Imaging a Beam with Synchrotron Light

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Beam Diagnostics Using Synchrotron Radiation:
Theory and Practice
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Power Radiated by the Beam

- A later lecture derives the spectrum of synchrotron radiation from a highly relativistic electron. For now, a few results:
  - Power radiated while traveling along an orbit with radius of curvature ρ

    $$P_s = \frac{2}{3}\frac{\gamma^4 r_e m_e c^3}{\rho^2}$$

    where the "classical electron radius" is $r_e = \dfrac{e^2}{4\pi\varepsilon_0 m_e c^2} = 2.818\times 10^{-15}$ m
  - The factor of γ^4 makes this power substantial. For example, 2 A of 9-GeV electrons in the PEP-II high-energy ring (HER) radiate

    $$P_s I_{\mathrm{HER}}/ec = 6.8\ \mathrm{kW/m}$$

    in the dipole magnets. The total power lost around the whole ring is

    $$2\pi\rho_{\mathrm{HER}} P_s I_{\mathrm{HER}}/ec = 7.0\ \mathrm{MW}$$

    where $\rho_{\mathrm{HER}} = 165$ m in the 192 arc dipoles.

The Electron's Power Spectral Density

- Power per unit frequency ω and solid angle Ω

$$\frac{d^2P}{d\Omega d\omega} = \frac{\gamma P_s}{\omega_c} F_s(\omega,\psi) = \frac{\gamma P_s}{\omega_c}\left[F_{s\sigma}(\omega,\psi) + F_{s\pi}(\omega,\psi)\right]$$

  - where the terms give the power in the two polarization components:
    - $F_{s\sigma}$: In the plane of the bend (typically the horizontal plane)
    - $F_{s\pi}$: Perpendicular to the bend plane (and so typically vertical)

$$F_{s\sigma} = \left(\frac{3}{2\pi}\right)^3 \left(\frac{\omega}{2\omega_c}\right)^2 \left(1+\gamma^2\psi^2\right)^2 K_{\frac{2}{3}}^2\left[\frac{\omega}{2\omega_c}\left(1+\gamma^2\psi^2\right)^{\frac{3}{2}}\right]$$

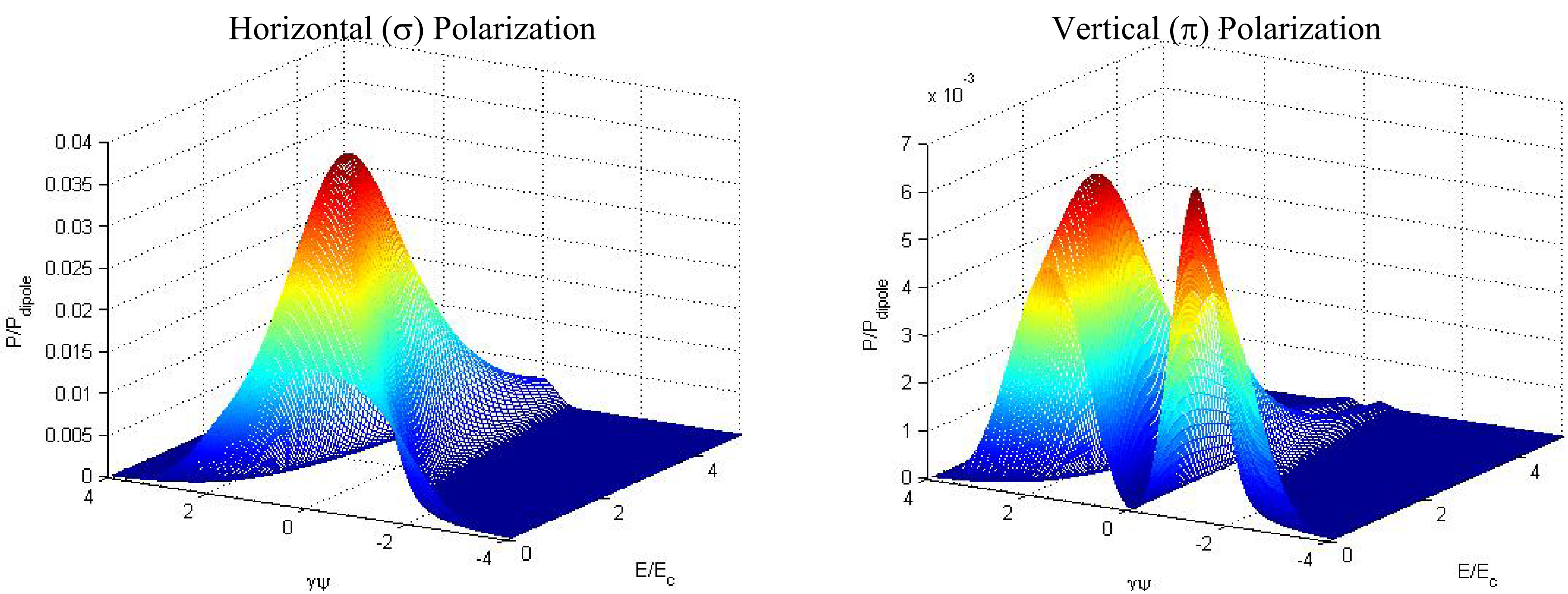
$$F_{s\pi} = \left(\frac{3}{2\pi}\right)^3 \left(\frac{\omega}{2\omega_c}\right)^2 \gamma^2\psi^2\left(1+\gamma^2\psi^2\right) K_{\frac{1}{3}}^2\left[\frac{\omega}{2\omega_c}\left(1+\gamma^2\psi^2\right)^{\frac{3}{2}}\right]$$

  - ψ is the vertical angle, and $K_{1/3}$ and $K_{2/3}$ are modified Bessel functions.

Critical Energy E_c

- The "critical energy" is $E_c = \hbar\omega_c = \frac{3}{2}\gamma^3\hbar\frac{c}{\rho}$
  - ω_c is the revolution frequency c/ρ, but scaled by γ^3.
    - This large factor moves the frequency from MHz to x-rays.
    - For PEP's HER dipoles, $E_c = 9.8$ keV.
- Half of the total energy is below E_c, half above.
  - Almost all power is emitted as hard photons.
  - Visible light is far into the tail.

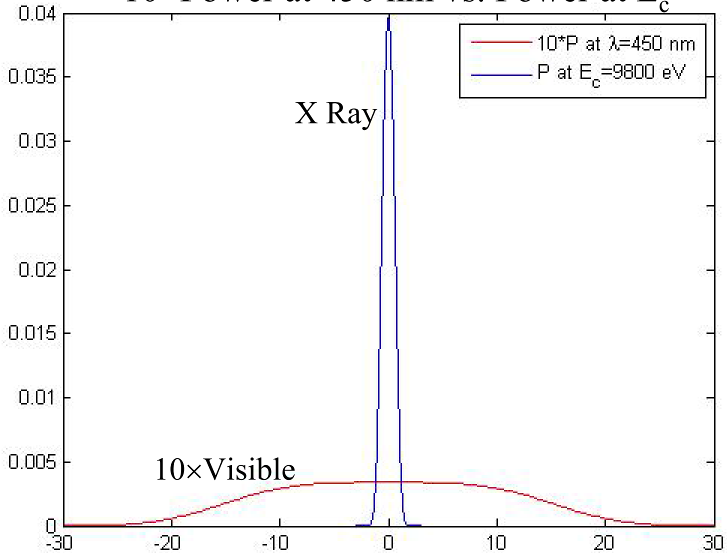
Polarization



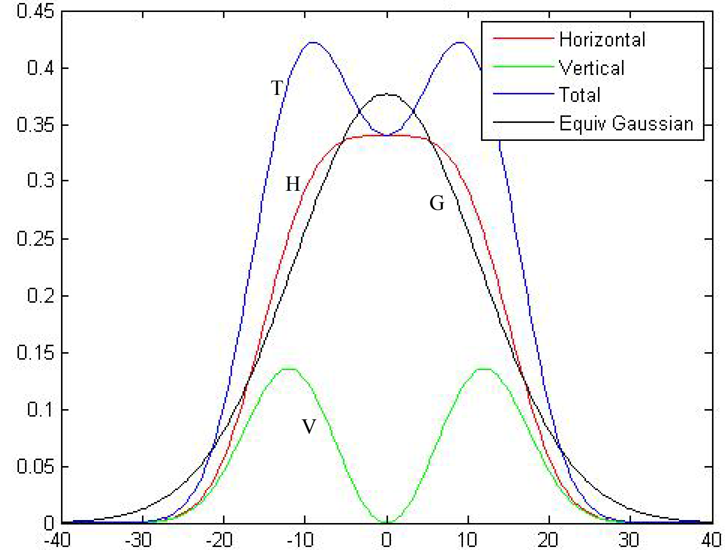
- The peak of the horizontal is 6 times higher than the vertical.
  - This peak is a little below the critical energy.
- The vertical component is zero on the midplane ($\psi = 0$).
  - Symmetry: No preference for upward versus downward field.
- The range of angles spanned is much wider at low energies.

Visible Light versus Vertical Angle

Horizontally Polarized Power:
10×Power at 450 nm vs. Power at E_c



Horizontally and Vertically Polarized Power
at 450 nm and Best-Fit Gaussian



Normalized Vertical Angle $\gamma\psi$

- Power in visible is much weaker and broader than for x rays (above left, for HER).
  - For x rays ($E \sim E_c$), the angular width (the "opening angle") is roughly $1/\gamma$.
  - For visible ($E << E_c$), a Gaussian approximation (above right) to the horizontal polarization has an RMS width of: $\sigma_\psi \approx \frac{0.75}{\gamma}\left(\frac{E_c}{E}\right)^{\frac{1}{3}}$
- Horizontally polarized power is roughly Gaussian, but with flatter top, smaller tails.
  - Area of horizontal is 3 times area of vertical.

