

Lecture 8:

Beam Optics and Emittance Growth: -Chromatic Aberration in the Solenoid -Emittance Optimization

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- *The objective of this lecture is to classify and describe the various sources of emittance growth in an electron gun and injector. In it the student will learn how to compute the chromatic aberration caused by the gun solenoid.*
- *A short review of emittance optimization is given at the end of the lesson.*



Introduction

There are four types of emittance in RF guns: thermal (aka cathode, $\epsilon_{thermal}$), rf (ϵ_{rf}), space charge (σ_{sc}) and aberrations due to the optical focusing elements (σ_{optics}). These emittances are usually assumed to be un-correlated such that the total emittance is given by,

$$(1) \quad \epsilon_{total} = \sqrt{\epsilon_{thermal}^2 + \epsilon_{rf}^2 + \epsilon_{sc}^2 + \epsilon_{optics}^2}$$

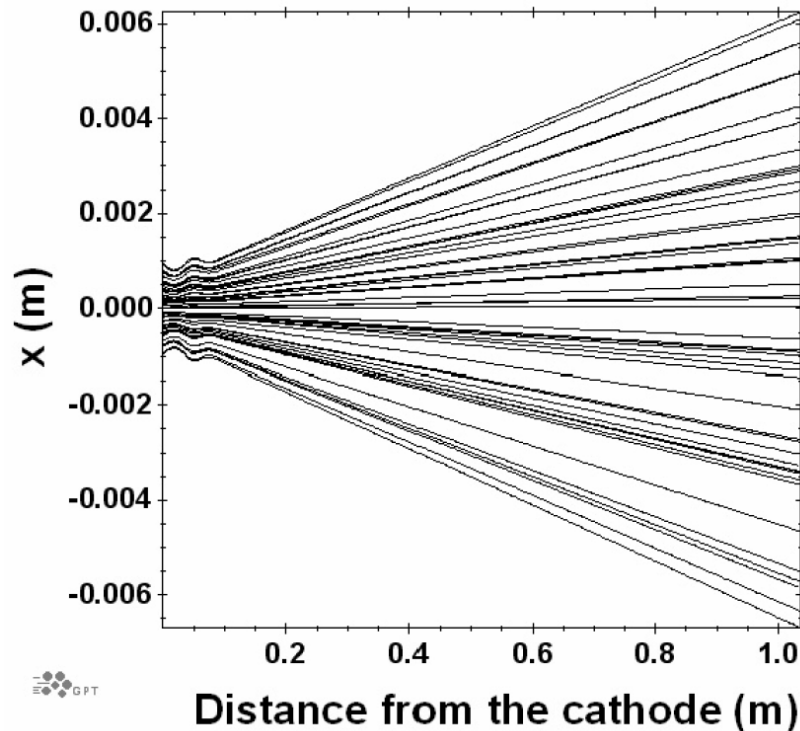
A previous lecture described the thermal emittance in its three forms of thermionic, photo-electric and field emission. This lecture discusses the rf emittance, geometric and chromatic aberrations of the beam, and space charge emittance growth. Along the way, the concepts of projected emittance and slice emittance are introduced.

+ wakefields



Chromatic Aberration of the Solenoid (1)

- This part of the lecture discusses the geometric and chromatic aberrations in the solenoid's long axial magnetic field. This solenoid is located either over or immediately after the gun to cancel the strong defocus at the gun's exit and to compensate for the space charge emittance.*



Chromatic Aberration of the Solenoid (2)

- *The transverse emittance which we wish to compute is given by,*

$$\epsilon_x = \sqrt{\langle p_x^2 \rangle \langle x^2 \rangle - \langle p_x x \rangle^2}$$

- *Consider a solenoid with a uniform axial magnetic field, $B(0)$, and a beam of rigidity, $B\rho_0$, moving along its axis with focusing strength K ,*

$$K = \frac{B(0)}{2B\rho_0}$$

- *The rigidity is given by the handy formula:*

$$B\rho_0 = 33.356p(\text{GeV}/c)\text{kG} - m.$$



Chromatic Aberration (3)

