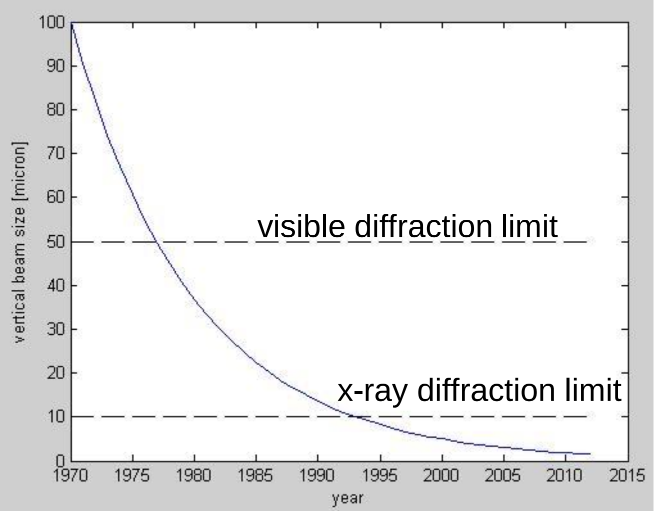
Michelson's Interferometer – Theory and Practice

US Particle Accelerator School
January 14-18, 2008

- Motivation
- Two-slit Interference – Young's experiment
- Diffraction from a single slit - review
- Extended Source – Partial Coherence
- The Mutual Coherence function
- Van-Cittert/Ziernike theorem
- Stellar Interferometers for SR applications

Motivation for Interferometry

- Electron beam size can be very small



$$\sqrt{\varepsilon\beta} = \sqrt{(1)(1\times10^{-9}/1000)} = 1\mu m!$$

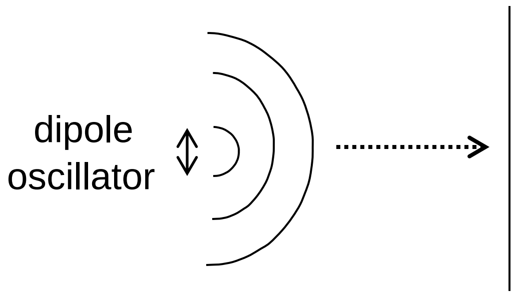
- Need to measure beam size for optics verification,
  machine monitoring and operation

- Conventional imaging diffraction limited
  σ_{res}~50 um visible
  σ_{res}~10 um x-ray pinhole

- What else can be used?

Motivation for Interferometry (cont'd)

➢ Take advantage of light *coherence* properties



$$e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

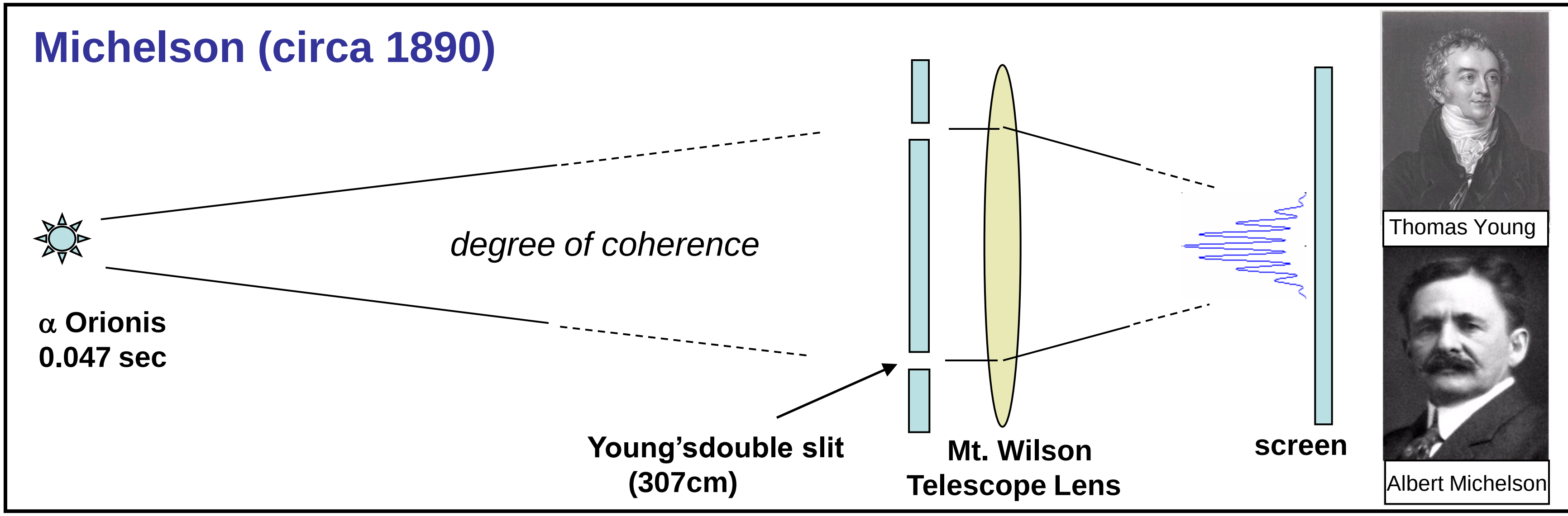
(plane waves at infinity)

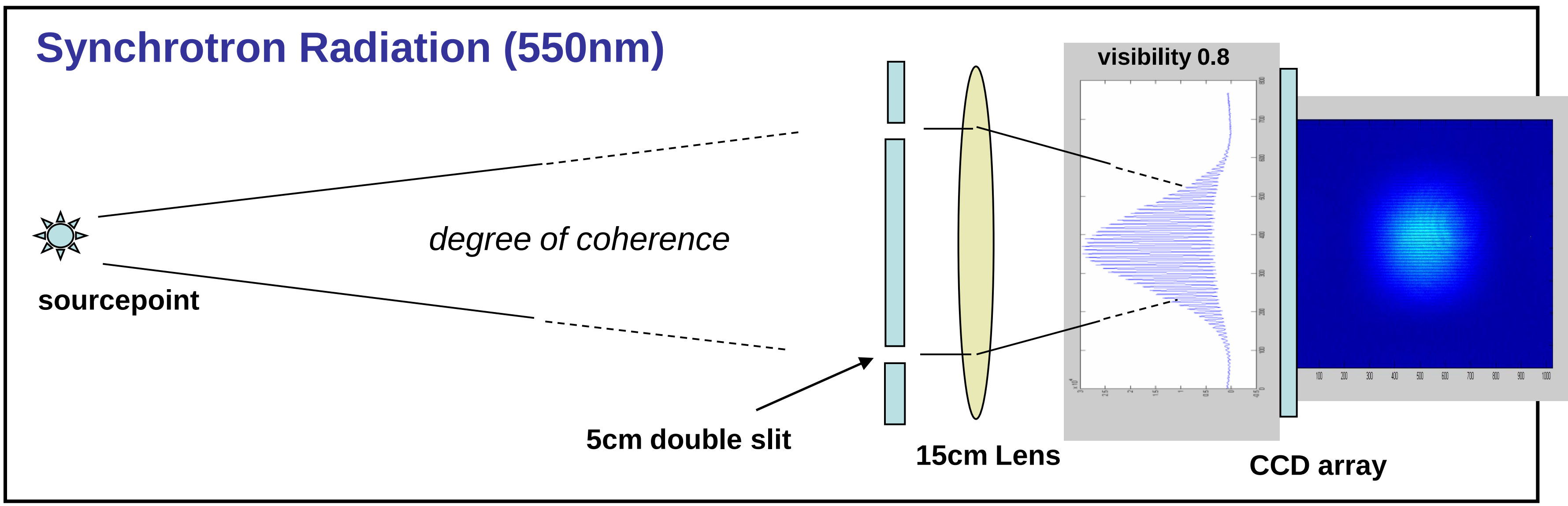
➢ For a *distributed* source at finite distance the light is only *partially coherent*

$$\vec{k} = k_x \hat{x} + k_z \hat{z}$$

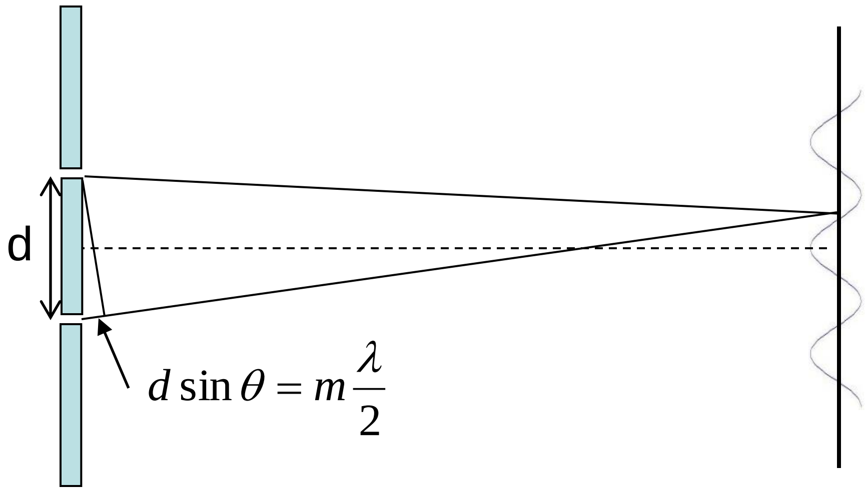
➢ Interferometry enters world of wavefront physics and statistical optics