Peculiarities of Synchrotron-Light Imaging

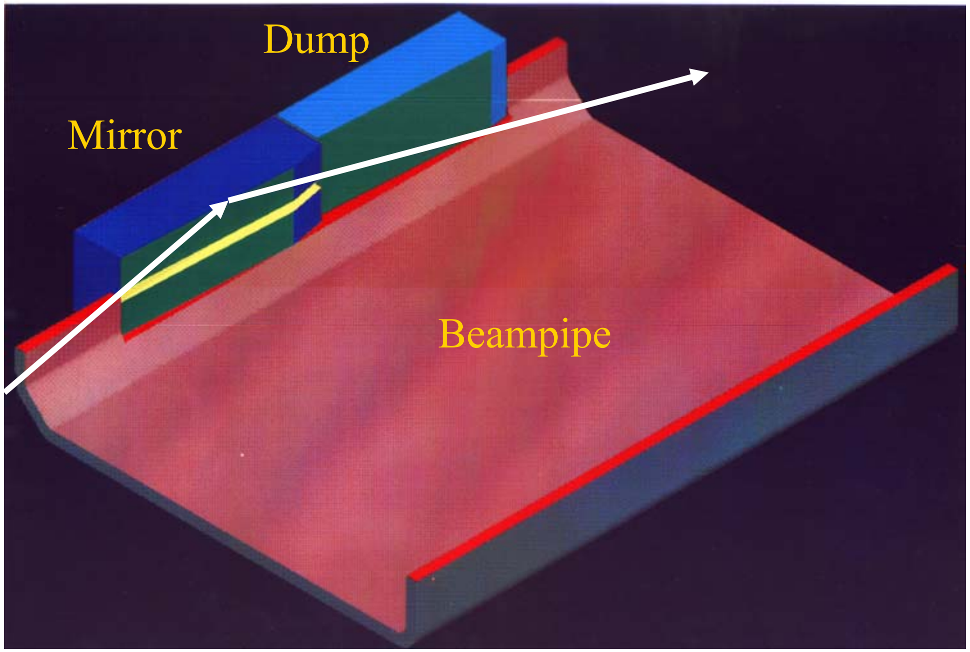
- Typical imaging situation:
  - Object reflects unpolarized incident light in all directions.
  - Lens catches some light from almost any angle.
  - Most objects have more transverse extent than depth.
- Synchrotron Light:
  - The source of the light is the object's own emission.
  - Light is radiated only in the forward direction, tangent to the beam's instantaneous circular path through a bend.
    - Vertically: The beam lights up narrow forward-directed cone.
    - Horizontally: The beam paints a stripe of light along the midplane of the vacuum chamber as bends.
    - Like a car rounding a bend in the dark with its bright headlights on.
  - Longitudinal profile is Gaussian at any instant.
    - But over the exposure time, the source goes around the ring many times.
    - We want to measure a glowing, curved string by imaging it from a tangent.

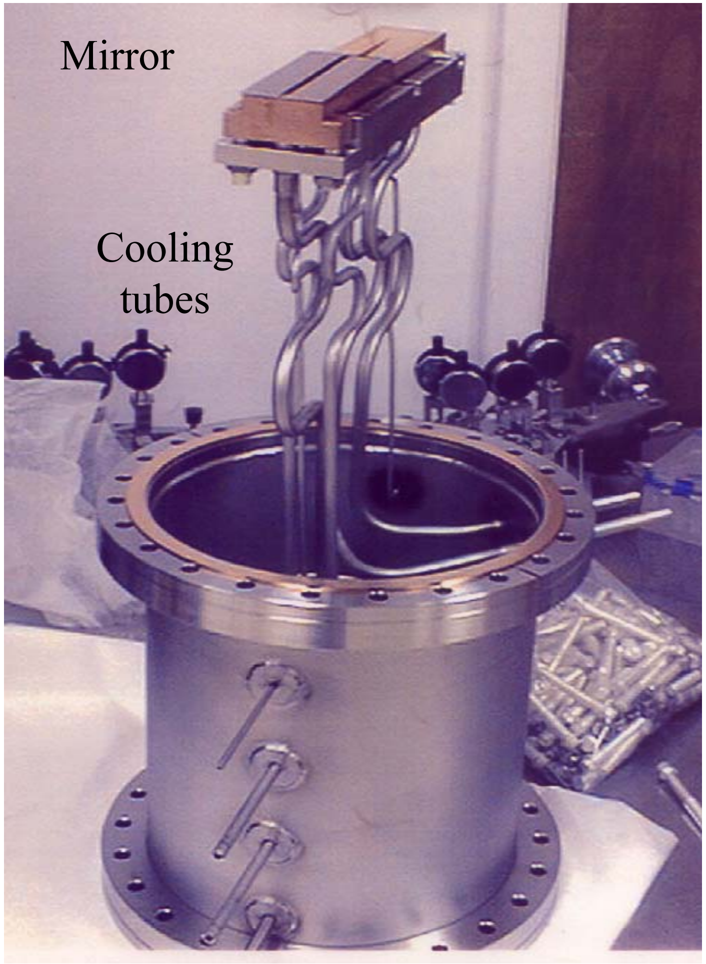
Choosing a Wavelength: Visible

- Visible light has advantages (in addition to being easy to see):
  - You can use common parts like windows, lenses, mirrors, video cameras, and specialized instruments like streak cameras.
  - The wide opening angle is wider than at the critical wavelength.
    - Remove the narrow fan with most of the heat, without much loss to the visible image, in order to avoid thermal distortion of the first mirror (M1).
      - “Cold finger”: a narrow cooled mask on the midplane, upstream of M1, that blocks the x-ray fan while casting a thin shadow across the mirror.
      - Slot along the middle of M1, so that the x-rays pass through and can’t heat it.
      - A thin, low-Z substrate (beryllium) for M1, to transmit most of the x-rays.
- But diffraction limits the resolution at longer wavelengths.
  - An important consideration when the beam is small.
    - Image a point near a defocusing quad, where the beam is large vertically.
  - A cold finger or slot also adds diffraction.
  - Drives the design toward shorter wavelengths.
    - At least blue rather than red, but sometimes ultraviolet or x rays.

Slotted First Mirror in PEP-II

- Midplane slot allows x-ray fan to bypass M1 and strike the thermal dump.
  - But slot is a mechanical weak point.
  - PEP's x-ray fan is very narrow and hot, making a cold finger difficult.





