring the THz Pulse is Really Possible!

UVSOR II

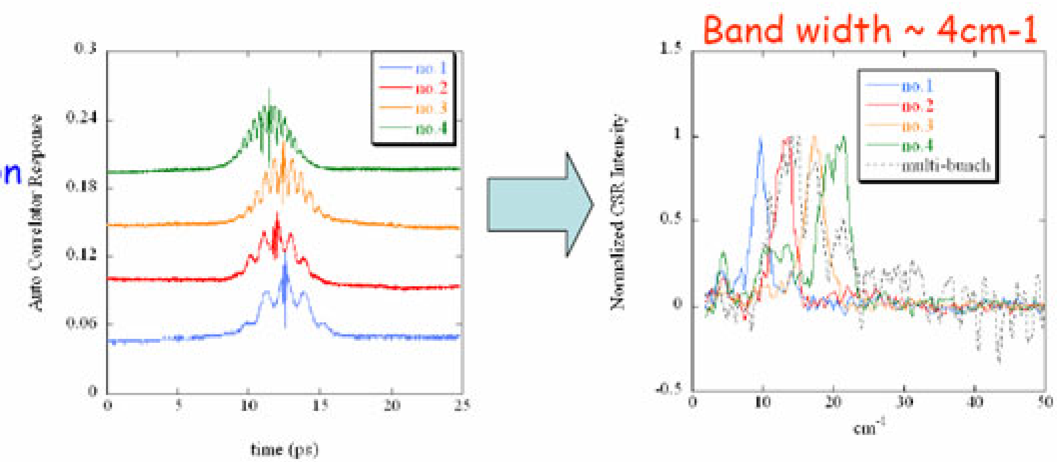
Output from auto-correlator

CSR spectrum

Laser pulse duration
60ps
(much beating)



Laser pulse duration
1~2 ps
(less beating)

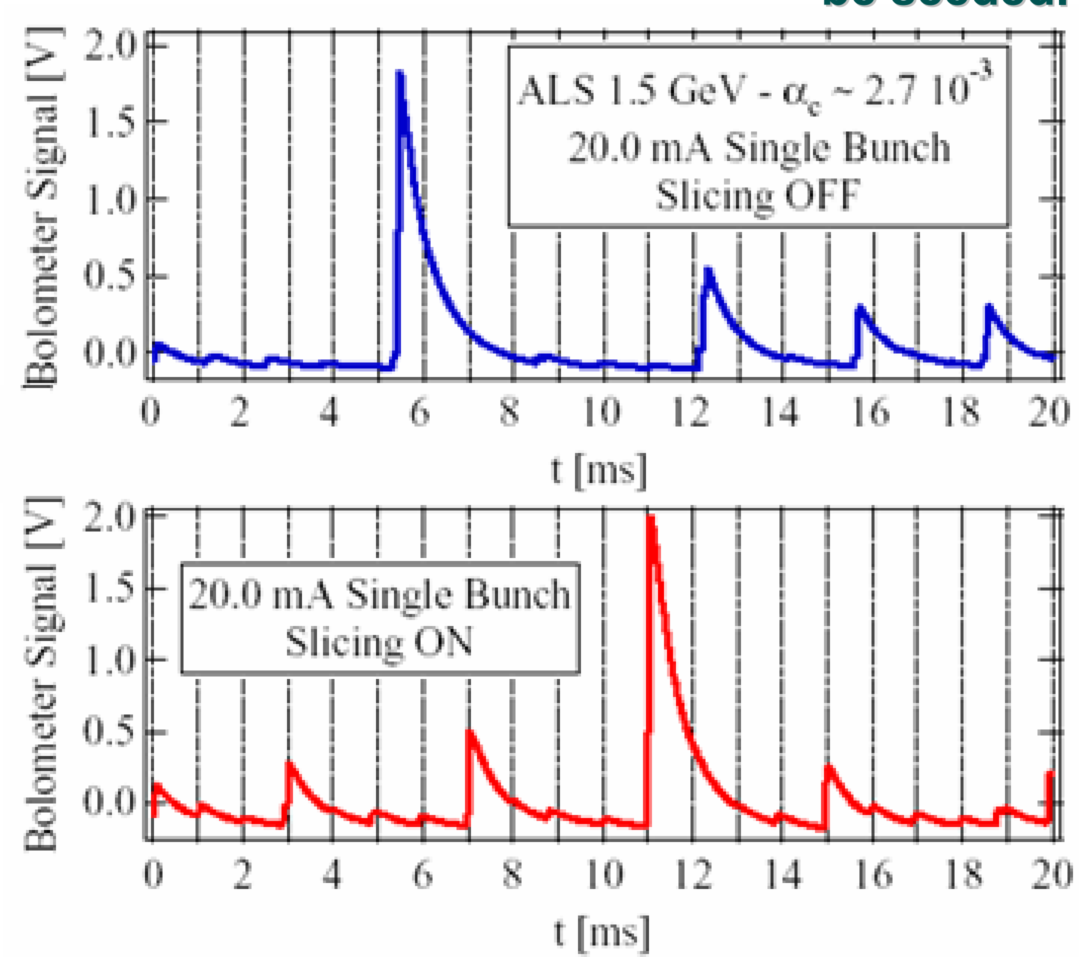


Institute for Molecular Science

A. Mochihashi *et al.*, UVSOR Workshop on THz CSR (September 2007)

Laser Seeding of the MBI

During the experiments for characterizing the CSR from femtoslicing, we discovered that if the beam is sliced above the MBI threshold the instability can be seeded.