Interferometric Beam Size Measurement





Two-Slit Interference: Young's Experiment



$$d\sin\theta = m\frac{\lambda}{2}$$

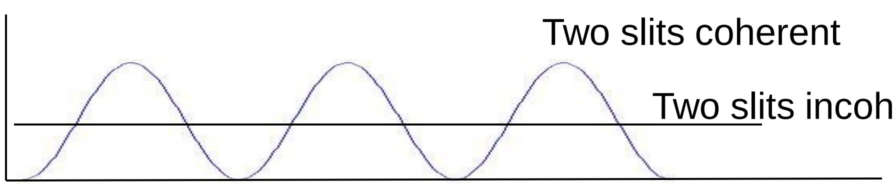
Field Pattern

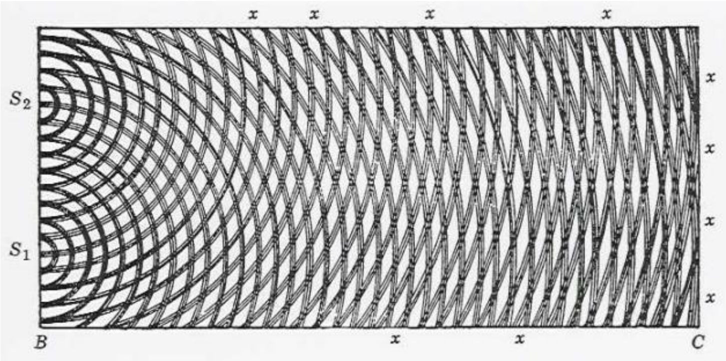
$$E = E_o e^{i(\Phi)} + E_o e^{i(-\Phi)} = 2E_o \cos(\Phi)$$

$$\int_{-\infty}^{\infty} (\delta(\Phi) + \delta(-\Phi)) e^{ikx} dx$$

monochromatic light!

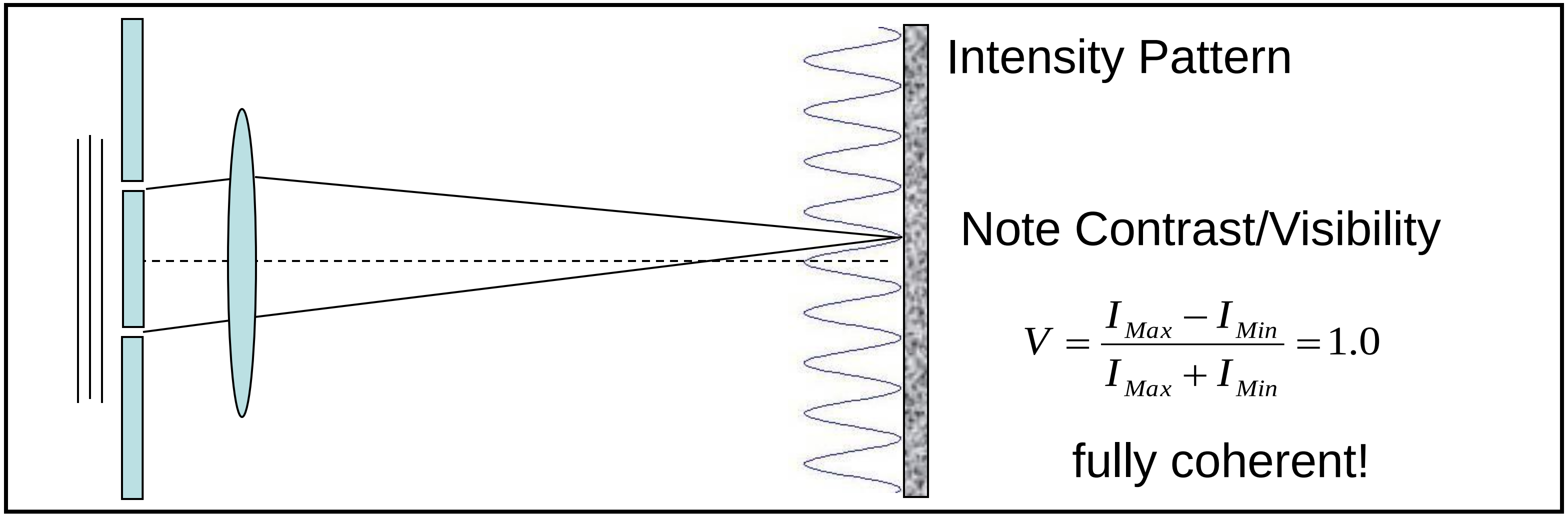
Intensity Pattern: $\cos^2(\Phi)$



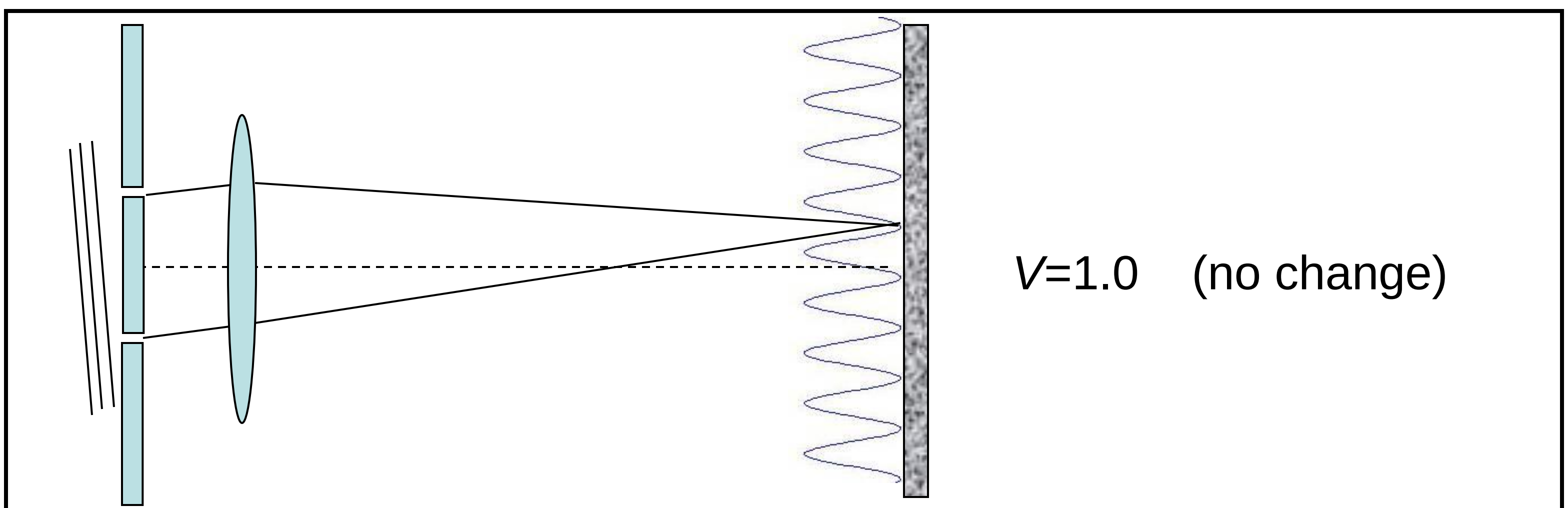


Two-Slit Interference (cont'd)