"Cold Finger" Mask in SPEAR-3

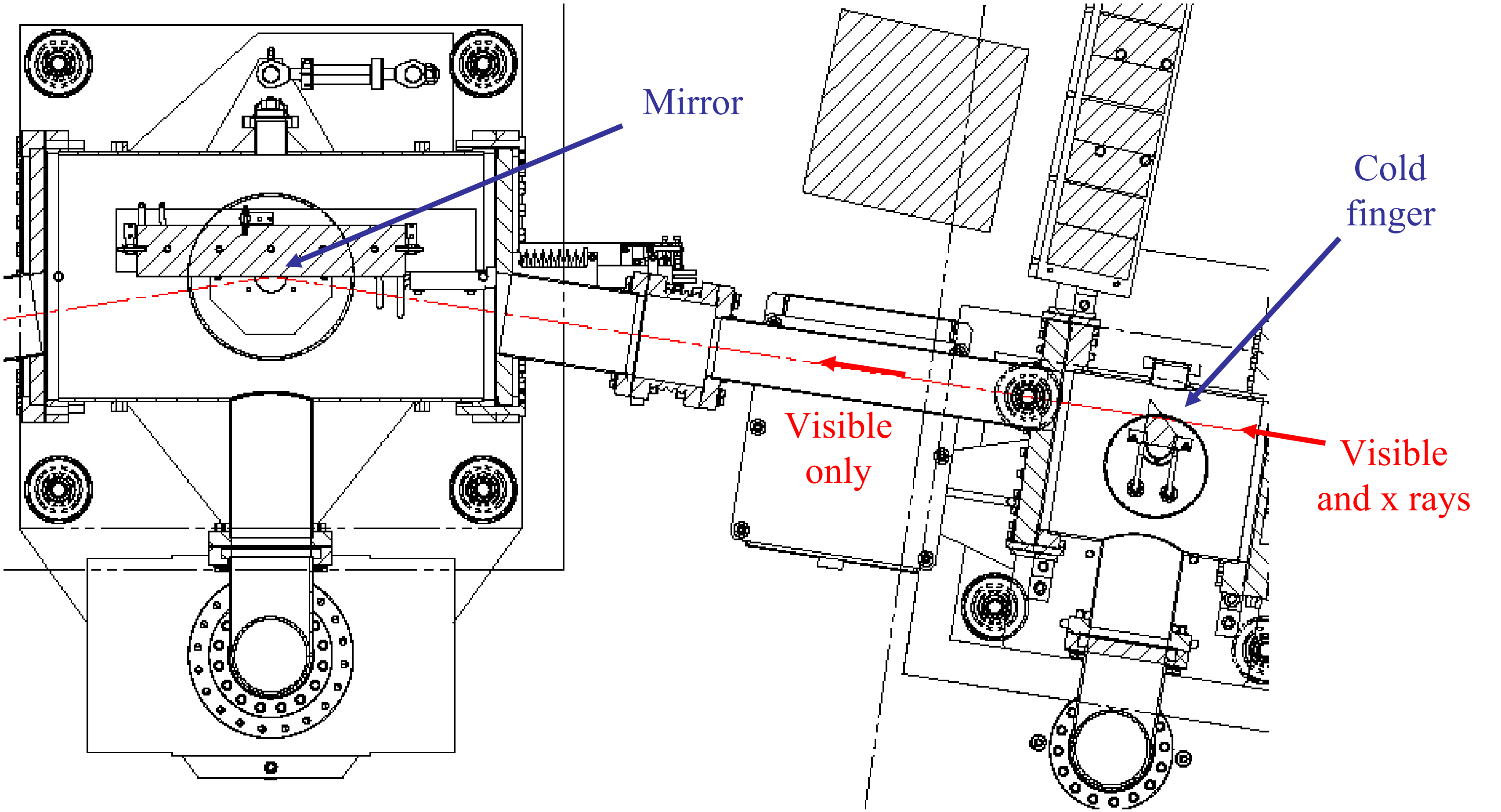
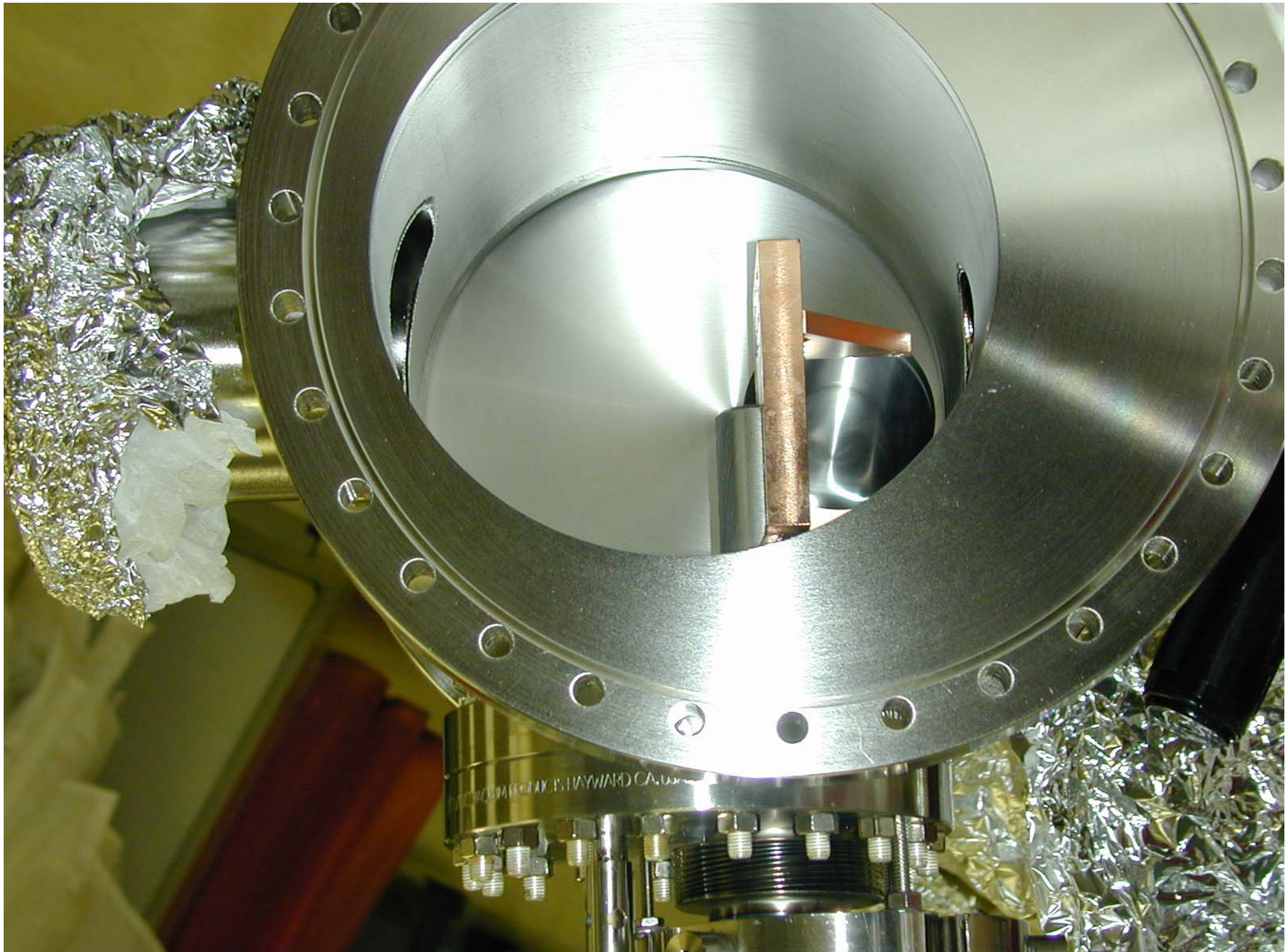
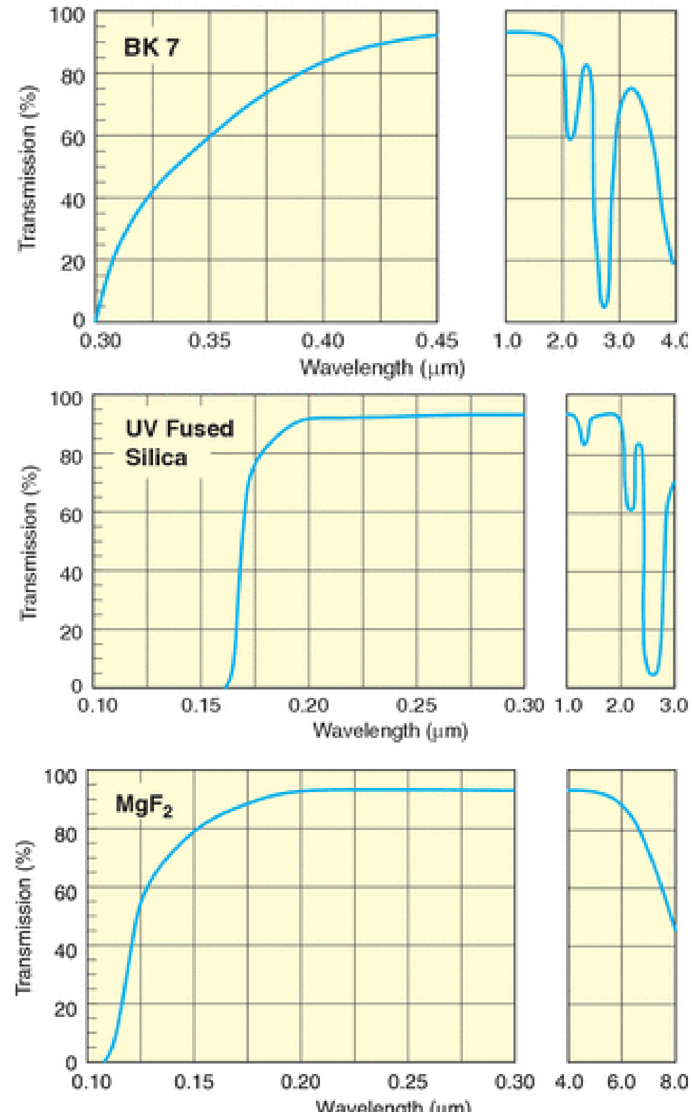


Photo of Cold Finger in SPEAR-3



Choosing a Wavelength: Ultraviolet

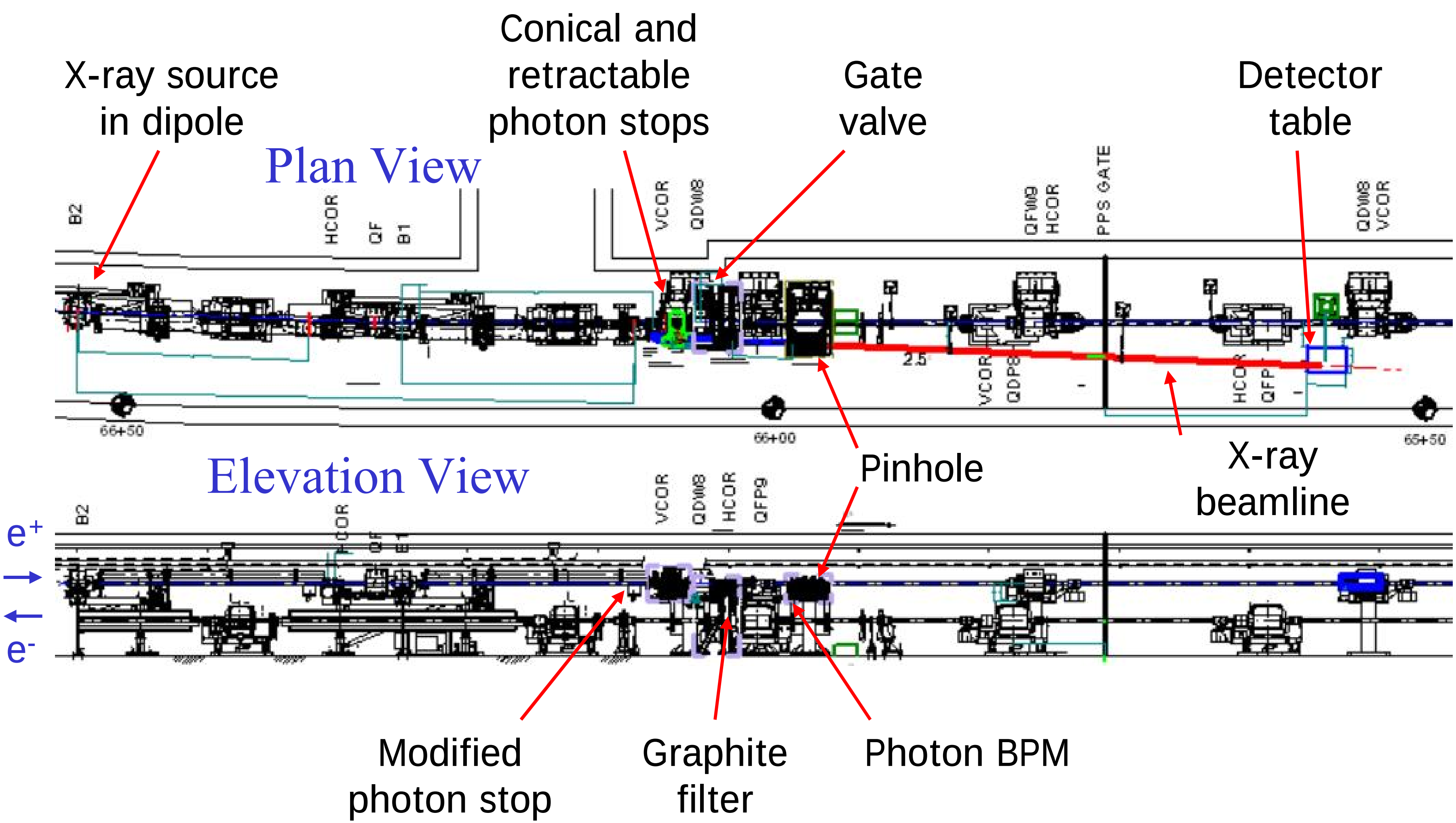
- Can't go far into the UV without problems.
  - Window and lens materials become opaque:
    - Glasses (like BK7 at right) are useful above ~330 nm.
    - Fused silica works above ~170 nm.
    - Special materials like MgF_2 work above ~120 nm.
  - Absorption in air below ~100 nm
    - Must use reflective optics in vacuum.