- *Due to the beam's rotation in the axial field, the x- and y-transverse trajectories become coupled and a 4x4 matrix is necessary for an optical calculation. However, if one rotates the x-y coordinates with the beam then the two planes decouple. In this rotating frame the transformation becomes (Transport Manual, slac-r-091, pp.104-106),*

$$R(-KL)R(\text{solenoid}) = \begin{pmatrix} C & S/K & 0 & 0 \\ -KS & C & 0 & 0 \\ 0 & 0 & C & S/K \\ 0 & 0 & -KS & C \end{pmatrix}$$

with $C = \cos(KL)$, $S = \sin(KL)$, L the magnetic field effective length, and the rotation matrix,

$$R = \begin{pmatrix} C & 0 & S & 0 \\ 0 & C & 0 & S \\ -S & 0 & C & 0 \\ 0 & -S & 0 & C \end{pmatrix}$$



Chromatic Aberration (4)

- Therefore in the beam frame, the focal length, R_{21} , is

$$\frac{1}{f_{sol}} = K \sin KL$$

- For small values of KL ,

$$\frac{1}{f_{sol}} = K^2 L = \left(\frac{B(0)}{2B\rho_0} \right)^2 L$$

- With these approximations, the electrons are deflected by the solenoid with an angle change of

$$x' = \frac{x}{f_{sol}} = xL \left(\frac{B(0)}{2B\rho_0} \right)^2$$



Chromatic Aberration (5)

- *Now compute the beam divergence change due to small deviations in momentum,*

$$\frac{dx'}{dp} = -xL \left(\frac{B(0)}{2B\rho_0} \right)^2 \frac{2}{B\rho_0} \frac{d(B\rho_0)}{dp}$$

$$\frac{dx'}{dp} = -\frac{xL}{2p} \left(\frac{B(0)}{B\rho_0} \right)^2$$

$$\Delta x' = \frac{dx'}{dp} \Delta p = -\frac{xL}{2} \left(\frac{B(0)}{B\rho_0} \right)^2 \frac{\Delta p}{p}$$

- *Switching to rms quantities gives*

$$\sigma_{x'} = \frac{L\sigma_x}{2} \left(\frac{B(0)}{B\rho_0} \right)^2 \frac{\sigma_p}{p}$$



Chromatic Aberration (6)

- *The normalized emittance is,*

$$\epsilon_n = \beta\gamma\sqrt{\langle x'^2 \rangle \langle x^2 \rangle - \langle x'x \rangle^2}$$

- *Since there is no overall displacement or steering, we can ignore the second term in the square root and assume*