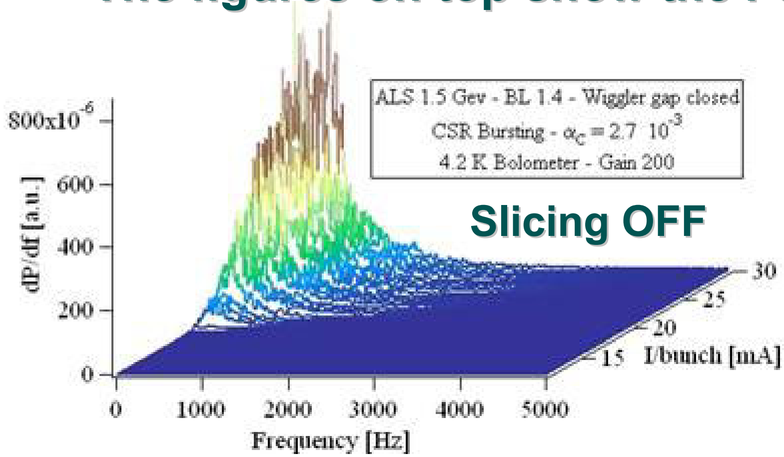
The figure shows the signal as measured from the bolometer. The CSR THz bursts associated with the microbunching instability (MBI) are clearly visible

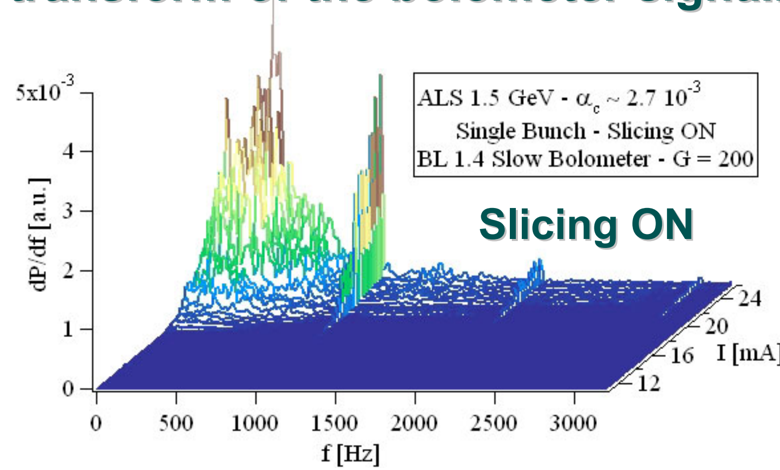
The bursts appearance is usually random (top part) but when the slicing laser is turned on most of the burst become synchronous with the laser (bottom part).

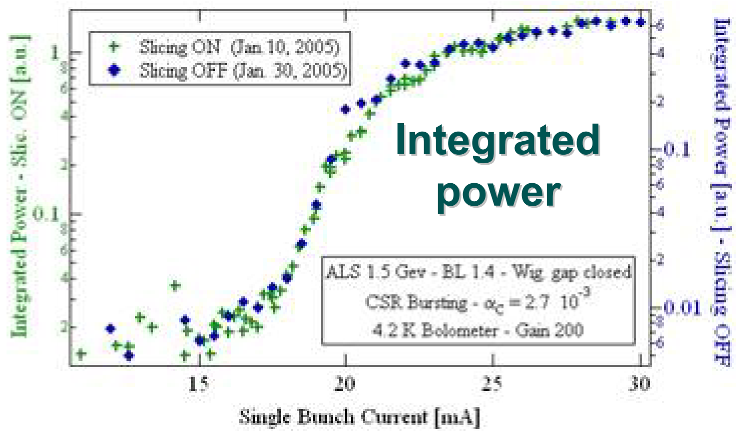
The slicing laser repetition rate is 1 kHz

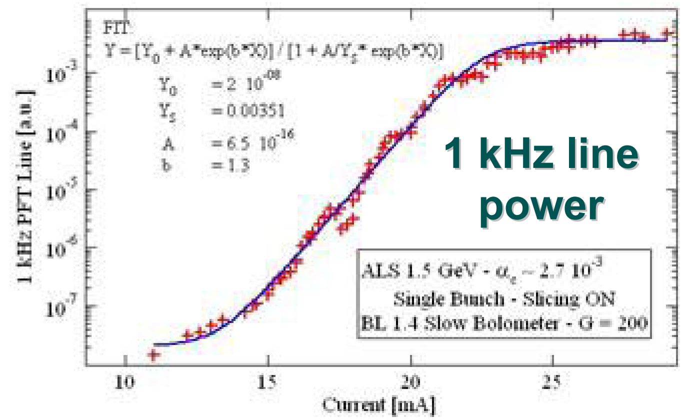
The Seeding in Frequency Domain

The figures on top show the Fourier transform of the bolometer signal.





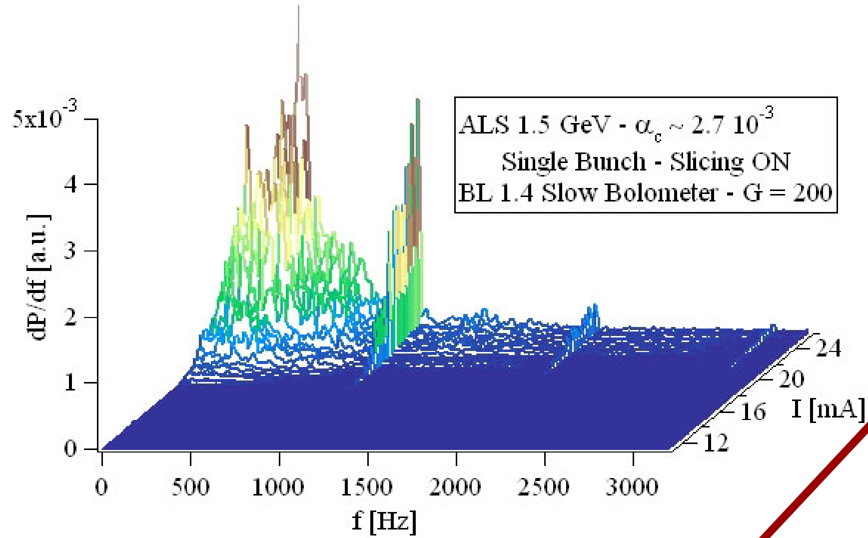




The slicing does not impact the dependency of the MBI from current but makes it synchronous with the slicing (1 kHz harmonics)