Use a lens to concentrate image on screen

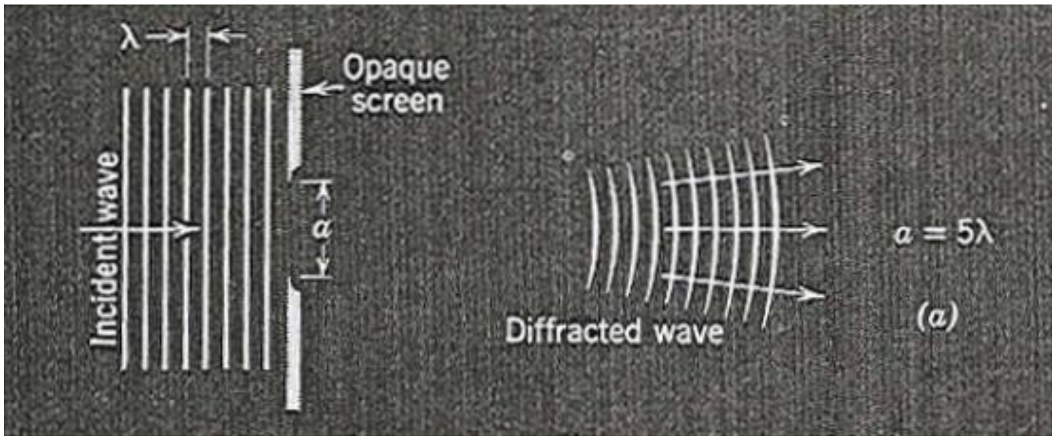


Change phase/incidence angle, shift pattern phase



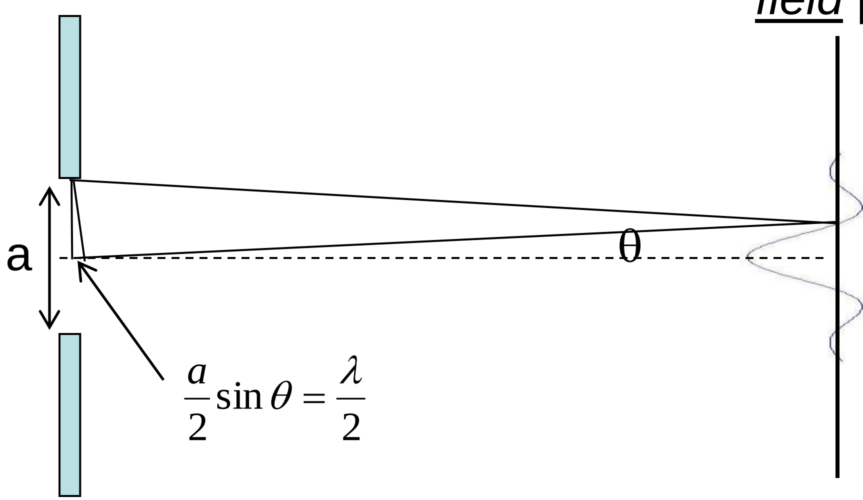
Single-Slit Diffraction - Review

Plane wave incident on aperture - diffraction

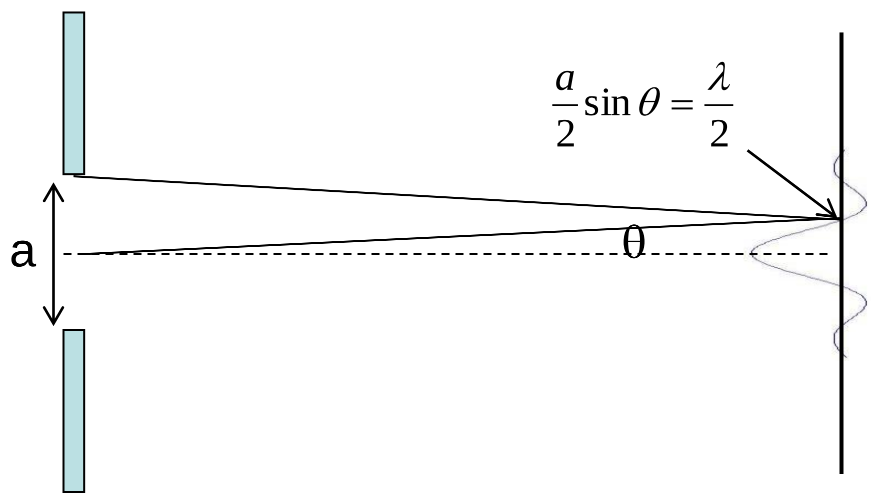


Condition for first diffraction minima

field pattern at screen



Single-Slit: Electric Field Pattern



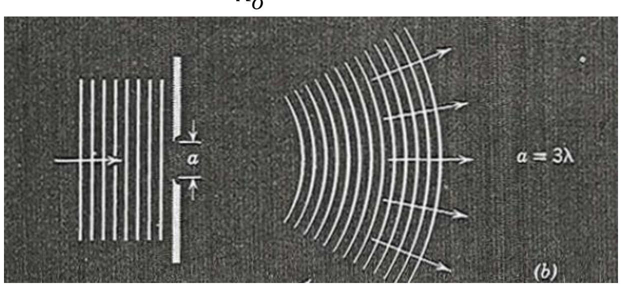
Field pattern

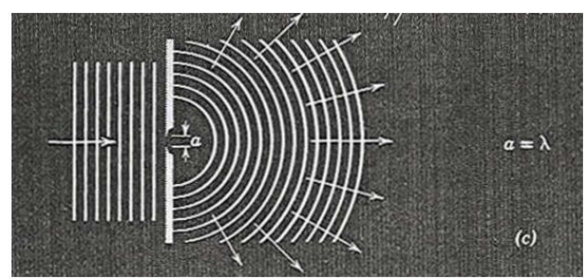
$$E_\theta = E_o \frac{\sin(\alpha)}{\alpha}$$

$$\alpha = \frac{\pi a}{\lambda}\sin\theta$$

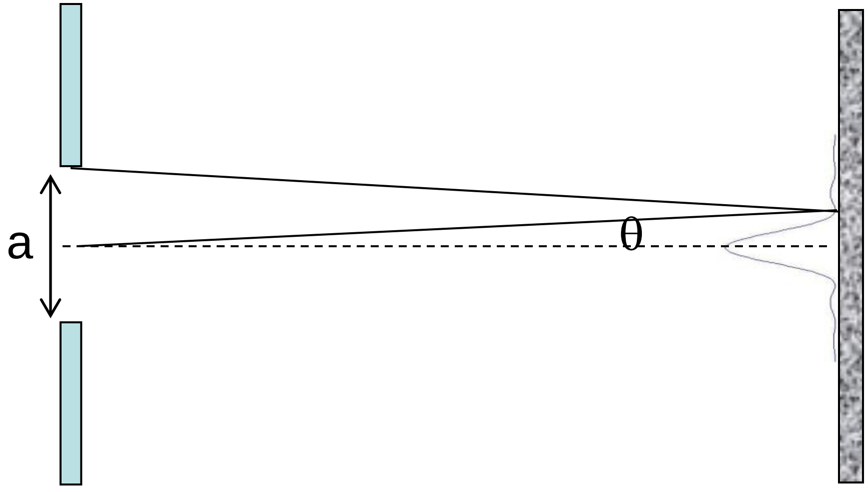
Field pattern is the *Fourier Transform* of the Aperture

$$\int_{-x_o}^{x_o} e^{ikx}dx = \frac{e^{ikx_o} - e^{ikx_o}}{ikx_o} = \frac{\sin(kx_o)}{kx_o}$$





Single-Slit: Intensity Pattern



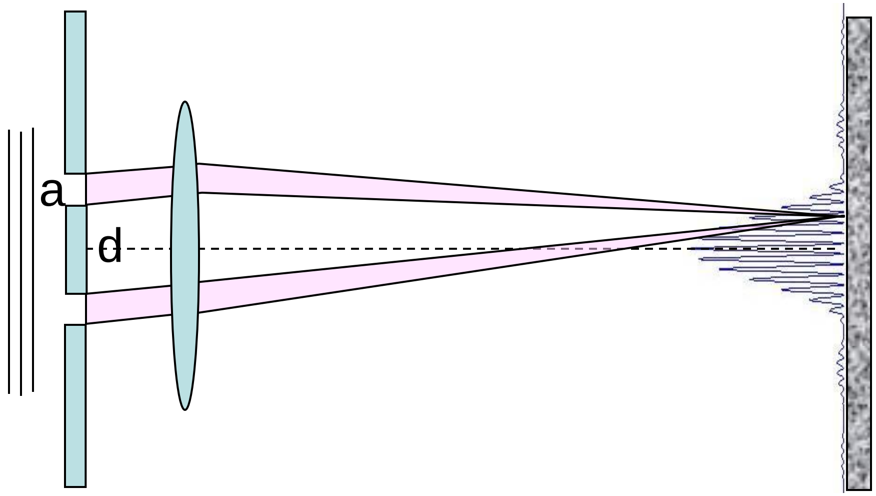
Intensity

$$I_\theta = E^* E = I_o \left(\frac{\sin(\alpha)}{\alpha} \right)^2$$

(measure laser diffraction through pinhole in afternoon)

Two-Slit Interference (cont'd)

Consider 2-slit interference with finite slit size:



Intensity Pattern

$$I_\theta = I_o\left(\frac{\sin(a\theta)}{a\theta}\right)^2 \times \cos^2(d\theta)$$

V=1.0 (no change)

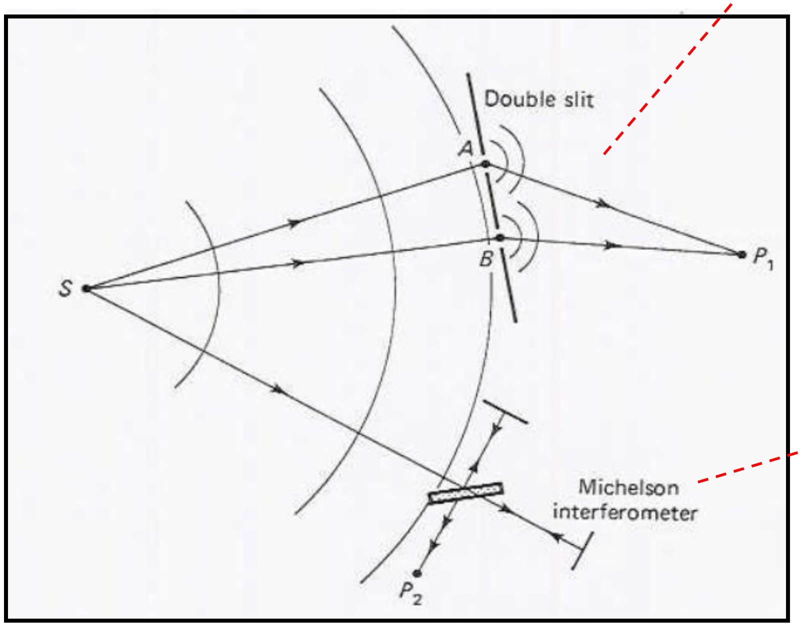
Important mathematical point: $1+\cos(\theta) = 2\cos^2(\frac{\theta}{2})$

Intensity pattern can be written: $$I_\theta = I_o\underbrace{\left(\frac{\sin(a\theta)}{a\theta}\right)^2}_{\text{Single-Slit}} \times \underbrace{(1+\cos(d\theta))}_{\text{Two-Slit}}$$

This is *approximately* the form we will work with (but visibility 1.0)

From Interference to Interferometry

Interferometry is used to measure *coherence* properties of light



Spatial Coherence

coherence length is inversely proportional to size

a star emits plane-waves randomly in direction
-emission is not coherent at close distance

a point source emits sphereical-waves in all directions
-emission is coherent at long distances