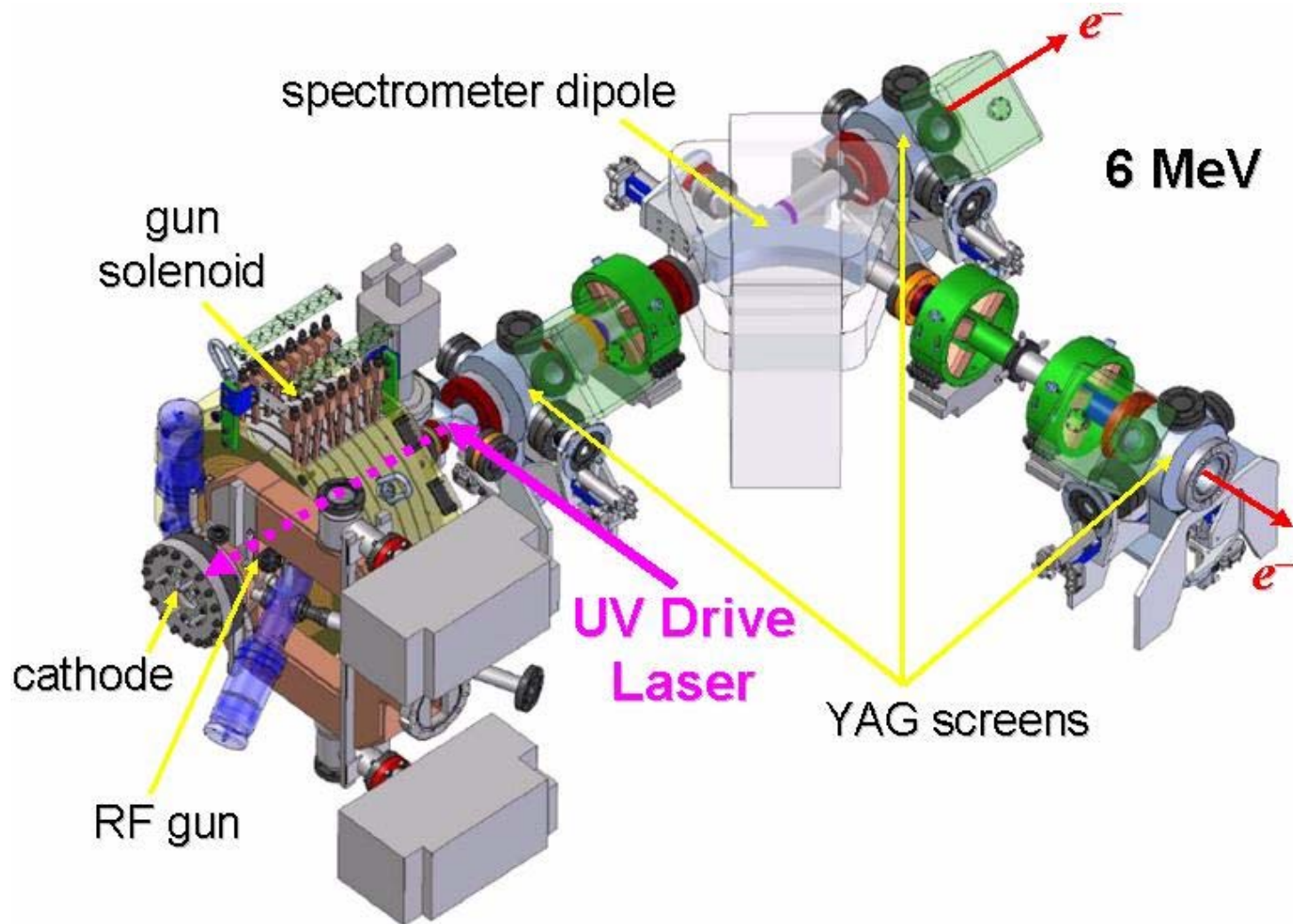
$$\epsilon_n = \beta\gamma\sigma_x\sigma_{x'}$$