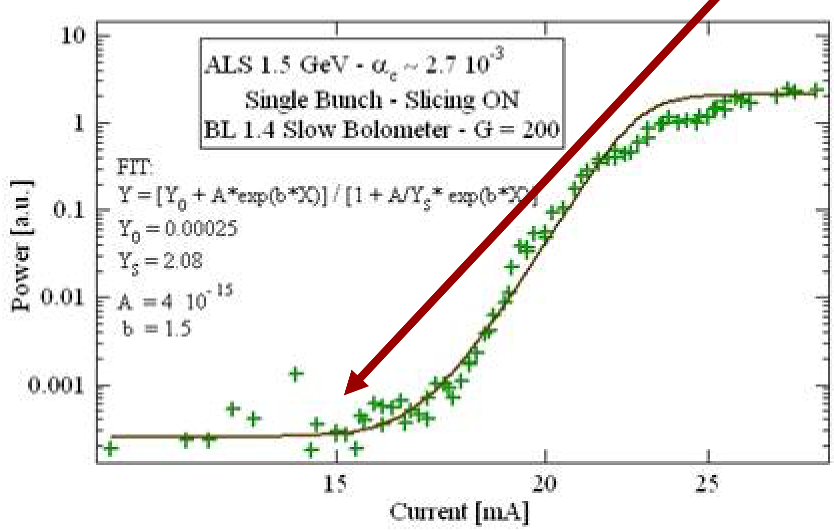
The CSR power correlated with the laser slicing scales <u>exponentially</u> with the current per bunch.

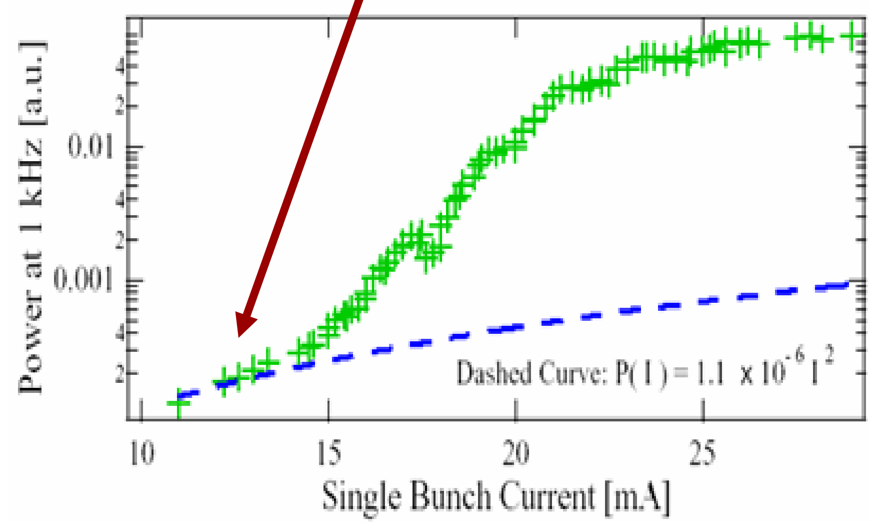
Behavior Below MBI Threshold



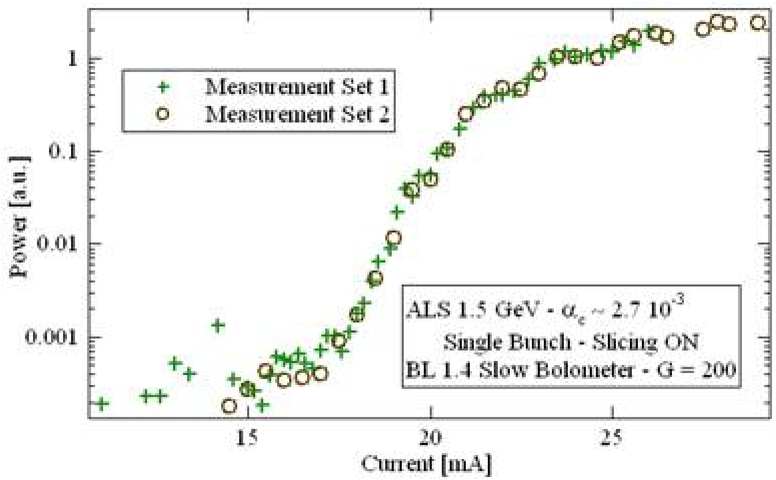
Total Power: ~ flat below MBI threshold

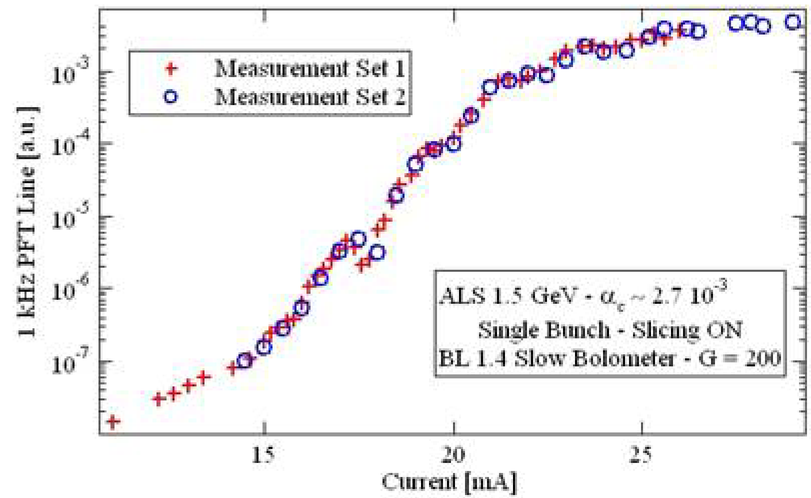
kHz line Power: ~ quadratic with current below MBI threshold