Temporal Coherence

coherence time is inversely proportional to line-width

thermal source emits wavepackets randomly in time
-emission is not coherent

laser source emits continuous wavetrain in time
-emission is coherent

Degree of Coherence

Light can be totally coherent, partially coherent or incoherent

The *degree of coherence* is found from a correlation function

Temporal Degree of Coherence

Consider two colinear light waves $E_1(t)$ and $E_2(t)$

the correlation function is $\Gamma_{12}(\tau) = \left\langle E_1(t) E_2^*(t+\tau) \right\rangle$

Γ_{12} *is the degree of self-coherence*

if $\tau < \tau_o$ (correlation time), then waves are coherent and light interferes

if $\tau > \tau_o$ then coherence is lost and light waves do not interfere

Spatial Degree of Coherence

Consider two waves $E_1(\vec{k})$ and $E_2(\vec{k})$ from two sources

the correlation function is now $\Gamma_{12}(r) = \left\langle E_1(P_1) E_2^*(P_2) \right\rangle$

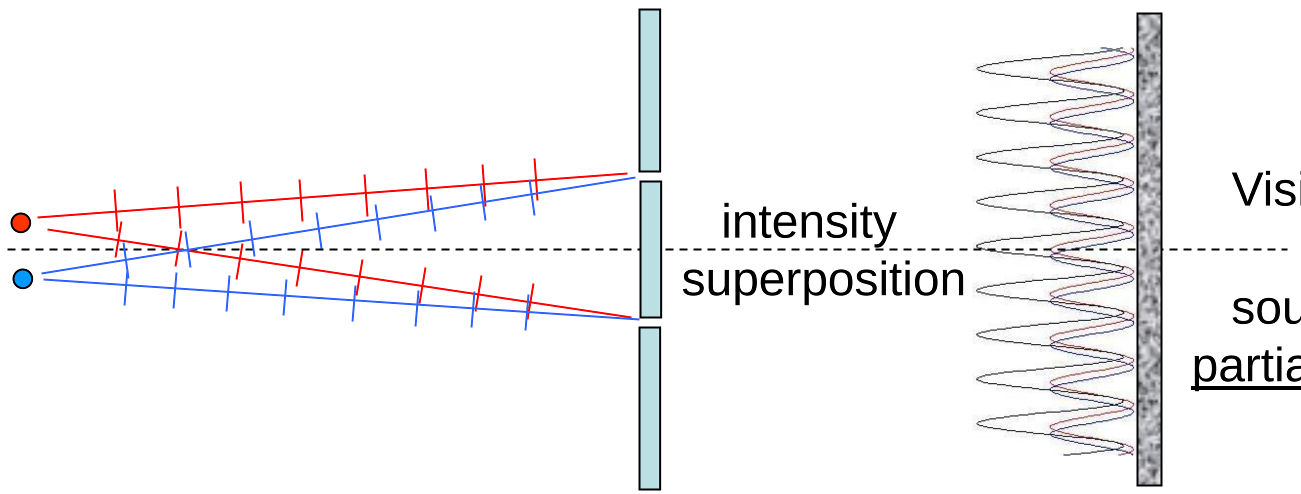
P_1 and P_2 are two points in space and $r=P_1-P_2$

Γ_{12} *is the degree of mutual-coherence*

if $r < r_o$ (correlation length), then waves are coherent and light interferes

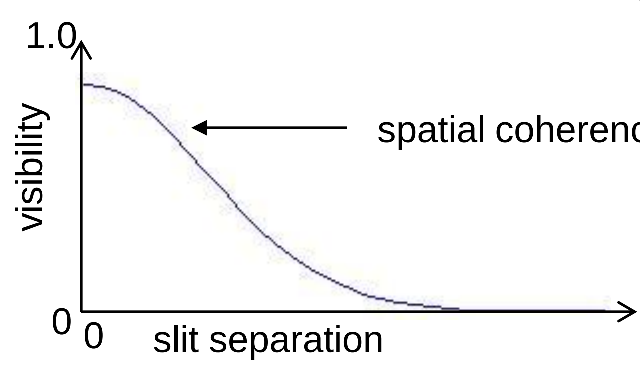
if $r > r_o$ then coherence is lost and light waves do not interfere

Spatial Coherence - Two Sources

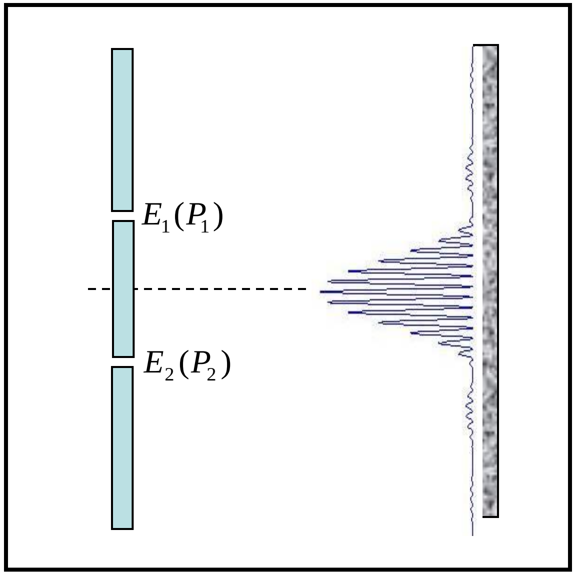


preview: visibility is Fourier Transform of source intensity

As slits separate
1) 'cosine' frequency goes up
2) superposition decoheres
3) visibility decreases



Mutual Coherence of field at two points



Solve for intensity on the screen

$$I(\theta) = E_T^* E_T = \left(E_1 + E_2 e^{+i\theta}\right)^* \left(E_1 + E_2 e^{-i\theta}\right)$$

$$I(\theta) = E_1^2 + E_2^2 + E_1^* E_2 e^{-i\theta} + E_1 E_2^* e^{+i\theta}$$

Now take the time average

$$I(\theta) = E_{01}^2 + E_{02}^2 + E_{01}E_{02}\left(|\gamma| e^{-i\gamma} e^{-i\theta} + |\gamma| e^{+i\gamma} e^{+i\theta}\right.$$

$$\boxed{I(\theta) = E_{01}^2 + E_{02}^2 + E_{01}E_{02}|\gamma| \cos(\gamma + \theta)}$$

$$Visibility = \frac{I_{Max} - I_{Min}}{I_{Max} - I_{Min}} = \frac{2E_{01}E_{02}|\gamma(P_1, P_2)|}{E_{01}^2 + E_{02}^2}$$

For E_{01}=E_{02}, $$Visibility = \frac{I_{Max} - I_{Min}}{I_{Max} - I_{Min}} = |\gamma(P_1, P_2)|$$

Mutual Coherence (cont'd)

For the case when $I_1=I_2$, we have

$$Visibility = \frac{I_{Max} - I_{Min}}{I_{Max} - I_{Min}} = \left|\gamma(P_1, P_2)\right|$$

Where $\gamma = \left|\gamma(P_1, P_2)\right| e^{i\gamma} = < E_1 * E_2 >$

has various descriptive labels

"mutual intensity"
"mutual coherence function"
"complex degree of coherence"
"correlation function"
"fringe parameter"

...fringe parameter measurement is central to all problems involving coherence

Putting it all together...

$$I(\theta) = E_{01}^2 + E_{02}^2 + E_{01}E_{02}|\gamma|\cos(\gamma + \theta)$$

$$I(y) = I_0 \left[\sin c\left(\frac{2\pi a}{\lambda R} y \right) \right] \cdot \left[1 + |\gamma| \cos\left(\frac{2\pi d}{\lambda R} y + \Phi \right) \right]$$

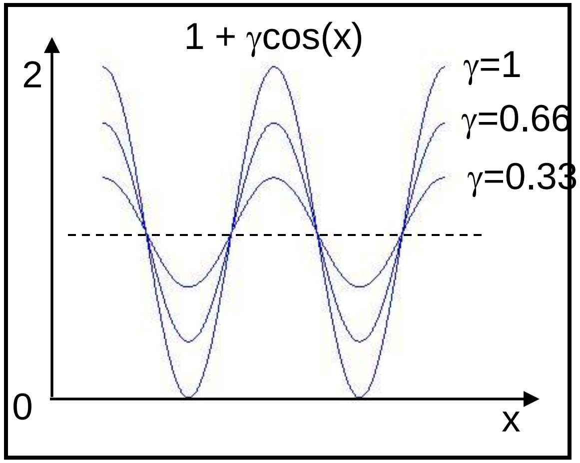
Single-Slit

Two-Slit

beam intensity
(equal both slits)

visibility factor (mutual coherence)

$$Visibility = \frac{I_{Max} - I_{Min}}{I_{Max} - I_{Min}} = |\gamma|$$



Alan will derive in-depth with chromatic effects

Van-Cittert/Zernike Theorem

"Visibility is the Fourier transform of source intensity"

$$\gamma(\nu) = \int I(y) e^{i2\pi\nu y} dy$$

I(y)=intensity distribution

$$\nu = \frac{d}{\lambda L} = spatial - frequency$$

d=slit width
λ=wavelength
L=source to slits

In two dimensions: $\gamma(\nu_x, \nu_y) = \iint I(x,y) e^{i2\pi(\nu_x x + \nu_y y)} dxdy$