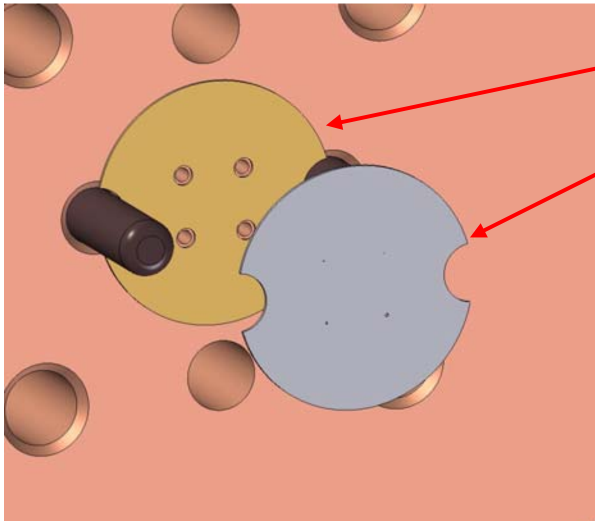
Choosing a Wavelength: X Rays

- The good news: Most of the beam’s emission is in the x-ray range.
- The bad news: How do you form an image?
  - You can use a simple pinhole camera.
    - But this throws out most of the light.
    - Must absorb this power before the pinhole, or it will get too hot.
  - Other imaging optics are difficult:
    - Grazing-incidence optics
    - Zone plates

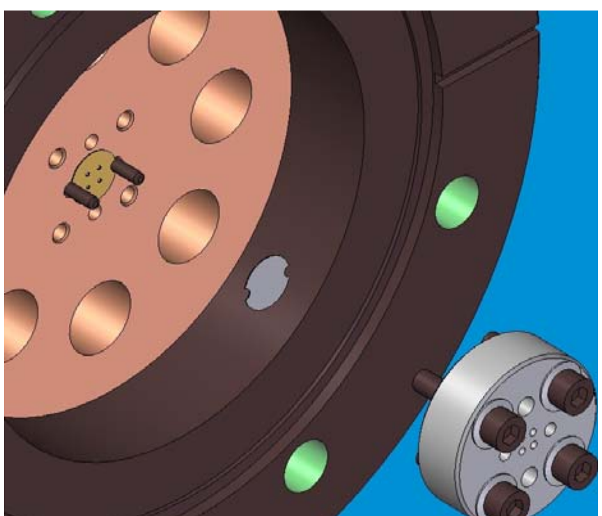
X-Ray Pinhole Camera in the PEP-II LER

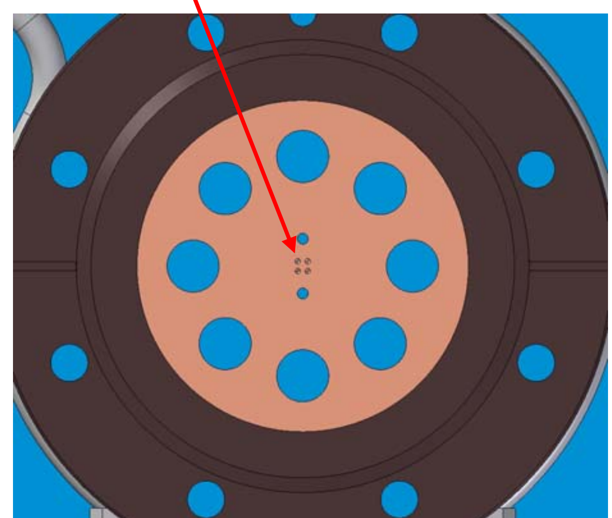


Design of the Pinhole Assembly

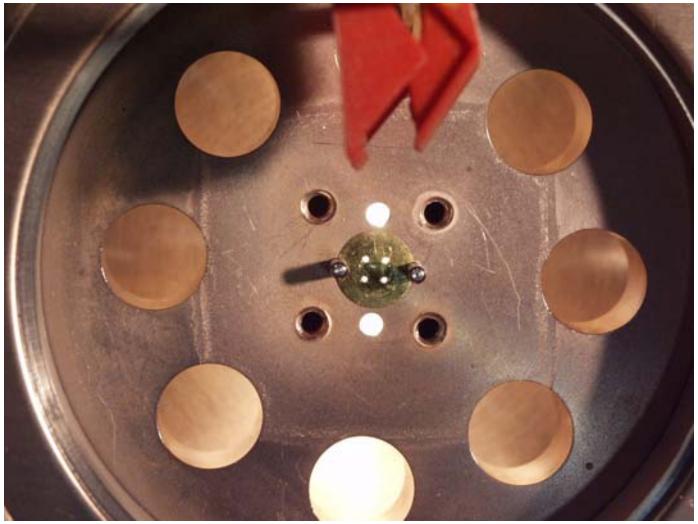
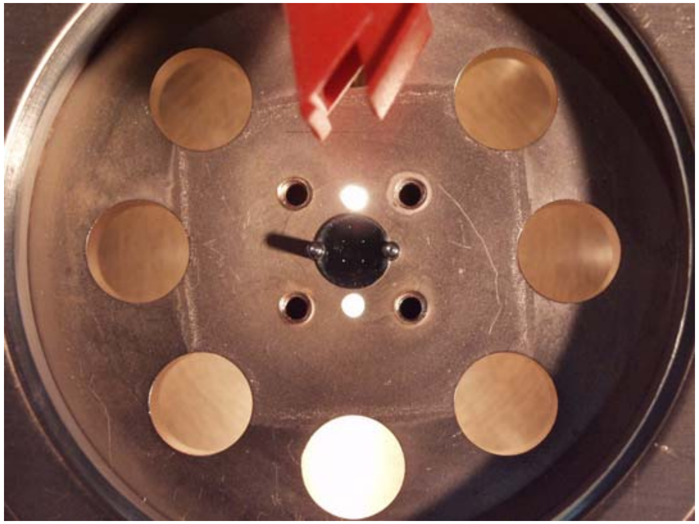
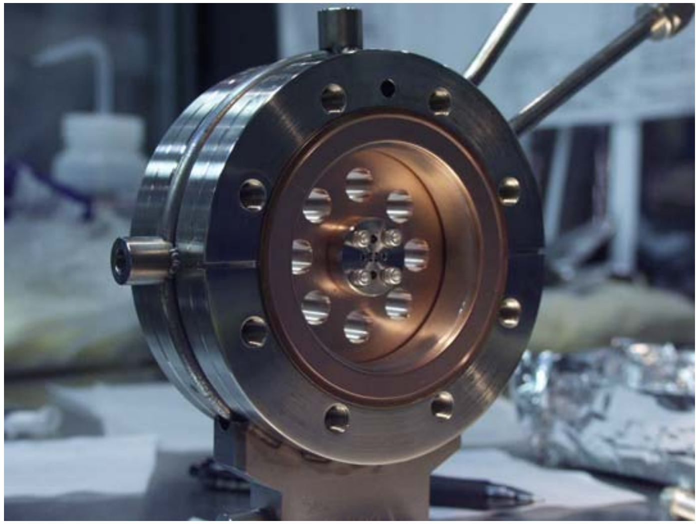
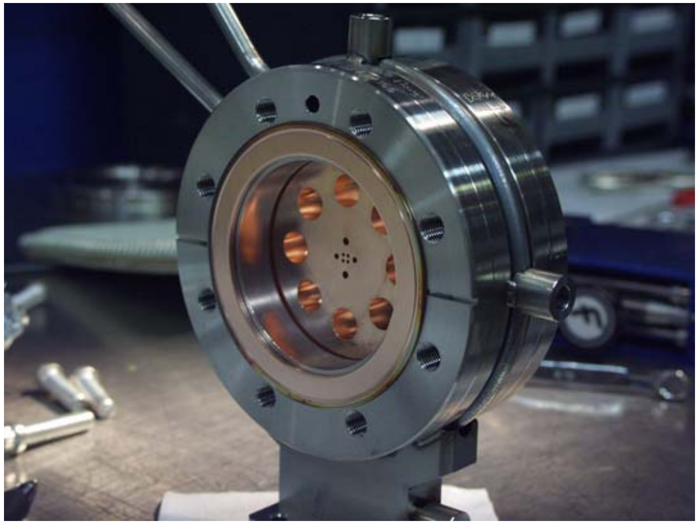


- Gold disk for heat transfer
- Pt:Ir (90:10) disk with 4 pinholes.
  - Diameters of 30, 50, 70, and 100 μm.
- Front: Glidcop with 4 larger holes