- *Therefore the chromatic aberration in the solenoid becomes*

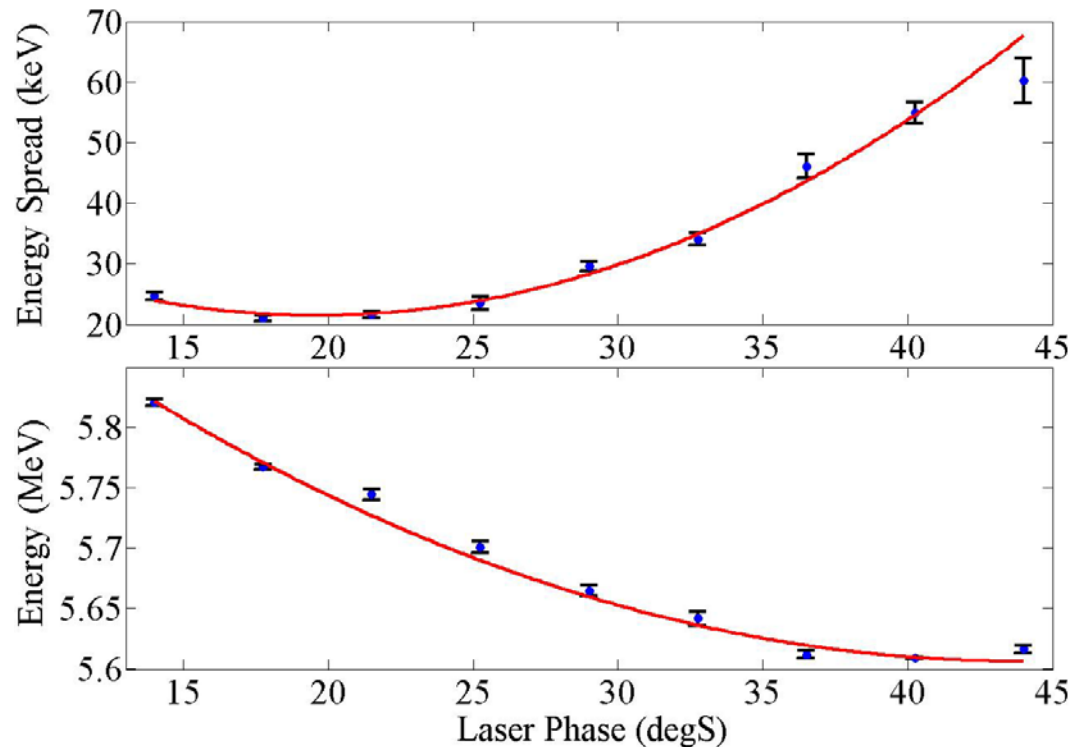
$$\epsilon_{sol} = \beta\gamma\sigma_x^2 L \left(\frac{B(0)}{B\rho_0} \right)^2 \frac{\sigma_p}{2p}$$



LCLS Gun-to-Linac Region



Gun Energy and Energy Spread



- *The usual LCLS parameters are $L=19.35$ cm, $p=5.75$ MeV/c, $\sigma_p=20$ keV, $B(0)=2.5$ KG, $\sigma_x=0.75$ mm, giving a chromatic emittance in the solenoid of:*

$$\epsilon_{sol} = 0.36 \text{ microns}$$



Overview of Emittance Growth



Fully Symmetric RF to Minimize Emittance Growth

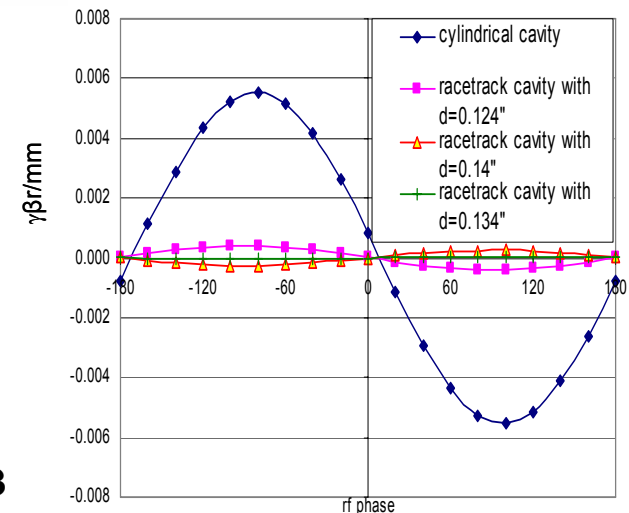
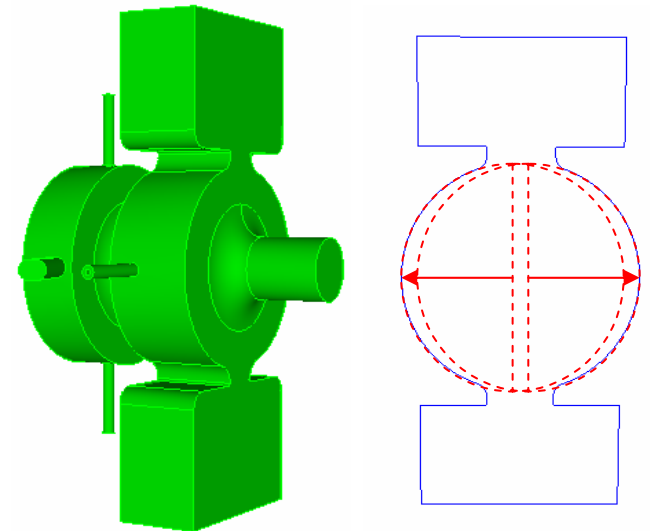
3D RF Design of Gun