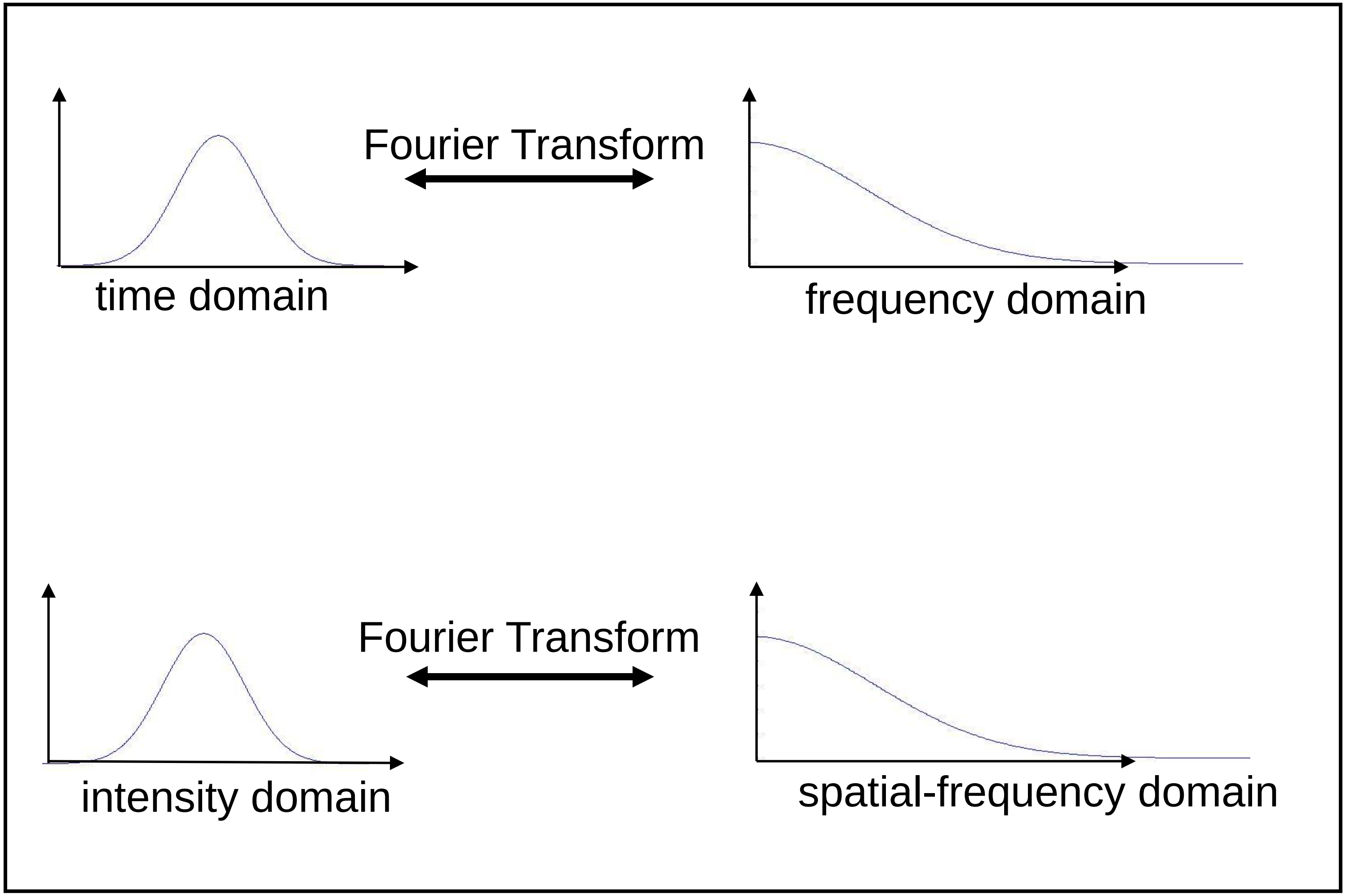
For a Gaussian, thermal-light source distribution

$$\gamma(d) = e^{\frac{-d^2}{2\sigma_d^2}}$$

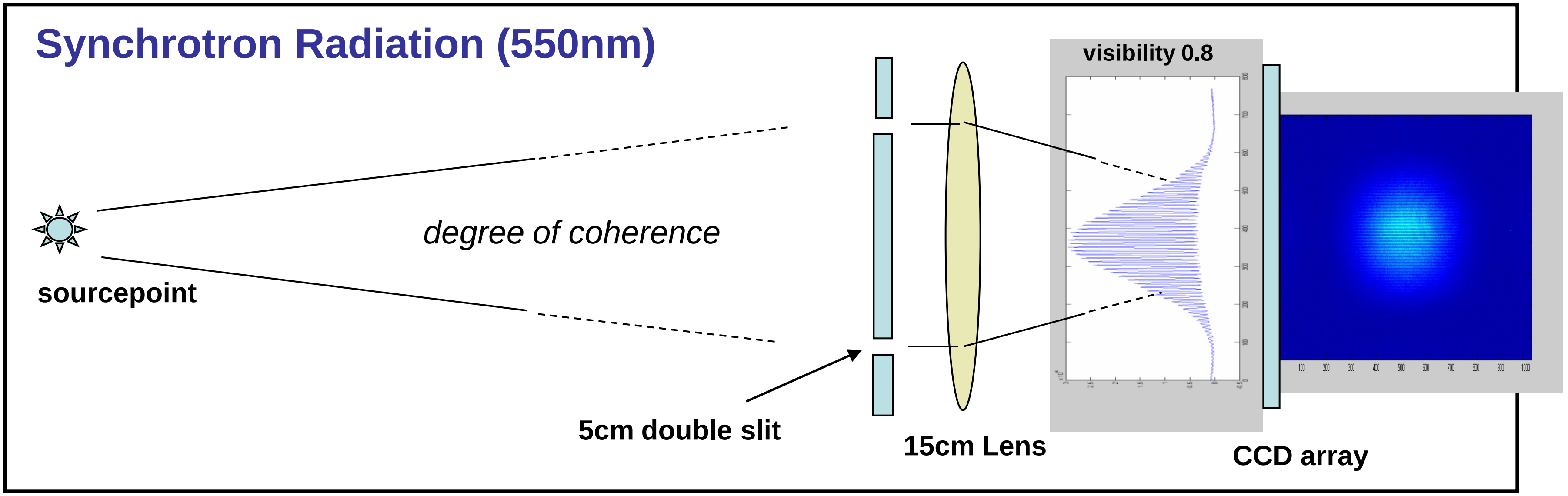
(one dimension)

where $\sigma_d = \frac{2\pi\sigma_y}{\lambda L}$ = spatial frequency characteristic

Fourier Transform Pairs



Interferometeric Beam Size Measurement

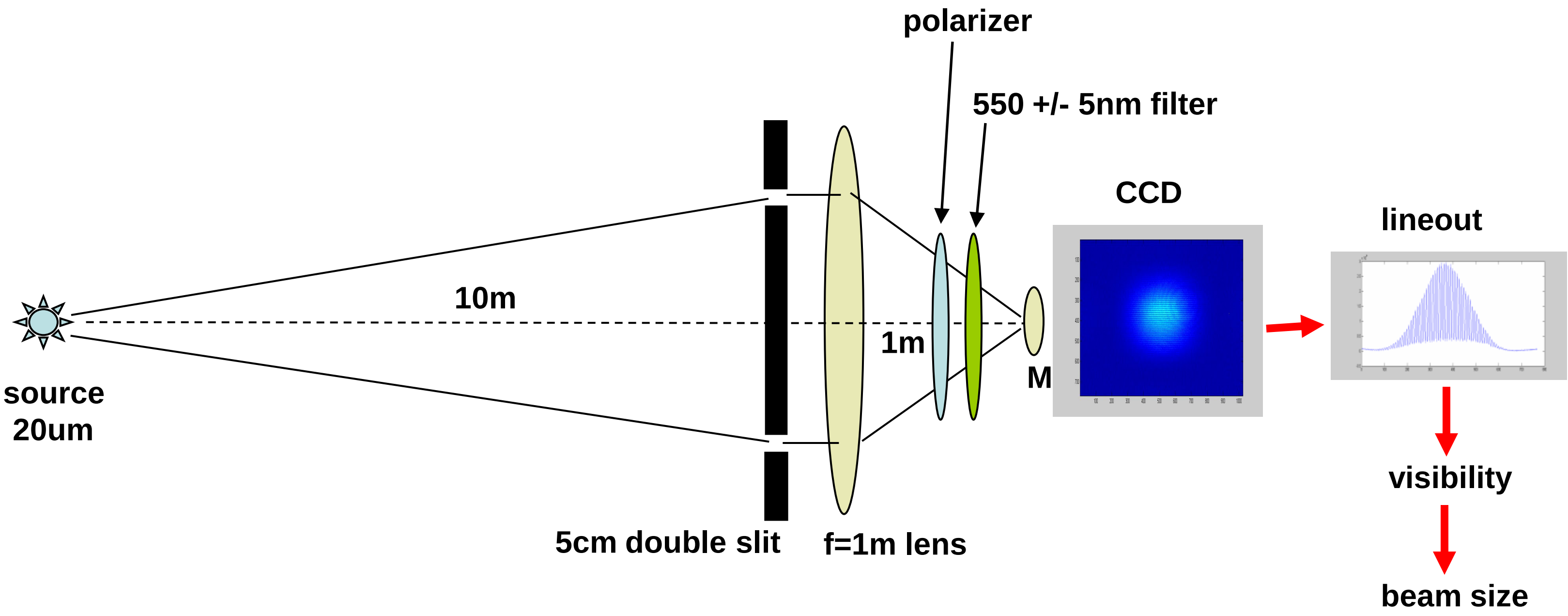


For a Gaussian source

$$\gamma(d) = e^{\frac{-d^2}{2\sigma_d^2}} \qquad \sigma_d = \frac{\lambda L}{2\pi\sigma_y}$$ = spatial frequency characteristic