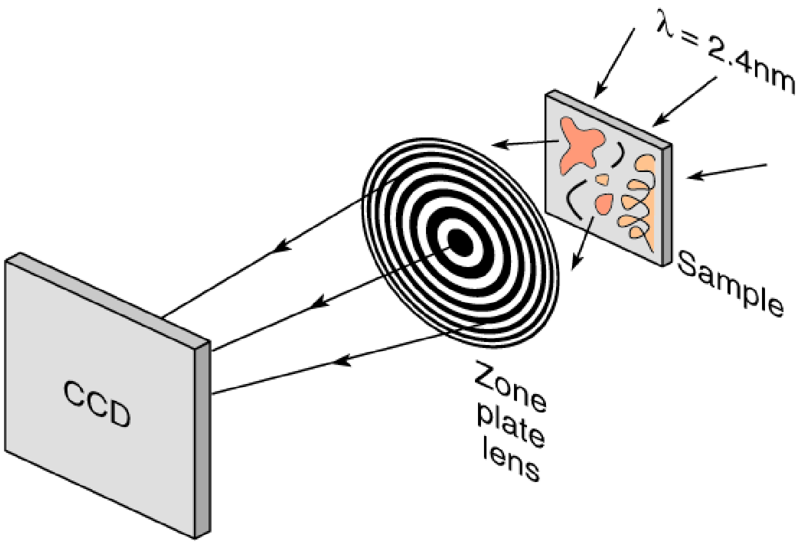


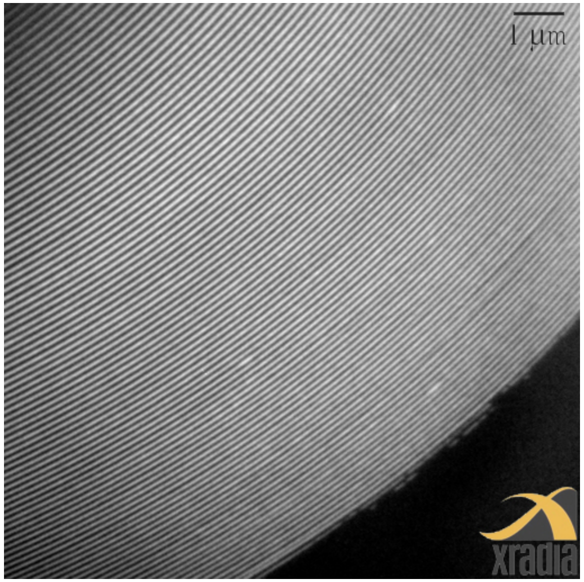


Photos of the Pinhole Assembly

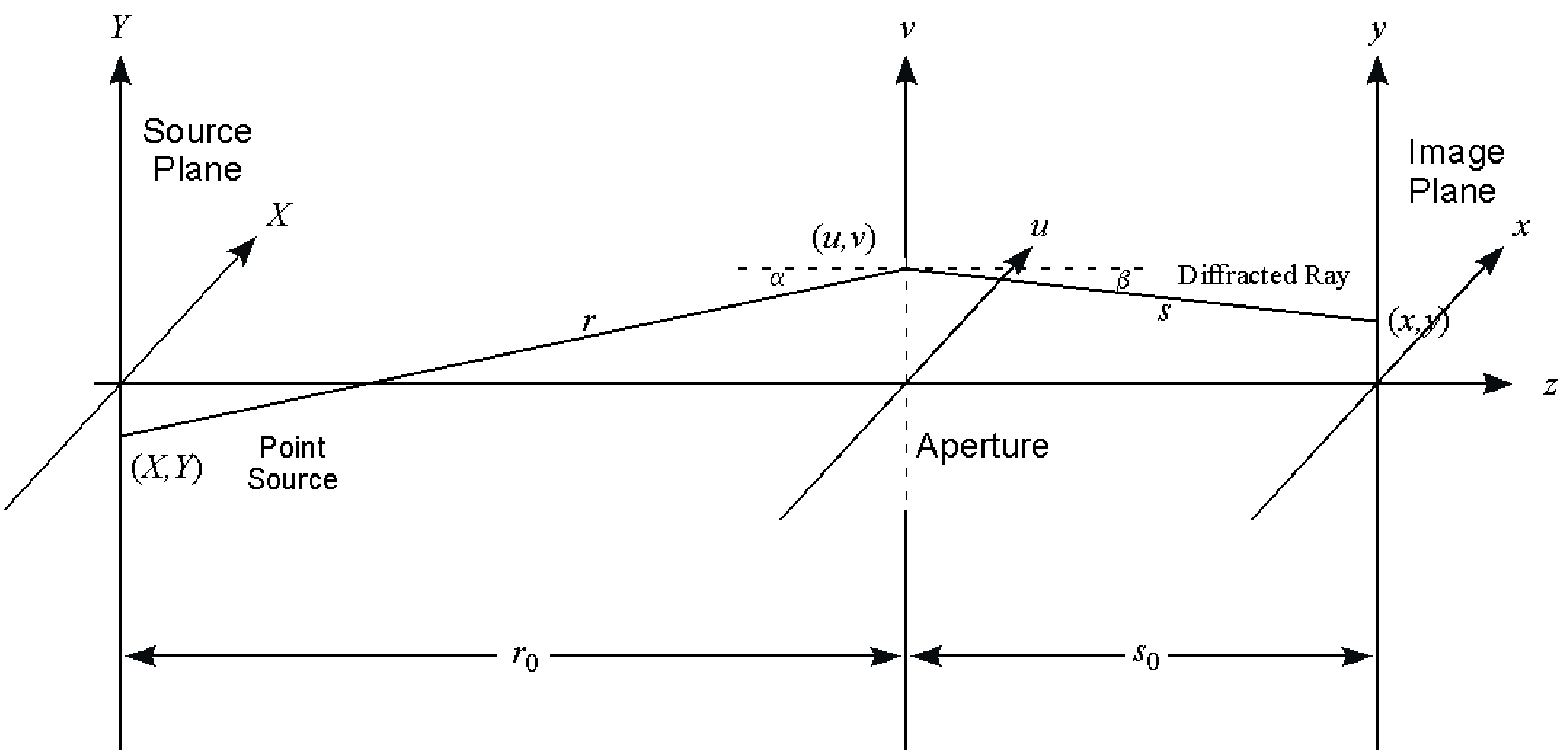
Zone-Plate Imaging





- A diffractive lens, made by lithography
- High-*Z* metal (gold) on thin membrane of low *Z* (SiN)
- Requires low power and a narrow bandwidth (≈1%)
  - Precede with a pair of multilayer x-ray mirrors, for narrow-band reflection and for absorption of out-of-band power.

Diffraction by an Aperture



Diffraction by an Aperture

- All points in an aperture are considered point sources, reradiating light incident from a point source
  - Wavelength is $\lambda = 2\pi/k$.
- The field at (x,y) is given by a Fresnel-Kirchhoff integral over the (small) aperture:

$$E(x,y) = -\frac{Ai}{2\lambda} \iint_{\text{aperture}} \frac{e^{ik(r+s)}}{rs} (\cos\alpha + \cos\beta)\, dS$$

$$\approx -\frac{Ai}{2\lambda r_0 s_0} (\cos\alpha + \cos\beta) \iint_{\text{aperture}} e^{ik(r+s)}\, dS$$

  - Everything is essentially constant except the phase from each point in the aperture.

Expanding the Phase

$$r = \sqrt{(X-u)^2 + (Y-v)^2 + r_0^2} \approx r_0 + \frac{(X-u)^2 + (Y-v)^2}{2r_0}$$

$$s = \sqrt{(x-u)^2 + (y-v)^2 + r_0^2} \approx s_0 + \frac{(x-u)^2 + (y-v)^2}{2s_0}$$

$$e^{ik(r+s)} \approx \exp\left[ik\left(r_0 + s_0 + \frac{X^2+Y^2}{2r_0} + \frac{x^2+y^2}{2s_0}\right)\right]\exp\left[ik\left(\frac{u^2+v^2}{2r_0} + \frac{u^2+v^2}{2s_0}\right)\right]\exp\left[-ik\left(\frac{Xu+Yv}{r_0} + \frac{xu+yv}{s_0}\right)\right]$$

- First term: Independent of the aperture coordinates *u*, *v*.
  - Contributes only an overall phase to the *uv* integral over the aperture.
- Second: Small (since aperture is small) quadratic terms in *u*, *v*.
- Third: Products of *u*, *v* with cosines (X/r_0, etc.) of ray angles from the source or measurement points to the *x* and *y* axes.
  - The only term that matters in the integral over the aperture is:

$$e^{ik(r+s)} \approx \exp\left[-ik\left(pu+qv\right)\right]$$

where $p = X/r_0 + x/s_0$ and $q = Y/r_0 + y/s_0$

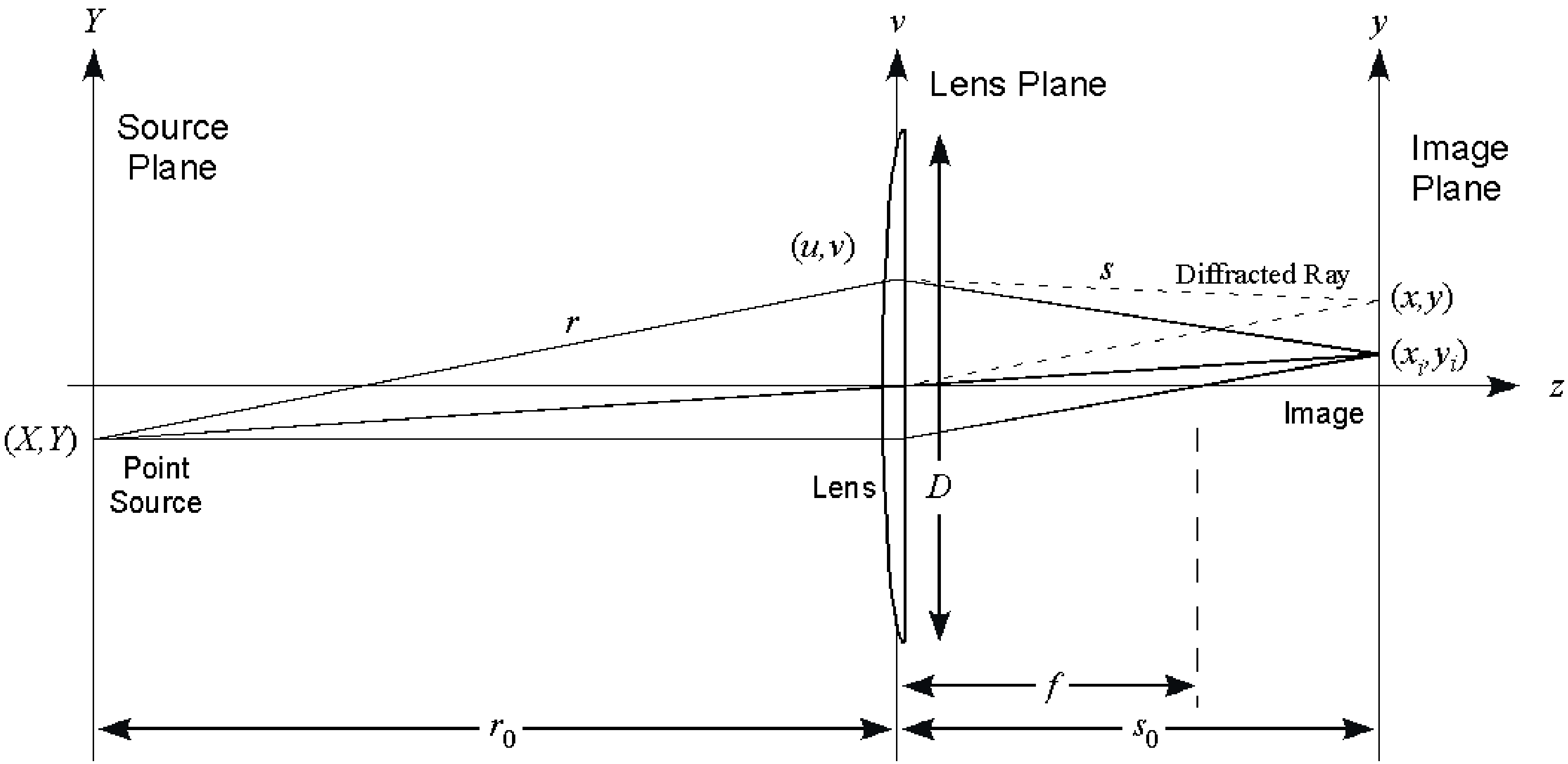
Spatial Fourier Transform

- The diffraction pattern on the *xy* plane becomes a Fourier transform in the spatial coordinates *uv* of the aperture:

$$E(x,y) = -\frac{Ai}{2\lambda r_0 s_0}(\cos\alpha + \cos\beta) \iint_{\text{aperture}} e^{-ik(pu+qv)} du dv$$

- Laser light is sometimes focused through a pinhole to remove noisy, non-Gaussian parts of the transverse profile.
  - The hole forms a *spatial filter*: since the noise is found at high *spatial frequencies*, which appear at larger values of *u* and *v*, it can be clipped by a properly sized hole.