- **Z-coupling:**
 - reduces pulsed heating
 - increases vacuum pumping
- **Racetrack to minimize quadrupole fields**
- **Deformation tuning to eliminate rf tuners**
- **Iris reshaped reducing field 10% below cathode**
- **Increase 0-pi mode separation to 15MHz**
- **All 3D features included in modeling:**
 - laser port and pickup probes
 - 3D fields used in Parmela simulation

RF Parameters	
f_0 (GHz)	2.855987
Q_0	13960
β	2.1
Mode Sep. Δf (MHz)	15
$E_0:E_1$	0.999:1

C. Limborg et al., "RF Design of the LCLS Gun", LCLS-TN-05-3

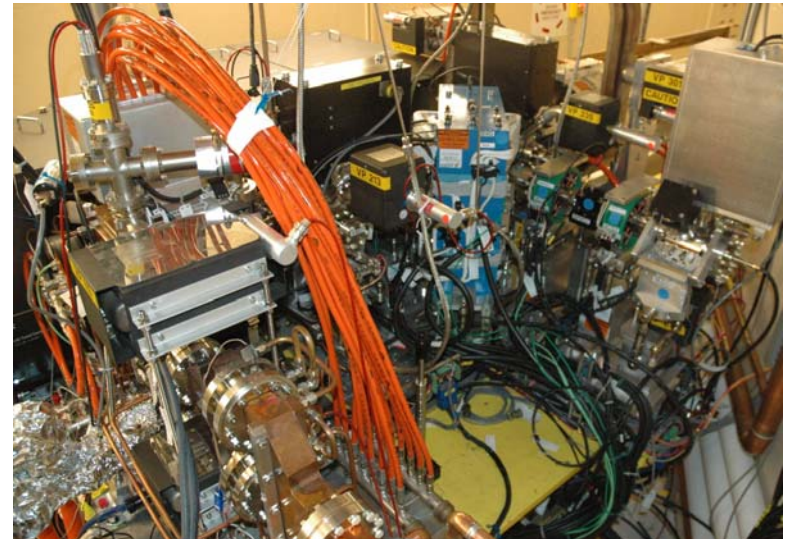
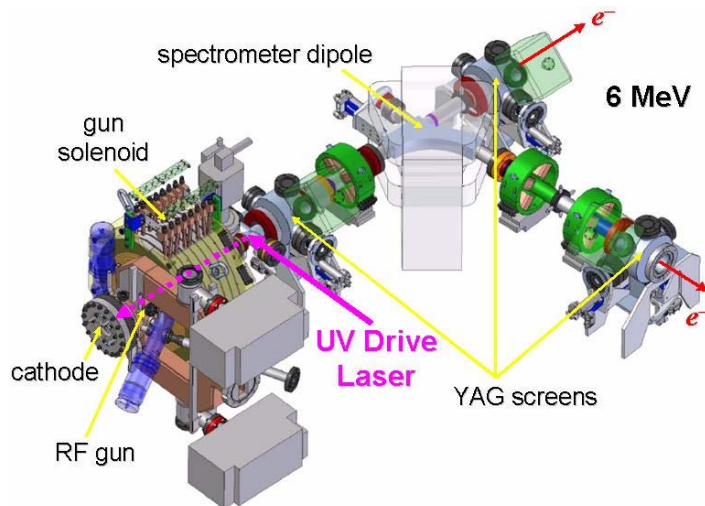
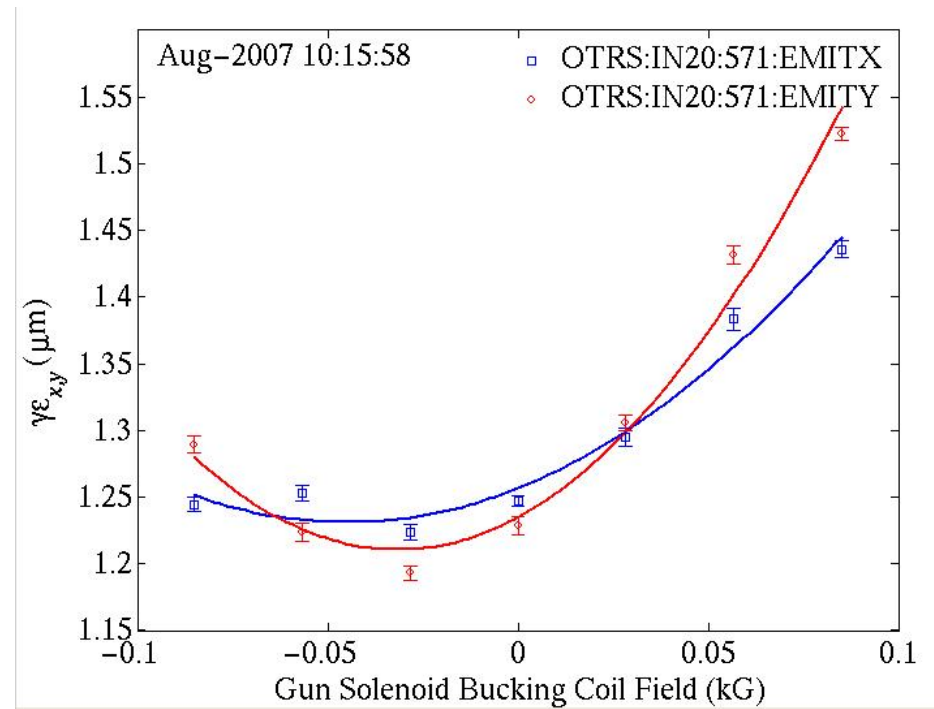
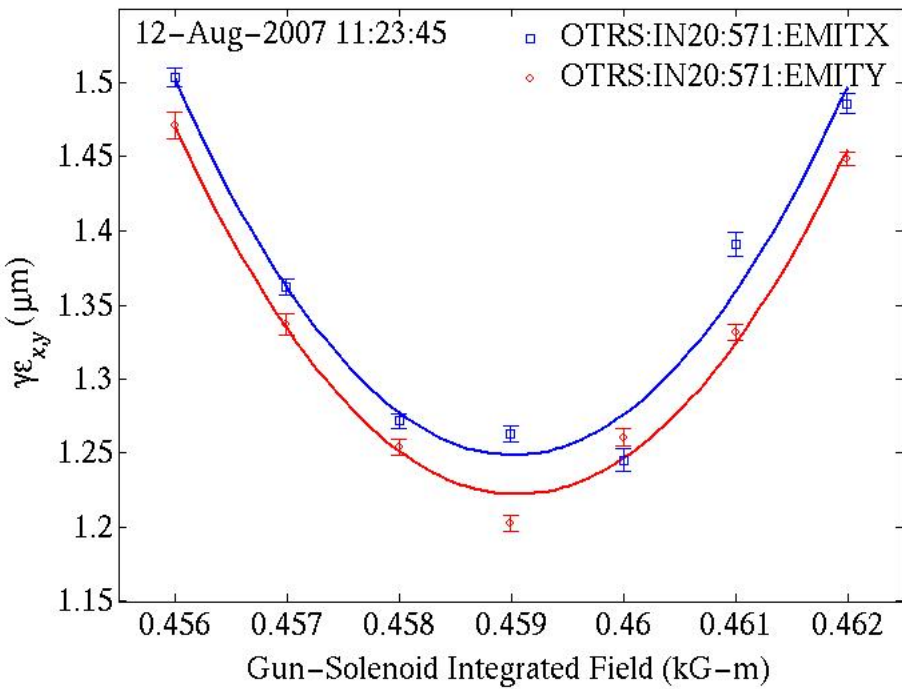
L. Xiao et al., "Dual feed rf gun design for the LCLS," Proc. 2005 Particle Acc. Conf.