1) Measure γ(d) [visibility as a function of slit separation]
2) Solve for characteristic width σ_d
3) Infer beam size from: $\sigma_y = \lambda L / 2\pi\sigma_d$

Typical Stellar Interferometer for SR Measurements



Typical System Parameters

Source size: σ_y=20um

Source-slit: L=10m

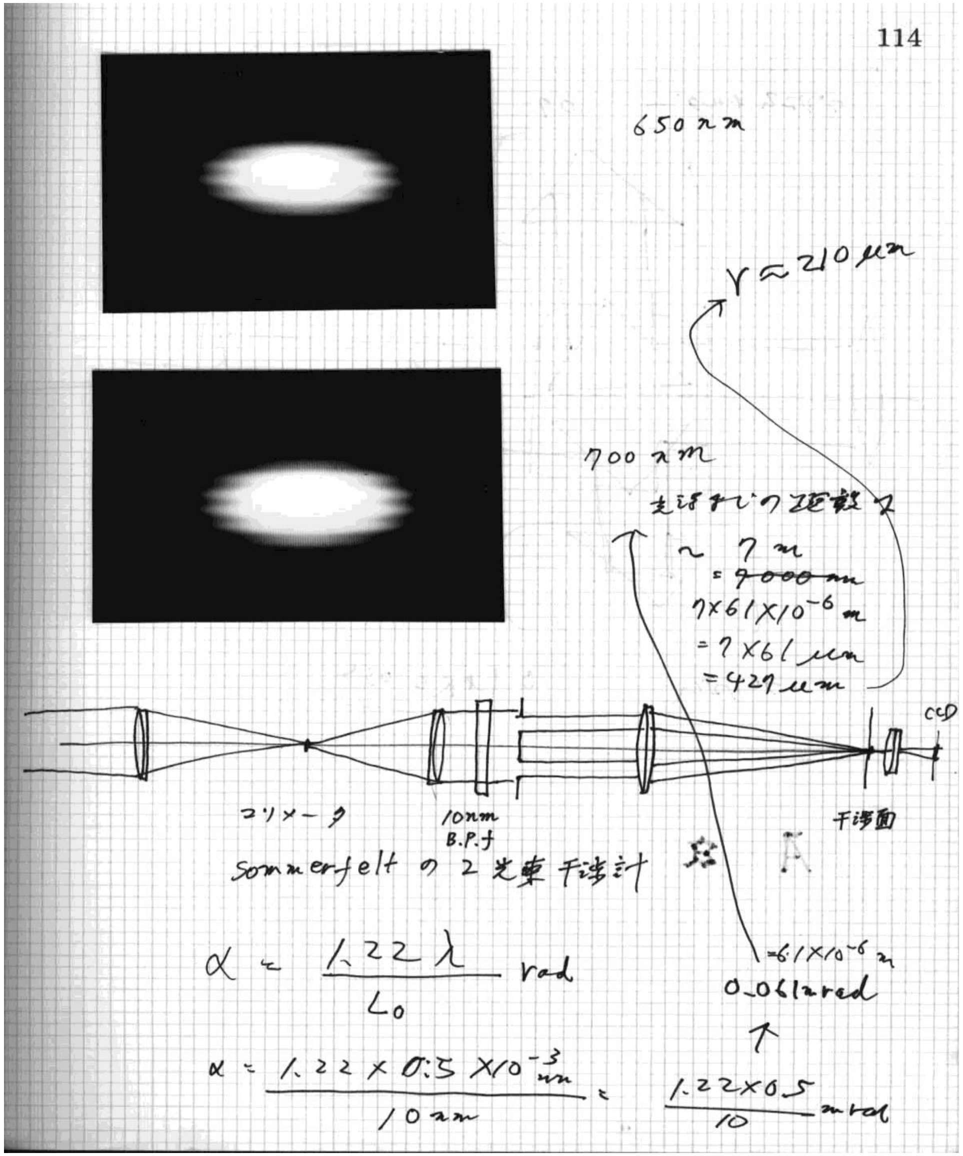
wavelength: λ=550nm

Visibility $$\gamma(d) = e^{\frac{-d^2}{2\sigma_d^2}}$$

$$\sigma_d = \frac{\lambda L}{2\pi\sigma_y} = \frac{550\times10^{-9}\cdot10}{2\pi\cdot20\times10^{-6}} = 44mm$$