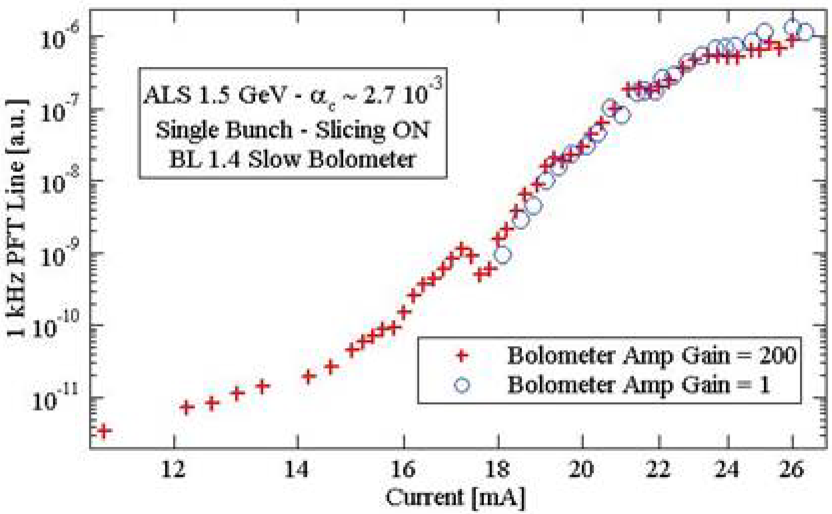


Reproducibility and Saturation





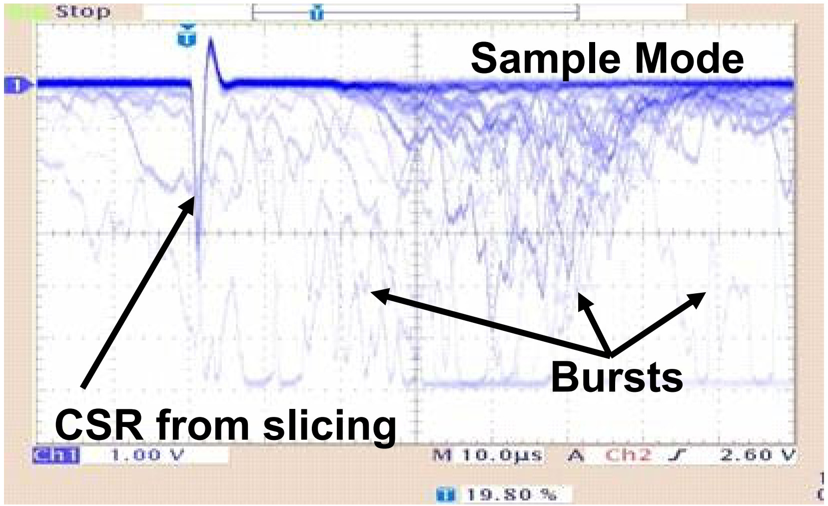
The measurement is very reproducible.

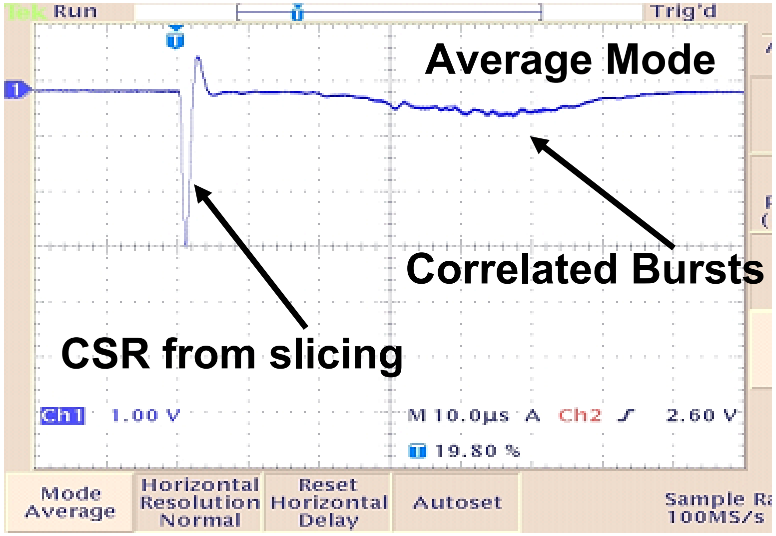


The measured saturation is real.
It is not due to the strumentation.

Cutting by ~ one half the signal at the bolometer input cuts the signal amplitude by the same amount