Diffraction by a Lens



Diffraction by a Lens: Path Length

- All paths from source (X,Y) to image (x_i,y_i) have *equal* length.
  - A fundamental property of geometric imaging.
  - The phase difference in the *uv* integral arises from the different paths from (X,Y) to (x,y), compared to the equal paths from (X,Y) to (x_i,y_i).
  - It is helpful to subtract this reference path, so that the phase difference becomes the difference between (u,v) to (x,y) and (u,v) to (x_i,y_i).

$$\sqrt{(x-u)^2+(y-v)^2+s_0^2}-\sqrt{(x_i-u)^2+(y_i-v)^2+s_0^2}$$
$$\approx -\frac{(x-x_i)u+(y-y_i)v}{s_0} = -\frac{\rho w}{s_0}\cos(\phi-\psi)$$

  - Here we used polar coordinates: $(u,v)\rightarrow(w,\psi)$ and $(x\text{-}x_i,y\text{-}y_i)\rightarrow(\rho,\phi)$

Diffraction by a Lens: Result

- The diffraction integral (neglecting constants) becomes:

$$E(x,y) = \int_0^{2\pi}\int_0^{D/2} \exp\left[-ik\frac{\rho w}{s_0}\cos(\phi-\psi)\right] w\,dw\,d\psi$$

$$= 2\pi\int_0^{D/2} J_0\left(\frac{k\rho w}{s_0}\right) w\,dw = \left(\frac{\pi D^2}{4}\right)\frac{2J_1\left(\dfrac{k\rho D}{2s_0}\right)}{\dfrac{k\rho D}{2s_0}}$$

where we have used two Bessel-function identities.

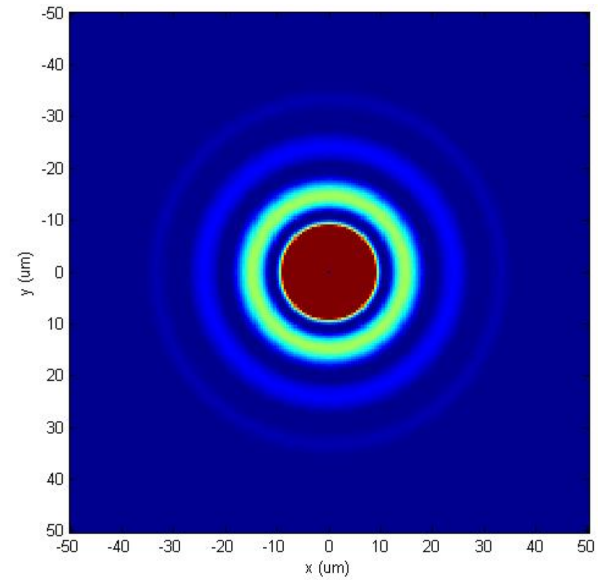
- This is called the Airy diffraction pattern.

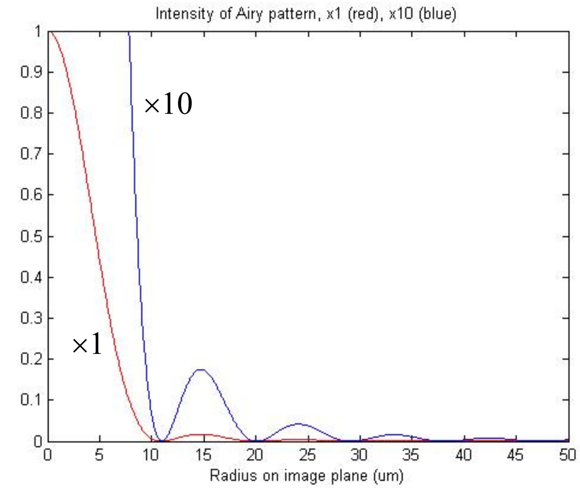
Diffraction by a Lens: Airy Pattern

- Concentric circles, with the first minimum at radius r_A:

$$r_A = 1.22\lambda\frac{L}{D} = 1.22F\lambda = 0.61\frac{\lambda}{\theta}$$

- F is called the "F-number" of the lens.
- θ is the half angle of the light cone illuminating the lens.
- Plot for $\lambda = 450$ nm, $D = 50$ mm, $s_0 = 1$ m
  - The central circle at right is saturated by a factor of 30 to highlight the faint rings.
  - The plot is expanded by 10 to show the rings.
- r_A is the resolution of the imaging system.
  - Compare it to the size of the geometric image to see if diffraction is a problem.





Diffraction of Synchrotron Radiation

- Approximating the half angle θ with 1 to 2 times the Gaussian angle σ_ψ gives a resolution (neglecting factors of order unity) of:

$$r_{SR} \approx 0.5 \frac{\lambda}{\sigma_\psi} \approx \rho^{\frac{1}{3}} \lambda^{\frac{2}{3}}$$

  - A difficult case: The HER of PEP-II has $\rho = 165$ m. For blue light at 450 nm, this resolution is

$$\rho^{\frac{1}{3}} \lambda^{\frac{2}{3}} = 0.32 \text{ mm}$$

- A more thorough treatment substitutes the SR power spectral density from the point source into a Fraunhofer diffraction integral over the area of the lens illuminated through the beamline aperture, finding the field at (x', y') on the image.

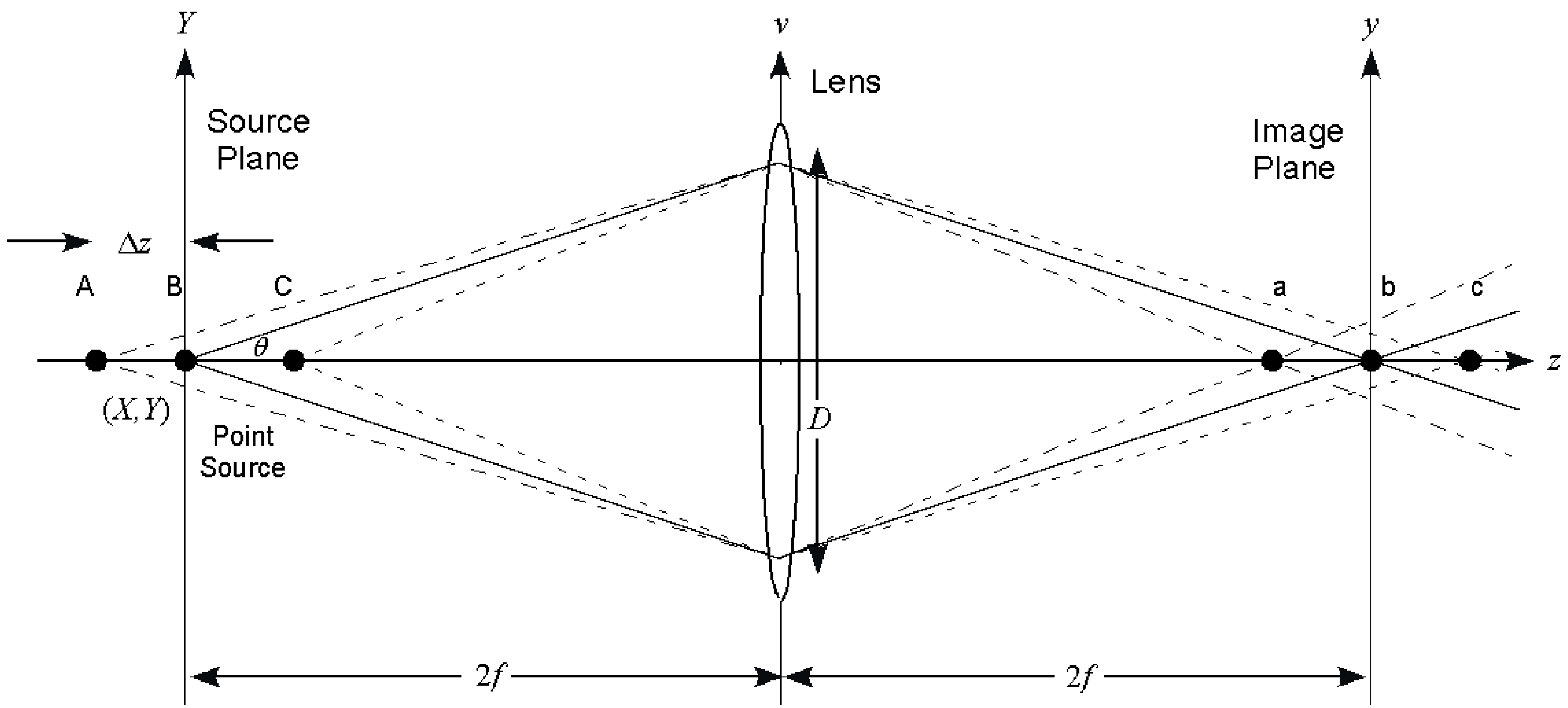
$$E(x', y') = A \int_{-x_a}^{x_a} dx \int_{-\infty}^{\infty} dy \frac{\gamma P_s}{\omega_c} F_s(\omega, \psi) e^{-ik(u'x + v'y)}$$

$$\text{with} \quad k = 2\pi/\lambda \quad u' = x'/L' \quad v' = y'/L'$$

- The first minimum of the intensity then gives the resolution.