a~2 mm

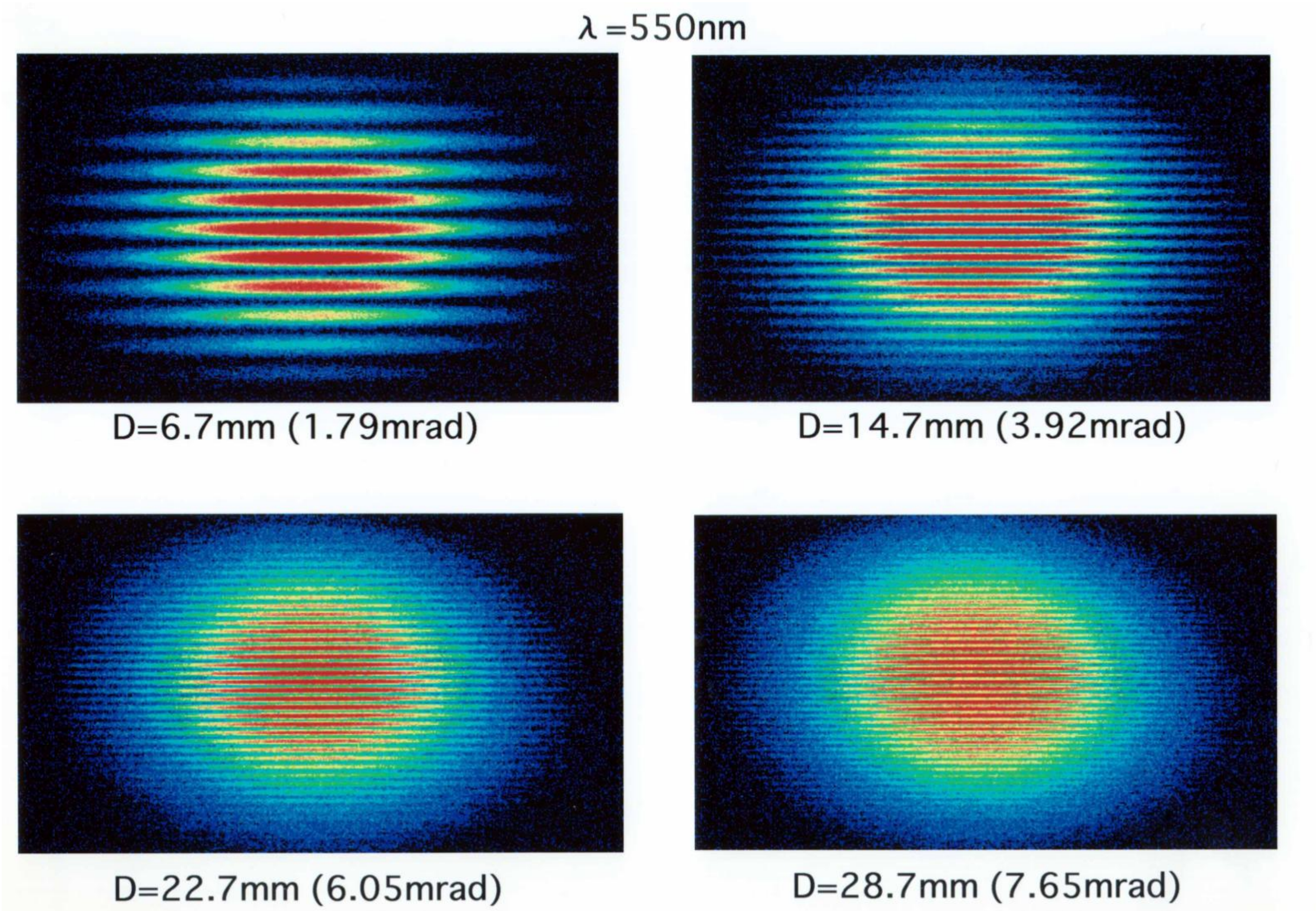
d~40 mm



Result of beam size is 210µm

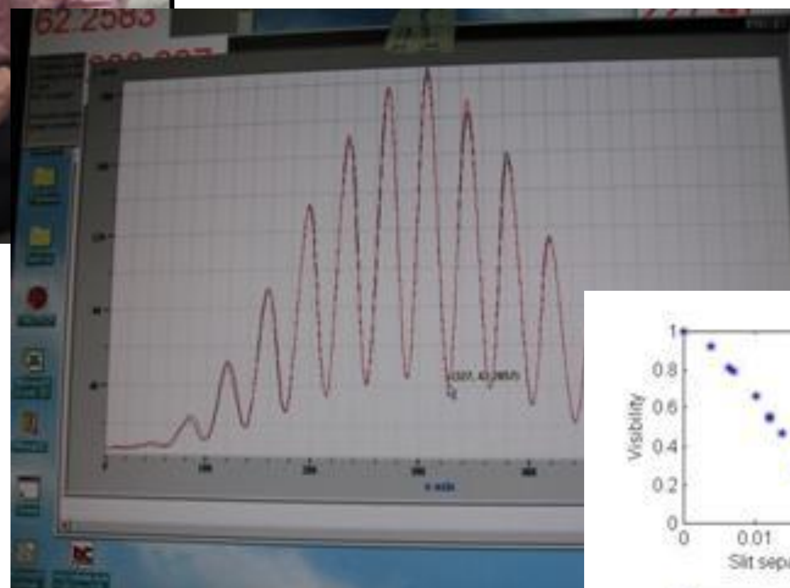
compliments: T. Mitsuhashi

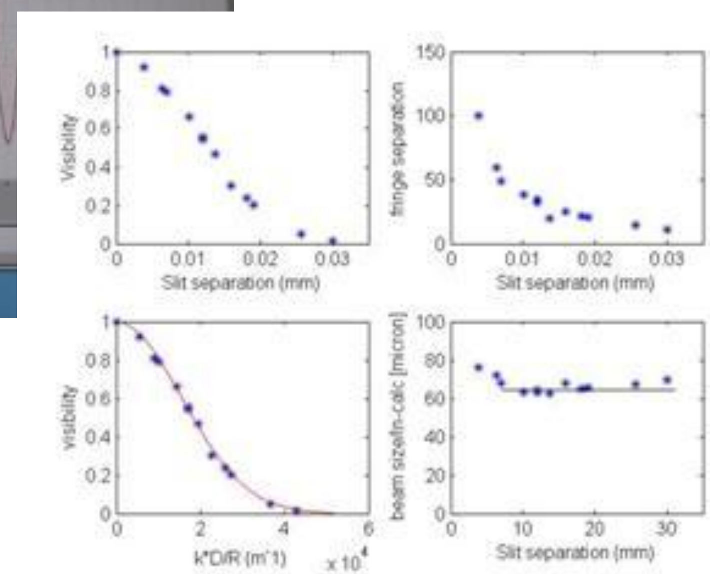
Vertical beam size at the SR center of Ritsumeikan university AURORA.



compliments: T. Mitsuhashi

Photon Factory Laboratory





Beam Size Measurement (cont'd)

From a single measurement at slit separation d_o

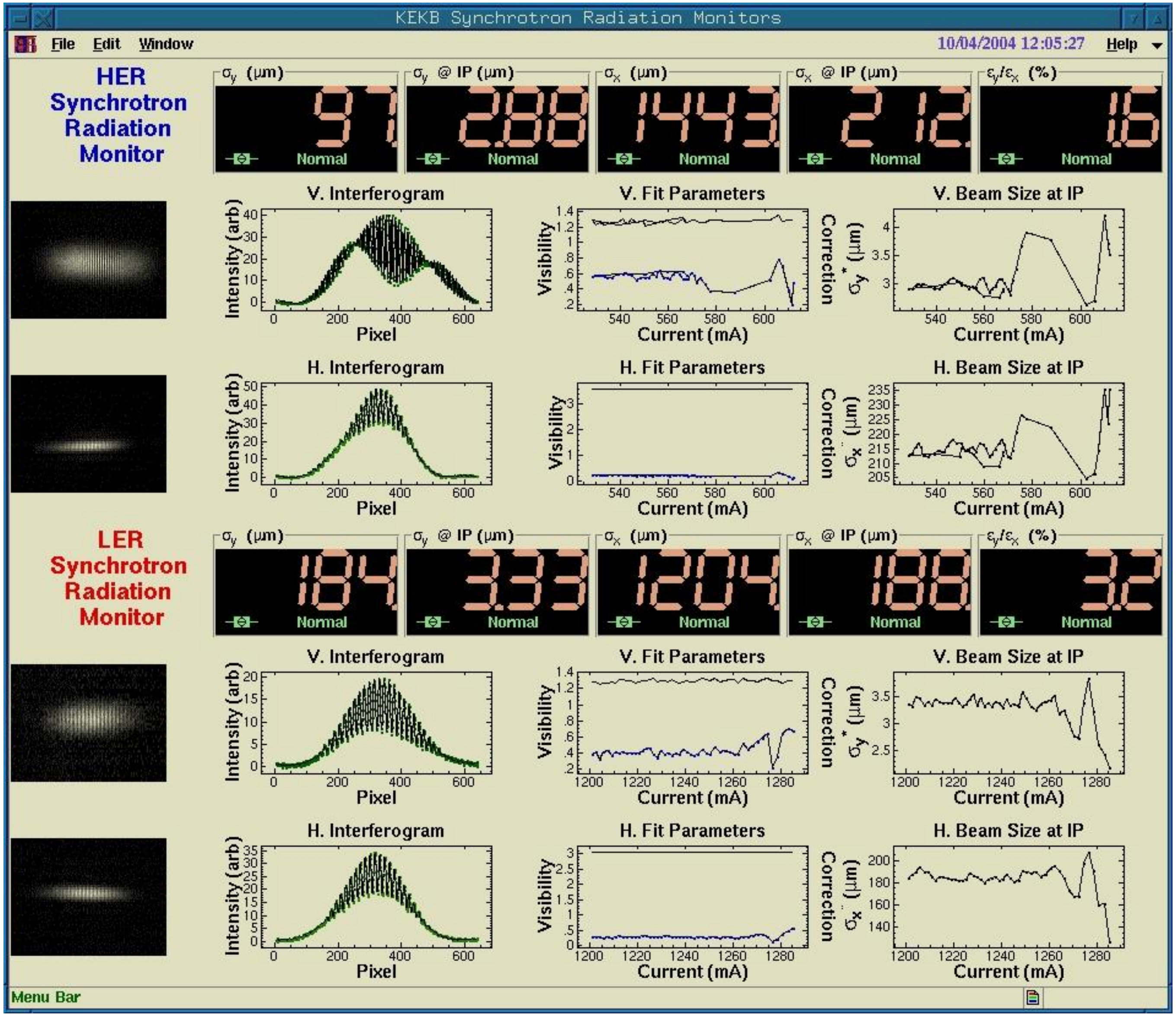
$$\gamma(d_o) = e^{\frac{-d_0^2}{2\sigma_d^2}}$$ solve for $$\sigma_d = \sqrt{\frac{d_o}{\ln(\frac{1}{\gamma})}}$$

From $$\sigma_y = \frac{2\pi}{\lambda L \sigma_d}$$

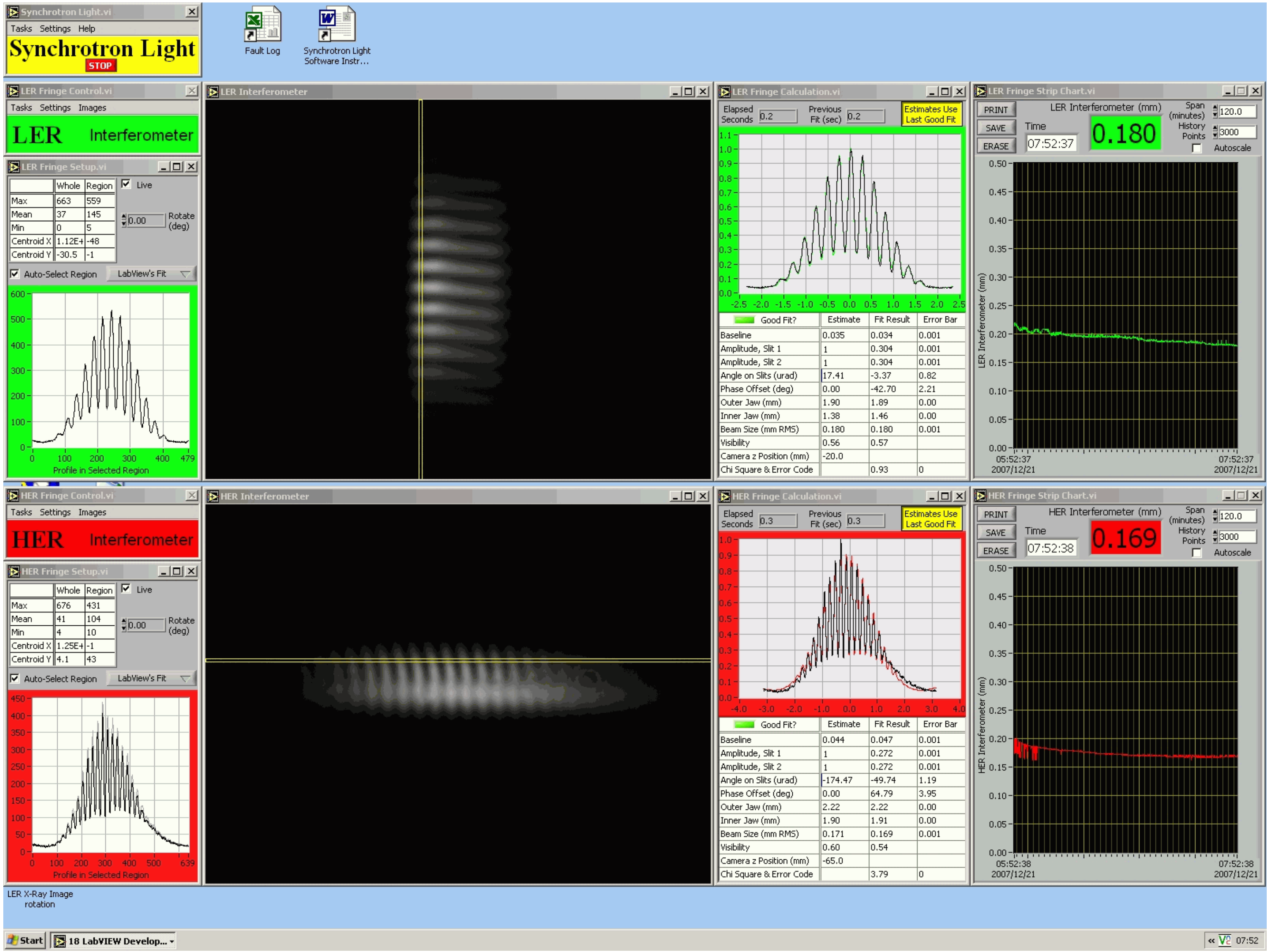
$$\sigma_y = \frac{\lambda L}{\pi d_o}\sqrt{\frac{\ln(\frac{1}{\gamma})}{2}}$$