Faster Detector Measurement

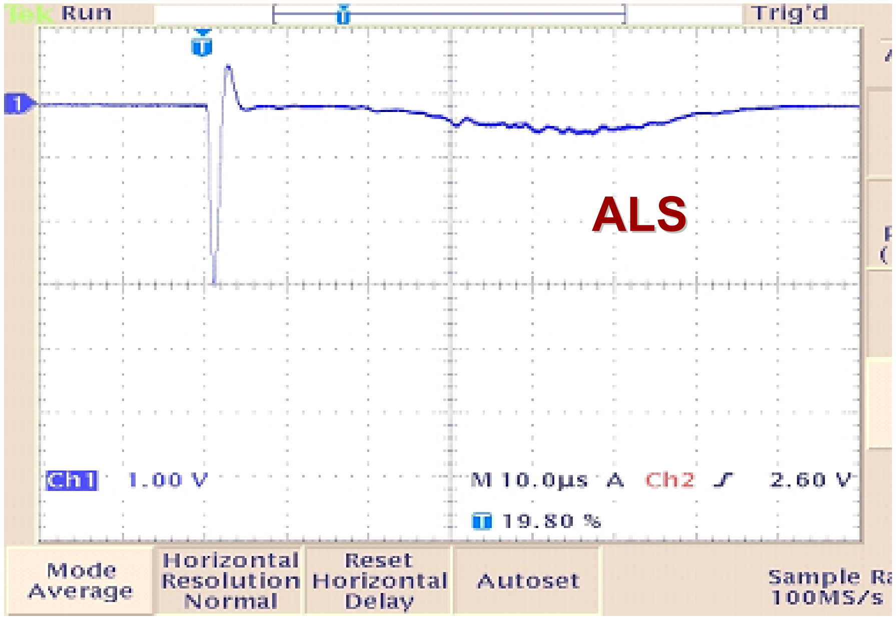


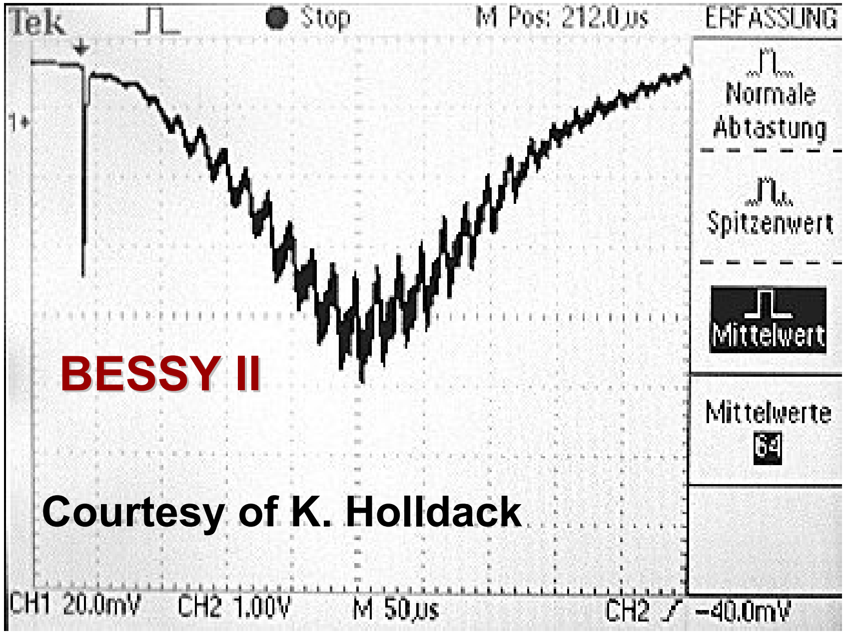


- **The correlated bursts do not start immediately after the slicing.**
- **In this particular case the correlated bursts peaks after ~ 45 μs.**

Hot electron bolometer ~800 kHz BW, ~10 – 1000 cm^{-1} frequency response (*Infrared Laboratories Inc.*)

Is the ALS a Special Case?





Single bunch 7 mA
MBI threshold ~ 3.5 mA.

BESSY II observed the same phenomenon.

A Theory for the Seeded MBI

In the framework of the Heifets-Stupakov MBI theory (PRST-AB 5, 054402, 2002) the micro-bunching can be represented by the linear combination of "modes" with shape:

$$\rho = \hat{\rho}\, e^{-i(\omega t - kz)}$$

In the "cold beam" approximation, the authors derived an analytical expression for the dispersion function between ω and $k=2\pi/\lambda$.

$$k < \sim\left[(2\pi r_0 n_b R^{1/3})/(\alpha_c \gamma \delta_0^2 L)\right]^{3/2}$$

$$\omega = c k^{2/3}\sqrt{\frac{\Gamma(2/3)}{3^{1/3}}(\sqrt{3}i - 1)\frac{2\pi r_0 n_b \alpha_c R^{1/3}}{\gamma L}}.$$

- $R \equiv$ *dipole bend radius*
- $L \equiv$ *ring length*
- $n_b \equiv$ *particle linear density*
- $\alpha_C \equiv$ *momentum compaction*
- $\delta_0 \equiv$ *relative energy spread*
- $\gamma \equiv$ *energy in rest mass units*
- $r_0 \equiv$ *classical electron radius*

From that, the growth rate a and the velocity v for the mode can be calculated:

$$a = \mathrm{Im}[\omega] \cong c k^{2/3}\sqrt{\frac{3^{2/3}\Gamma(2/3)}{2}\frac{2\pi r_0 n_b \alpha_c R^{1/3}}{\gamma L}}.$$

$$v = \mathrm{Re}\left[\frac{d\omega}{dk}\right] \cong c k^{-1/3}\sqrt{\frac{2\Gamma(2/3)}{3^{7/3}}\frac{2\pi r_0 n_b \alpha_c R^{1/3}}{\gamma L}},$$

These last two equations show that the unstable mode when excited (by noise or seeding) starts moving along the beam with velocity v and grows exponentially with rate a.

A Theory for the Seeded MBI

The accurate analysis of the microbunch evolution should be done (and in fact has been done by Heifets and Stupakov, SLAC-PUB-11815, 2006) by considering the sum of all the unstable modes at that current.

$$\rho = \hat{\rho}\, e^{-i(\omega t - kz)}$$

In a more simplified analysis, we will assume that one mode is dominant (the one with the highest growth rate) and we will investigate the evolution of this mode only.

In a bunch, the charge density n_b depends on the position z along the bunch and because the perturbation moves, its position z depends on t.