Depth of Field

- Our light source is a long, gradual arc, not a plane.
  - What is the source distance?
  - Can it all be in focus?
  - How do you avoid blurring the measurement?



Depth of Field: A Quick Derivation

- Diameters of A and C images as they cross the *xy* plane, based on typical rays at angles $\pm\theta/2$:

$$d = 2\left|\frac{D/4}{2f \mp \Delta z}(\pm\Delta z)\right| \approx \frac{D\Delta z}{4f} = \theta\Delta z$$

- The vertical angle θ lighting the lens is roughly $2\sigma_\psi$.
- If we capture a similar portion of a horizontal arc:

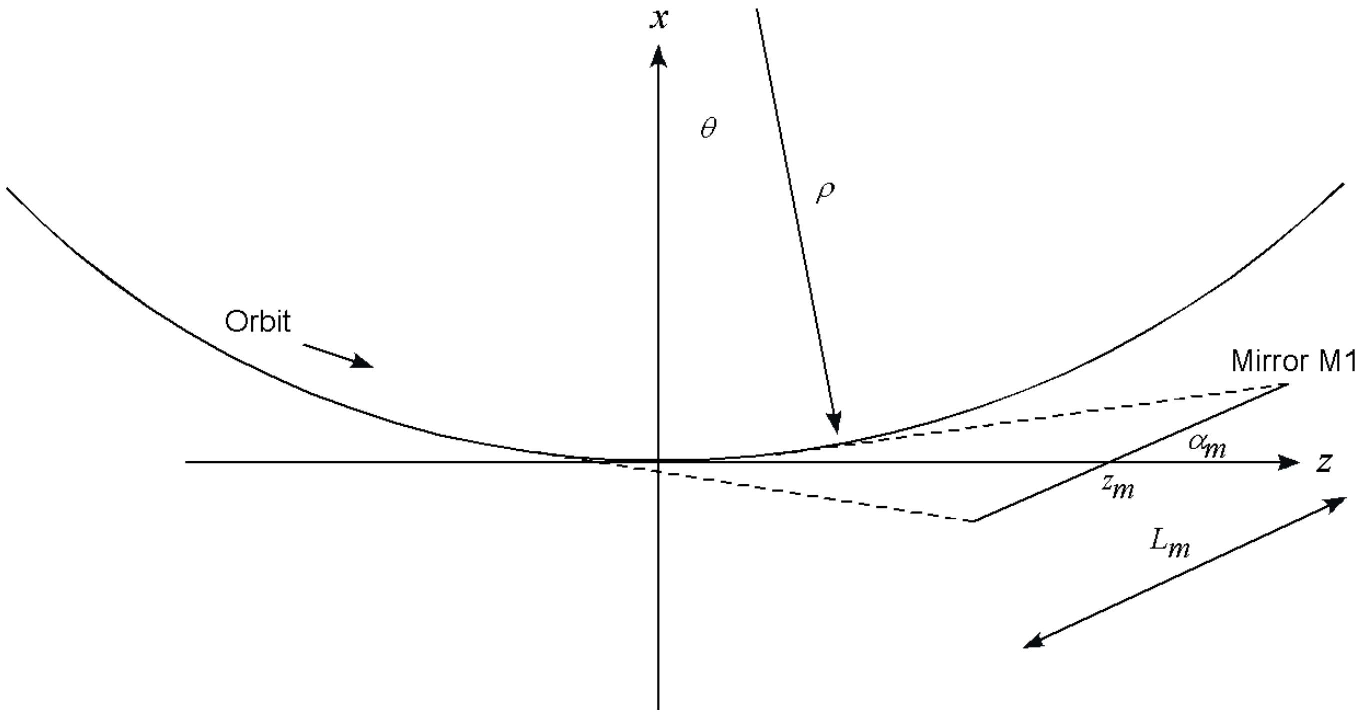
$$\Delta z = \rho\sigma_\psi$$

$$d = \theta\Delta z = 2\rho\sigma_\psi^2 = 0.4\rho^{\frac{1}{3}}\lambda^{\frac{2}{3}}$$

  - This expression is similar to the diffraction resolution.
- But how much of the orbit do we actually capture?

From Horizontal Space to Phase Space

- Consider the beam's orbit both in the horizontal plane (*xz*) and in horizontal *phase space* (*xx′*).
  - x' is the beam's angle to the direction of motion z.
- Which rays, at which angles, are reflected by M1?



Horizontal Phase Space

- A point on the orbit near the *xz* origin is given by:

$$(x,z) = (\rho - \rho\cos\theta, \rho\sin\theta) \approx (\tfrac{1}{2}\rho\theta^2, \rho\theta) = (\tfrac{1}{2}\rho x'^2, \rho x')$$

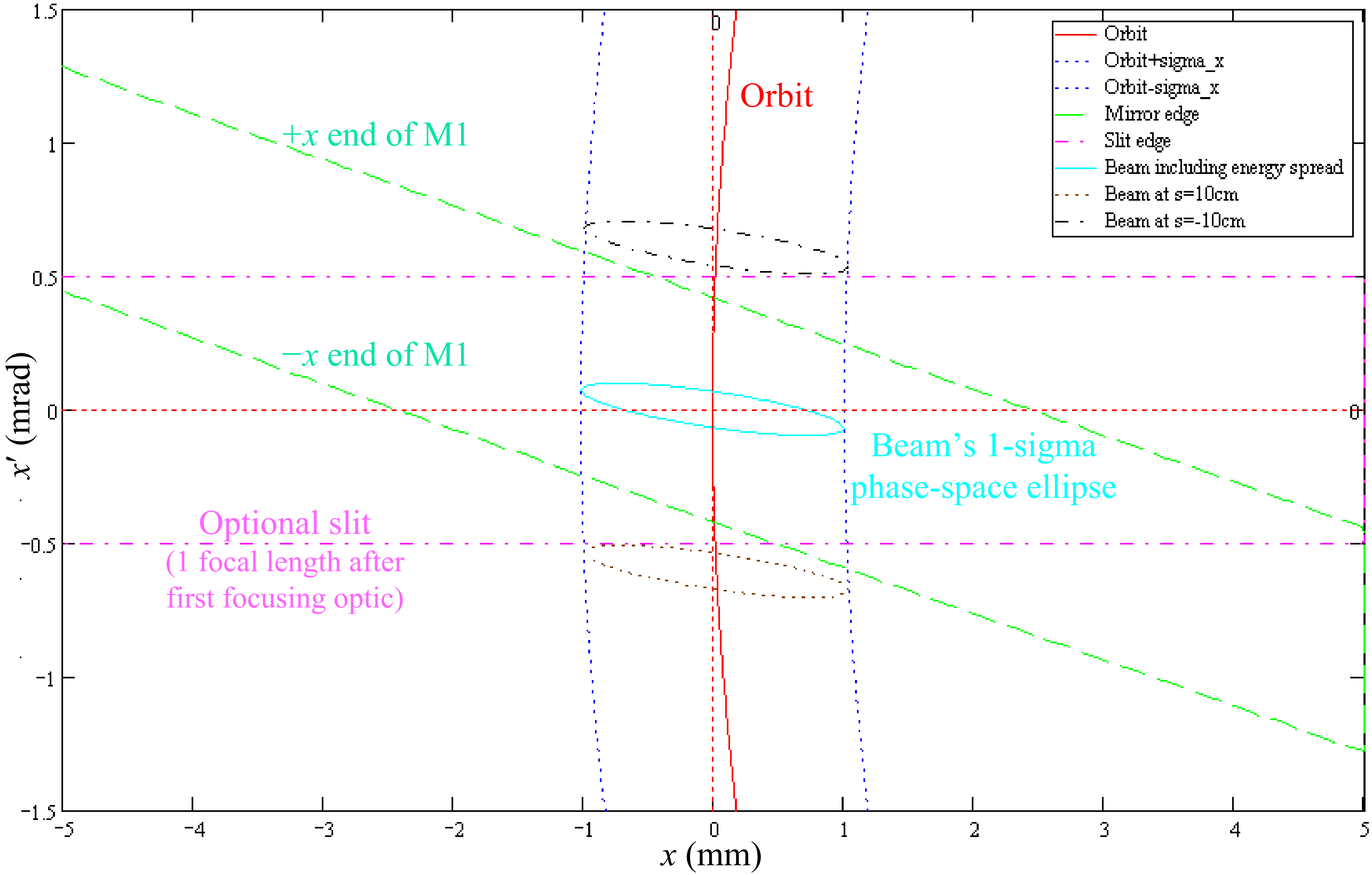
  - For a point on the orbit, the angle x' to the z axis is equal to θ.
- The rays striking the $+x$ and $-x$ ends of M1 are given by:

$$x + x'\left(z_m \pm \frac{L_m}{2}\cos\alpha_m - z\right) = \pm\frac{L_m}{2}\sin\alpha_m$$

$$x + x'z_m \approx \pm\frac{L_m}{2}\sin\alpha_m$$

- We plot these curves in phase space, along with the beam's 1-sigma ellipse at three points along its orbit.