on-line diagnostic for slit separation d_o

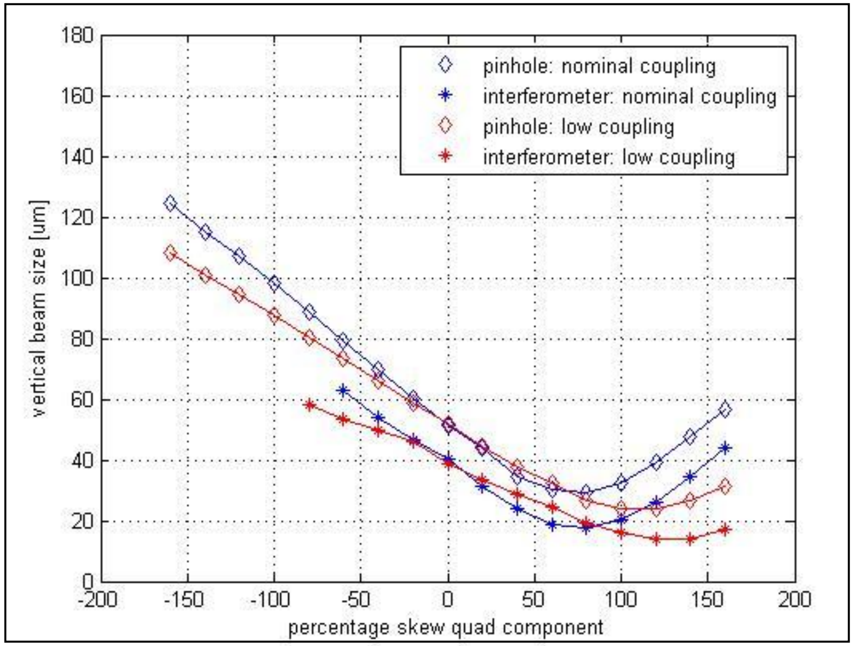
KEK-B on-line system

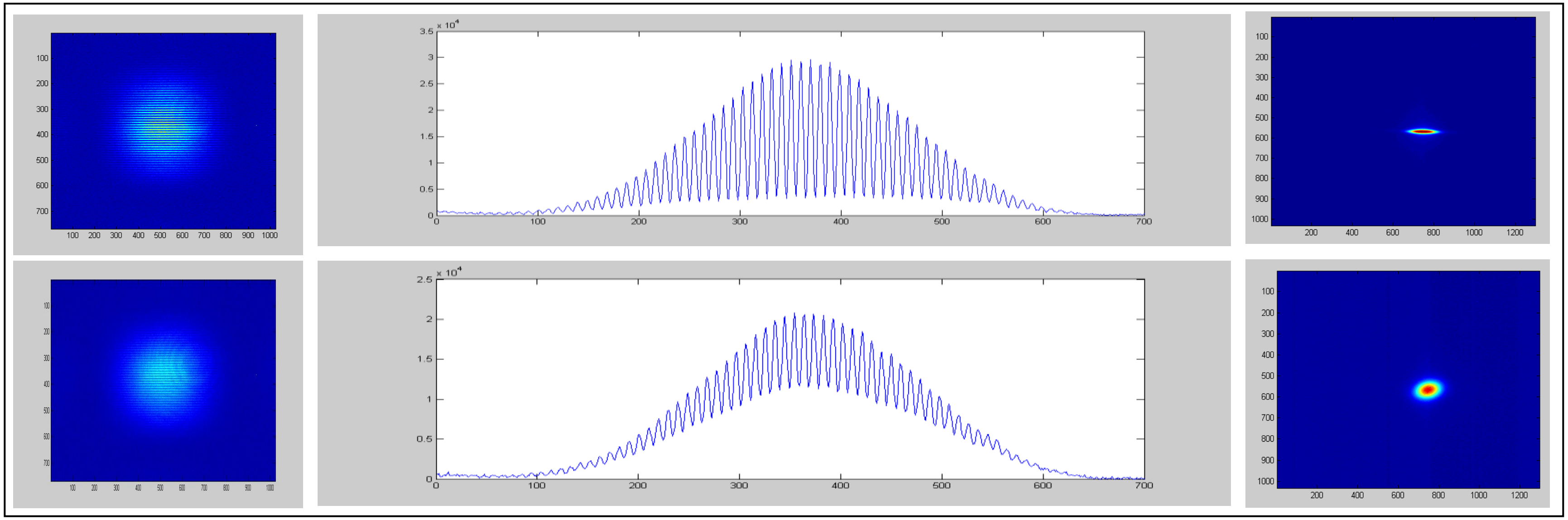


PEP-II on-line system

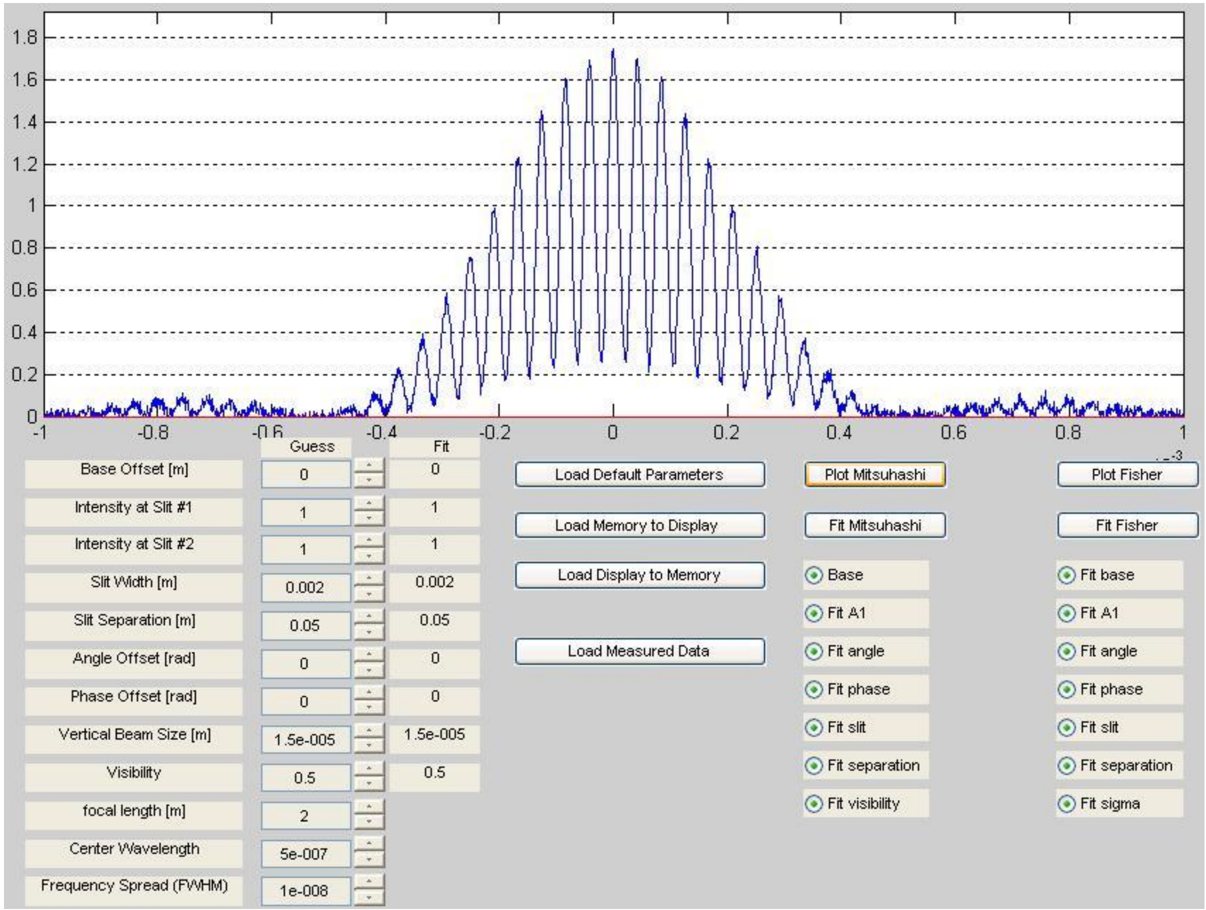


SPEAR-3 coupling measurements





USPAS Simulator



Practical Issues

- Thermal distortion of mirrors – wavefront distortion

- Precision control of slit width (I_1 and I_2)

- Depth of field effects

- CCD camera linearity

- Table vibrations

- Readout noise

- Beam stability

- Numerical fitting

Michelson's Interferometer – Summary

- Interferometers useful below the diffraction limit
- Two-slit Interference – Young's experiment
- Diffraction from a single slit
- Extended Source – Partial Coherence
- Visibility and the Mutual Coherence function
- Van-Cittert/Ziernike theorem: Fourier XFRM
- Stellar Interferometers for SR applications