If n_b at the perturbation position changes slowly with t, the amplitude of the mode can is obtained as the solution of:

$$d\rho / dt = a[z(t)]\rho$$

$$\Rightarrow \quad \rho(z) = \hat{\rho} \exp\left[\int_0^z \frac{a(\tilde{z})}{v(\tilde{z})} d\tilde{z} \right]$$

and using the previous results

$$\Rightarrow \quad \rho(z) = \hat{\rho} \exp[\sqrt{27/4}kz]$$

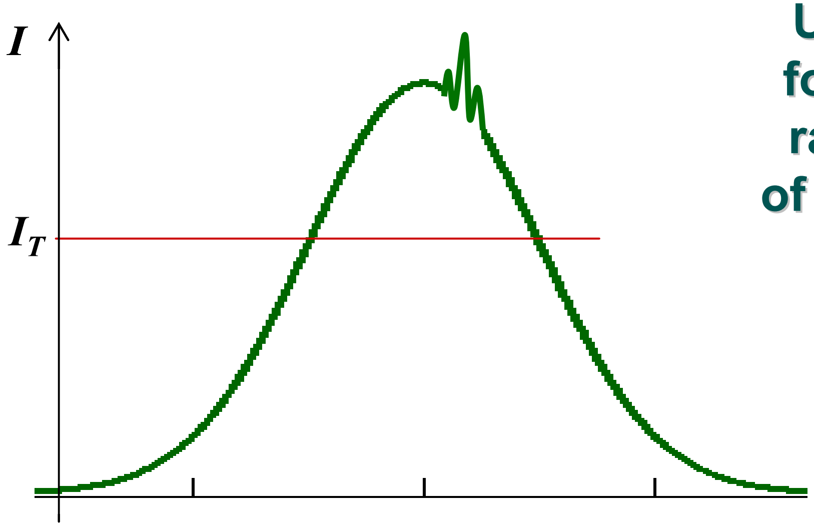
A Theory for the Seeded MBI

The perturbation typically originates at the bunch peak and then starts moving and growing exponentially, but when it arrives at a bunch position where n_b < than the MBI threshold the CSR wake cannot sustain the amplitude growth anymore and the modulation starts to decrease and is gradually reabsorbed by the bunch.

For the case of a Gaussian bunch with rms length σ_z the instability threshold is situated at the distance z_T from the bunch peak:

$$z = z_T = \sigma_z\sqrt{2\ln(\hat{n}_b/n_b^T)} = \sigma_z\sqrt{2\ln(I/I_T)},$$

where I is the bunch current and I_T is the MBI current threshold.

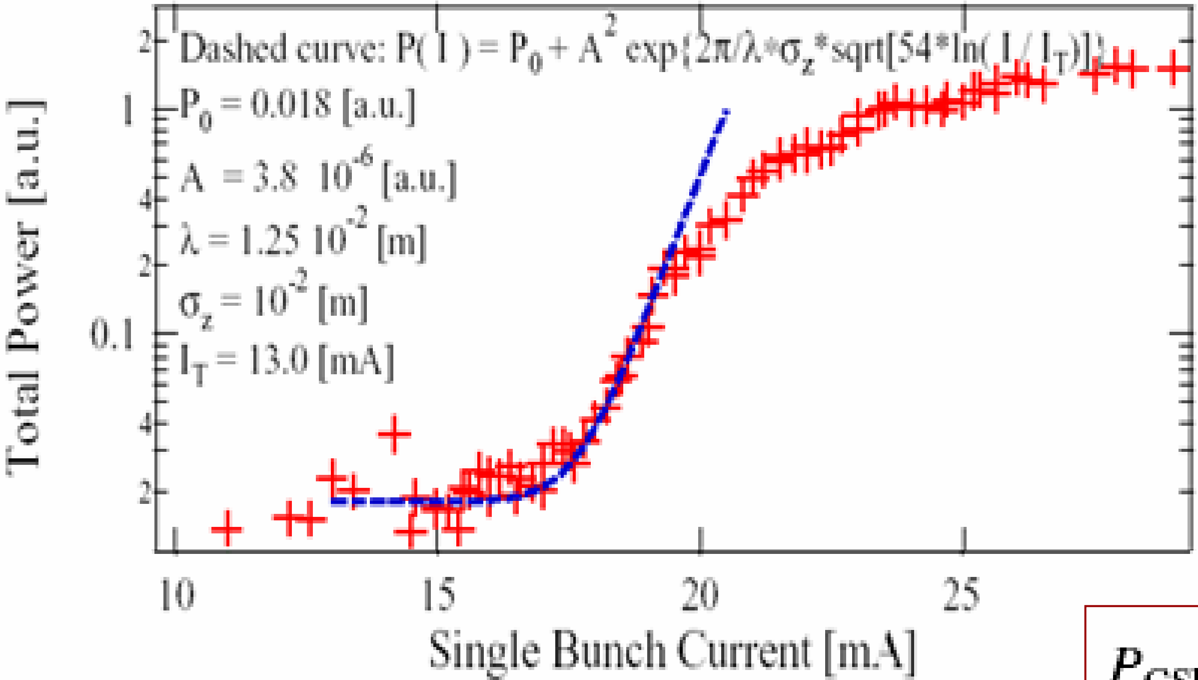


Using this result in the previous expression for the perturbation and considering that the radiated power is proportional to the square of the perturbation amplitude one finally finds:

$$P_{\mathrm{CSR}} \propto \hat{\rho}^2 \exp[k\sigma_z\sqrt{54\ln(I/I_T)}]$$

Comparing with Data

J.M.Byrd *et al*, PRL **97**, 074802, 2006



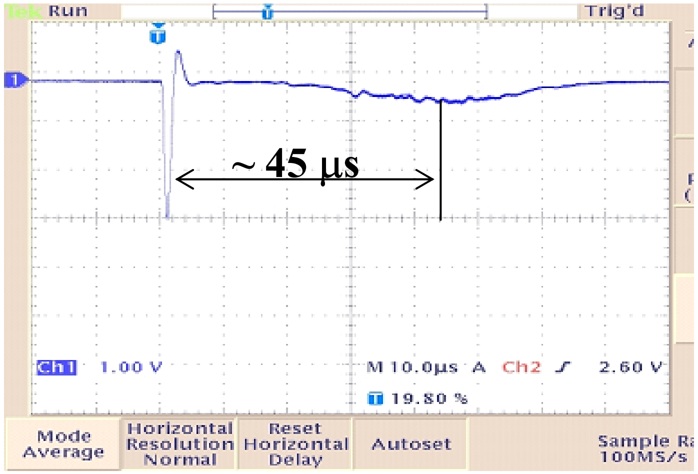
The comparison of this simple model predictions with the ALS experimental data showed a good agreement up to a certain current (~19 mA).

$$P_{\mathrm{CSR}} \propto \hat{\rho}^2 \exp\left[k\sigma_z\sqrt{54\ln(I/I_T)}\right]$$

Above this value, the experimental points show a saturation effect that we think is due to the fact that at high currents, the MBI goes in the nonlinear regime before the perturbation arrives at the threshold point.