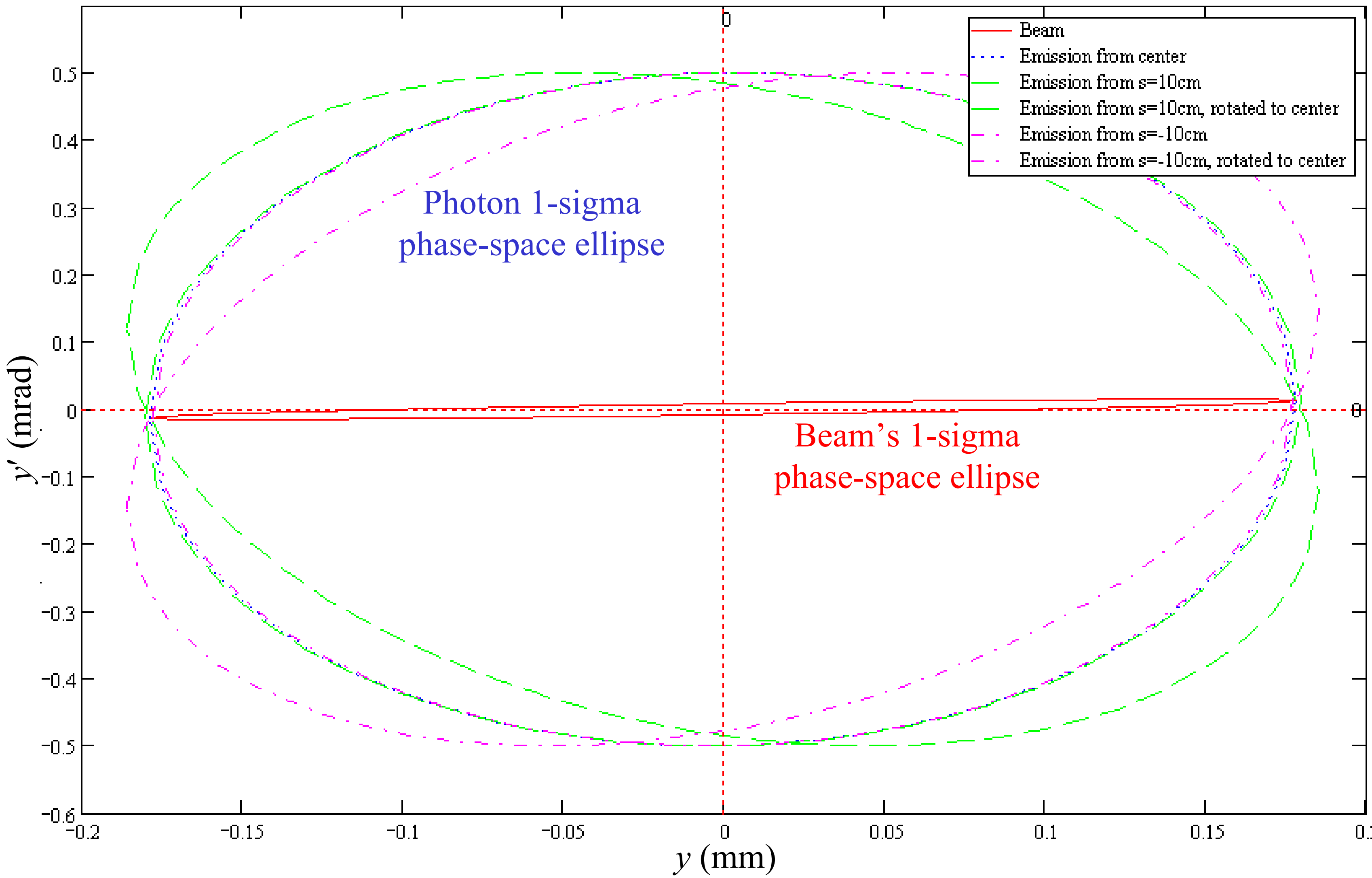
Horizontal Phase Space: PEP-II HER



Vertical Phase Space

- Two source of vertical angle for the light rays:
  - The opening angle σ_ψ of each electron's emission
  - The electrons' phase space, which gives each electron its individual angle to the z axis
- Compare to the horizontal axis:
  - Only the phase space matters: each electron emits along the tangent to its orbit.
- The opening angle is by far the bigger contributor.

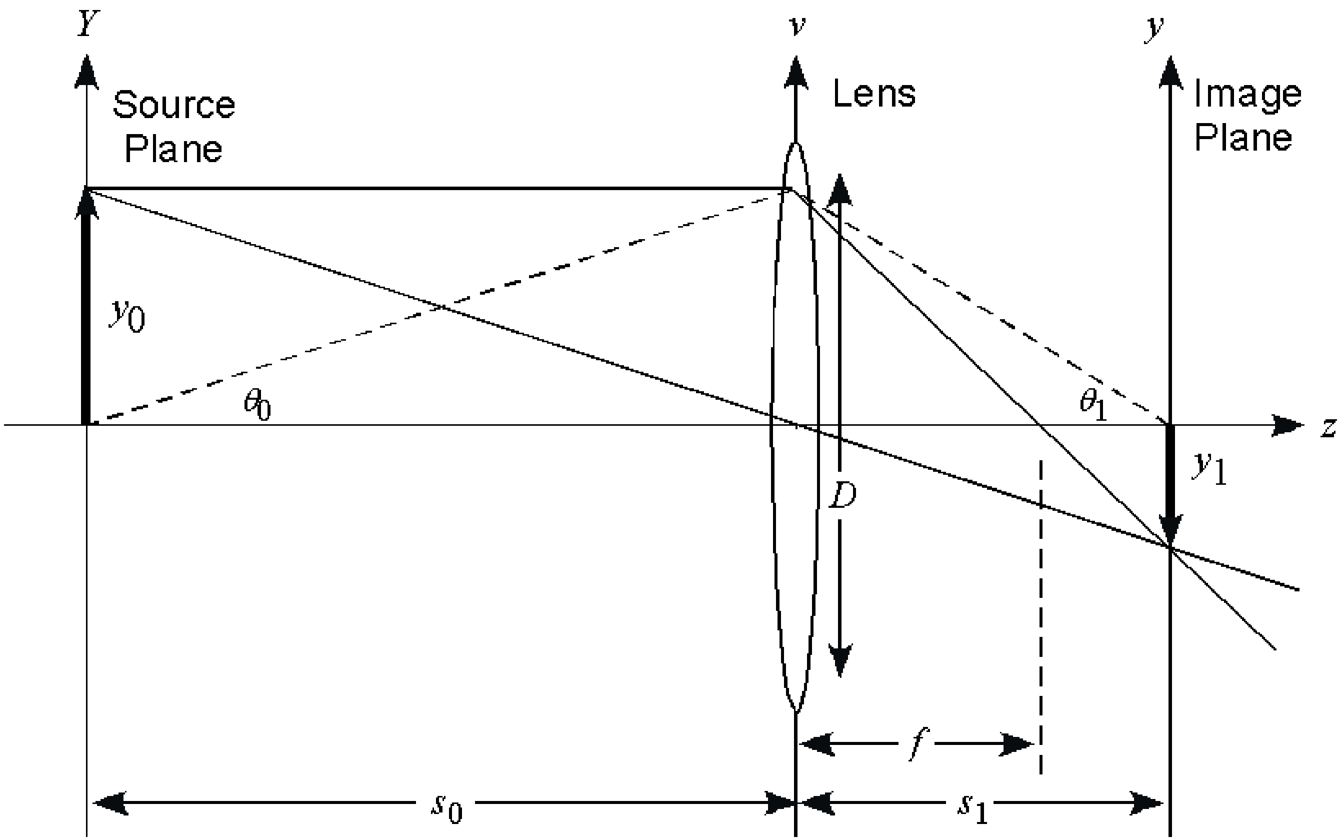
Vertical Phase Space: PEP-II HER



Photon Emittance (Brightness)

- Accelerator people know that Liouville's theorem conserves the emittance of a beam in a transport line.
  - The phase-space ellipse changes shape, but not area.
    - At each waist, the size-angle product $\sigma_x \sigma_{x'}$ is constant.
  - (But in a ring, dissipation by synchrotron radiation allows damping that "cheats" Liouville.)
- Light in an optical transport line has an emittance too.
  - At each image, the product of size and opening angle (light-cone angle) is constant.
    - Magnification makes the image bigger, but the angle smaller.
  - The area of the light's phase-space ellipse—the brightness of the source—is conserved.

Conservation of Brightness



Lens Equations

$$\frac{1}{f} = \frac{1}{s_0} + \frac{1}{s_1}$$

$$m = s_1 / s_0$$

$$y_1 = y_0 m$$

$$\theta_1 = \theta_0 / m$$

Minimum Photon Emittance

- The minimum emittance for a light beam is that of the lowest-order Gaussian mode (TEM_{00}) of a laser.
  - ω is the beam radius.
    - In the usual definition (where ω is not the one-sigma value):
      - The electric field follows $E(r) = E_0 \exp(-r^2/\omega^2)$
      - The intensity (power) is the square: $I(r) = I_0 \exp(-2r^2/\omega^2)$
  - ω_0 is the radius at the waist (the focus).
    - This size is nonzero due to diffraction.
  - $z_R = \pi\omega_0^2/\lambda$ is the Rayleigh range.
    - Characteristic distance for beam expansion due to diffraction.
  - The expansion is given by $\omega^2(z) = \omega_0^2(1+z^2/z_R^2)$
  - The angle (for $z >> z_R$) is $\theta = \omega/z = \omega_0/z_R = \lambda/\pi\omega_0$
  - The product of waist size and angle is then $\omega_0\theta = \lambda/\pi$
  - One-sigma values for the size and angle of I give an emittance of $\lambda/4\pi$

Constraints on the Beamline

- Distance to the first mirror
  - Flush with the beampipe wall?
  - Far down a synchrotron-light beamline?
  - Ports and M1 itself introduce wakefields and impedance.
  - The heat load on M1 is reduced by distance.
- Distance to the imaging optics
  - In a hutch: adds distance to get outside shielding
  - In the tunnel: not accessible, but often necessary for large colliders.
- Size and location of the optical table.

Constraints on the Optical Design

- Choose a source point with a large *y* size, to lessen effect of diffraction.
- Magnification: Transform expected beam size to a reasonable size on the camera.
  - $6\sigma <$ camera size $< 10\sigma$: Uses many pixels; keeps the image and the tails on the camera; allows for orbit changes.
  - Needs at least two imaging stages: Since the optics are generally far from the source, the first focusing element strongly demagnifies.
- Optics: Use standard components whenever possible.
  - For example, adjust the design to use off-the-shelf focal lengths from the catalog of a high-quality vendor.
  - Use a color filter to avoid dispersion (or use reflective optics).
  - Correct the focal length (specified at one wavelength) for your color.

Basic Design Spreadsheet

- You can iterate a lot of the basic design in a simple spreadsheet.
  - Enter the fixed distances.
  - Specify the desired magnifications.
  - Solve the lens equations, one stage at a time, to find lenses giving the ideal magnifications.
  - Change the lenses to catalog focal lengths.
  - Correct their focal lengths (using the formula for each material as found in many catalogs).
  - Adjust magnifications and distances.