Comparing with Data

The model can also account for the time structure shown by the CSR power signal in measured by the fast bolometer.



As previously said, the slicing seeds the perturbation that starts at the bunch peak, assumes its maximum amplitude at z_T, and after that is gradually reabsorbed. The CSR power radiated by the perturbation must show the same time structure, and the figure seems to confirm this scenario.

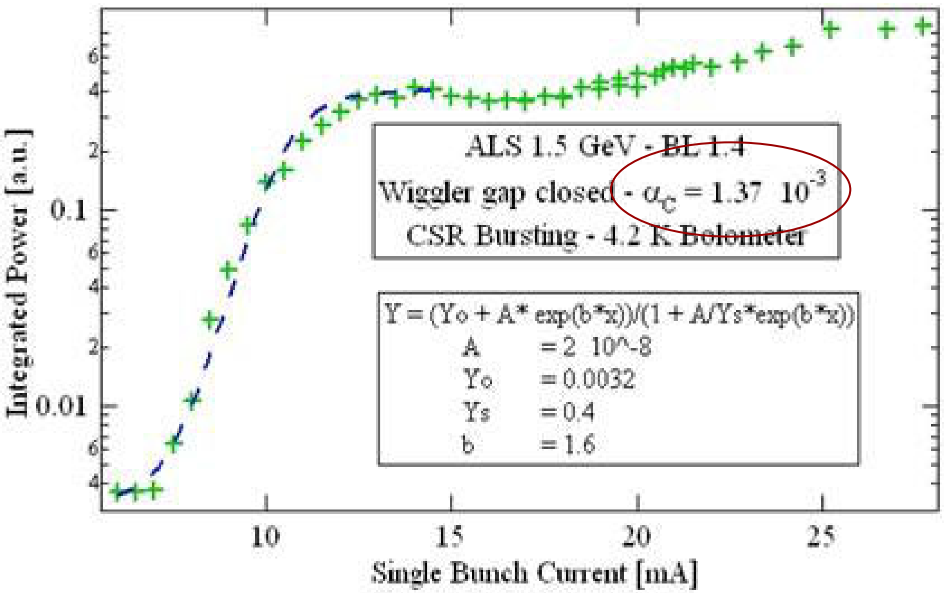
In a more quantitative comparison, we can use the fact that the distance in time between the peaks of the two pulses in the bottom part of the figure should coincides with the value t_T that takes for the perturbation for going from the bunch peak to the point z_T.

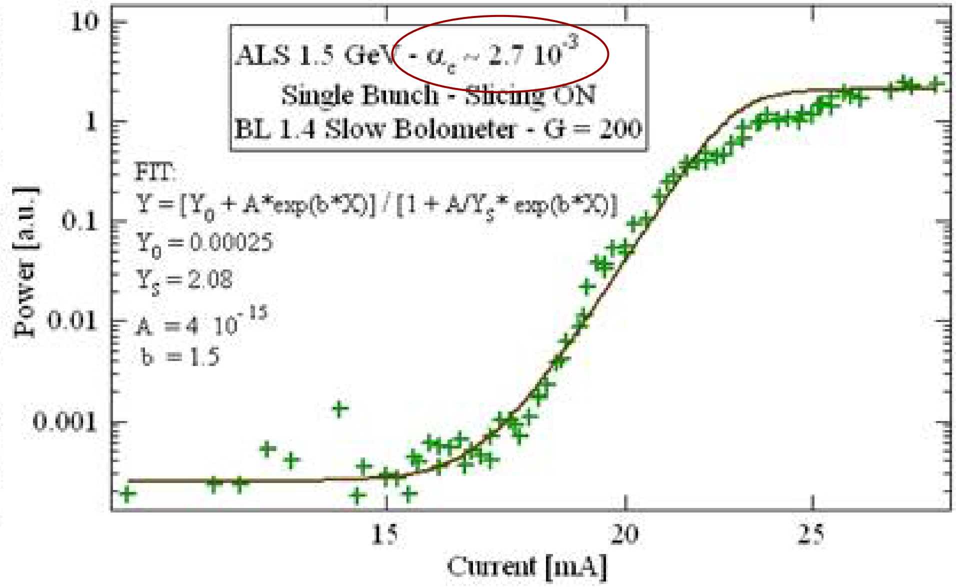
$$t_T = \int_0^{z_T} dz/v$$

By calculating this value for our case, we estimate $t_T \sim 31\ \mu s$ not far from the $45\ \mu s$ of the figure. Apart from the accuracy of the model, we think that the discrepancy is also due to the fact that the signal in the figure was taken with a current higher than the ~ 19 mA at which the linear regime breaks.

Saturation Can Minimize the Effect

At the ALS we also tried to characterize the MBI power dependence on current using a different lattice with the momentum compaction reduced to about half its value (1.37×10^{-3})



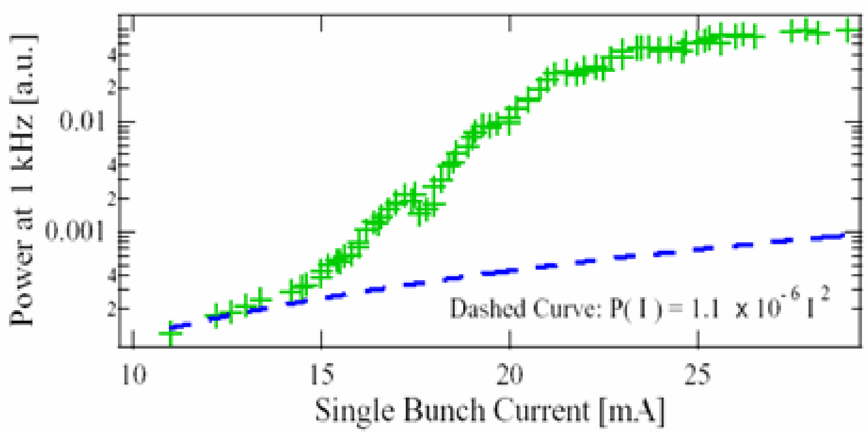


The exponential growth above the MBI threshold is still observed but it saturates much earlier than in the larger momentum compaction case.

As a consequence, we did not observe any measurable enhancement of the intensity of the CSR pulses induced by the slicing.

Possible Applications

The graph shows how the 1 kHz power (synchronous with the laser) loses the quadratic dependence for currents above the MBI threshold.



At saturation, the average power of the seeded CSR burst is about two orders of magnitude larger than for the "conventional" slicing case, but shows very large power fluctuations. Pump and probe and other experiments not requiring shot to shot intensity stability could benefit from this several orders of magnitude increase in power.

In a more speculative scenario, part of the THz signal could be brought back into the ring to co-propagate the bending magnet with a subsequent electron bunch, modulating its energy and seeding the MBI that generates a new burst that is then used in the loop for seeding a new fresh bunch. By this process, that continues involving all the bunches, one can in principle bring the CSR emission to a stable high power saturation regime where all the bunches radiate coherently.

Other FEL-like schemes exploiting the MBI gain are possible as well.

Used References

CSR by Laser Slicing

J.M.Byrd *et al*., PRL 96, 164801, (2006.)

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CSR from Seeded MBI:

S. Heifets, G. Stupakov PRST-AB 5, 054402, (2002)

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S. Heifets and G. Stupakov, SLAC-PUB-11815, (2006)

Homework

In a slicing experiment, define the energy that a laser pulse at λ=852 nm must have in order to generate in an electron beam of 1.9 GeV an energy modulation six time larger of the beam relative energy spread. Assume that you are using a wiggler with parameter K=2. Assume also that the relative energy spread of the beam is 10^{-3} , that the laser pulse length is 50 fs (FWHM), and that the wiggler relative bandwidth is $1/N_W$ (where N_W is 50 and is the number of periods in the wiggler).

For a slicing experiment, list the system parameters that allows to control the CSR spectrum and explain their effect.

Calculate the growth rate and the perturbation velocity for the MBI mode with λ=6 mm for a beam in a storage ring with the following characteristics. Beam energy 1.5 GeV, momentum compaction 2,7 x 10^{-3}, ring length 197 m, bending radius R=55m and particle density n_b= 10^9 electrons per cm. $\Gamma(3/2)\sim 1.354$.

Physical Constants (SI Units)

Quantity	Symbol	Value	Unit	Relative std. uncert. u_r
speed of light in vacuum	c, c_0	299 792 458	$\mathrm{m\,s^{-1}}$	(exact)
magnetic constant	μ_0	$4\pi \times 10^{-7}$	$\mathrm{N\,A^{-2}}$	
		$= 12.566\,370\,614... \times 10^{-7}$	$\mathrm{N\,A^{-2}}$	(exact)
electric constant $1/\mu_0 c^2$	ϵ_0	$8.854\,187\,817... \times 10^{-12}$	$\mathrm{F\,m^{-1}}$	(exact)
Newtonian constant of gravitation	G	$6.674\,28(67) \times 10^{-11}$	$\mathrm{m^3\,kg^{-1}\,s^{-2}}$	1.0×10^{-4}
Planck constant	h	$6.626\,068\,96(33) \times 10^{-34}$	J s	5.0×10^{-8}
$h/2\pi$	$\hbar$	$1.054\,571\,628(53) \times 10^{-34}$	J s	5.0×10^{-8}
elementary charge	e	$1.602\,176\,487(40) \times 10^{-19}$	C	2.5×10^{-8}
magnetic flux quantum $h/2e$	Φ_0	$2.067\,833\,667(52) \times 10^{-15}$	Wb	2.5×10^{-8}
conductance quantum $2e^2/h$	G_0	$7.748\,091\,7004(53) \times 10^{-5}$	S	6.8×10^{-10}
electron mass	m_e	$9.109\,382\,15(45) \times 10^{-31}$	kg	5.0×10^{-8}
proton mass	m_p	$1.672\,621\,637(83) \times 10^{-27}$	kg	5.0×10^{-8}
proton-electron mass ratio	m_p/m_e	$1836.152\,672\,47(80)$		4.3×10^{-10}
fine-structure constant $e^2/4\pi\epsilon_0\hbar c$	α	$7.297\,352\,5376(50) \times 10^{-3}$		6.8×10^{-10}
inverse fine-structure constant	α^{-1}	$137.035\,999\,679(94)$		6.8×10^{-10}
Rydberg constant $\alpha^2 m_e c/2h$	R_∞	$10\,973\,731.568\,527(73)$	$\mathrm{m^{-1}}$	6.6×10^{-12}
Avogadro constant	N_A, L	$6.022\,141\,79(30) \times 10^{23}$	$\mathrm{mol^{-1}}$	5.0×10^{-8}
Faraday constant $N_A e$	F	$96\,485.3399(24)$	$\mathrm{C\,mol^{-1}}$	2.5×10^{-8}
molar gas constant	R	$8.314\,472(15)$	$\mathrm{J\,mol^{-1}\,K^{-1}}$	1.7×10^{-6}
Boltzmann constant R/N_A	k	$1.380\,6504(24) \times 10^{-23}$	$\mathrm{J\,K^{-1}}$	1.7×10^{-6}
Stefan-Boltzmann constant $(\pi^2/60)k^4/\hbar^3 c^2$	σ	$5.670\,400(40) \times 10^{-8}$	$\mathrm{W\,m^{-2}\,K^{-4}}$	7.0×10^{-6}

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