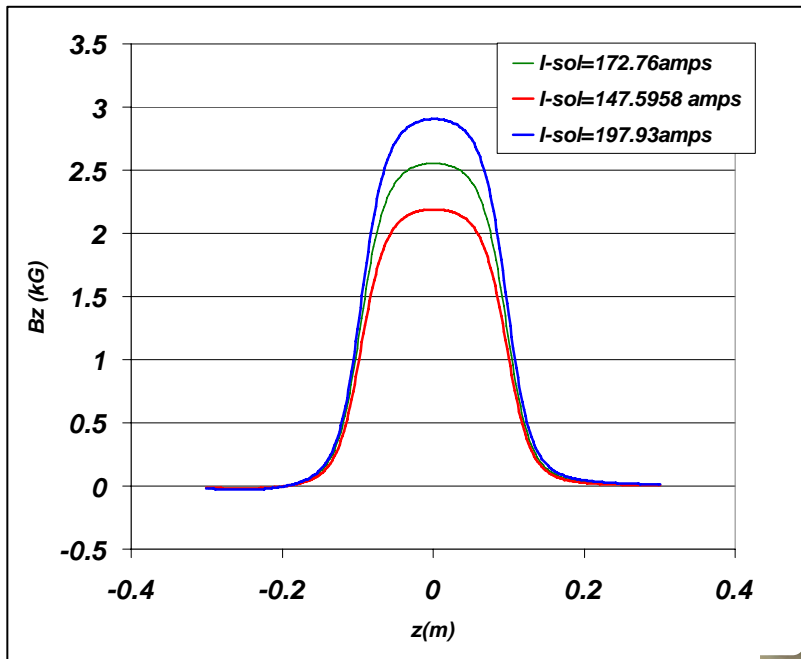
Compliments of Z. Li & L. Xiao



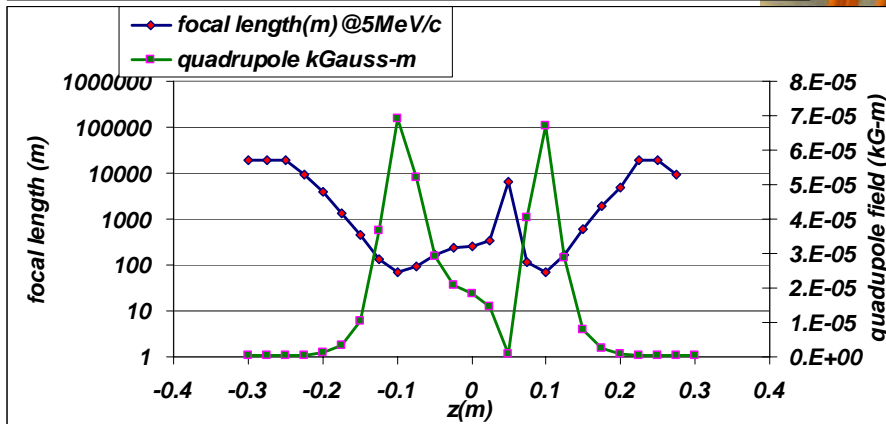
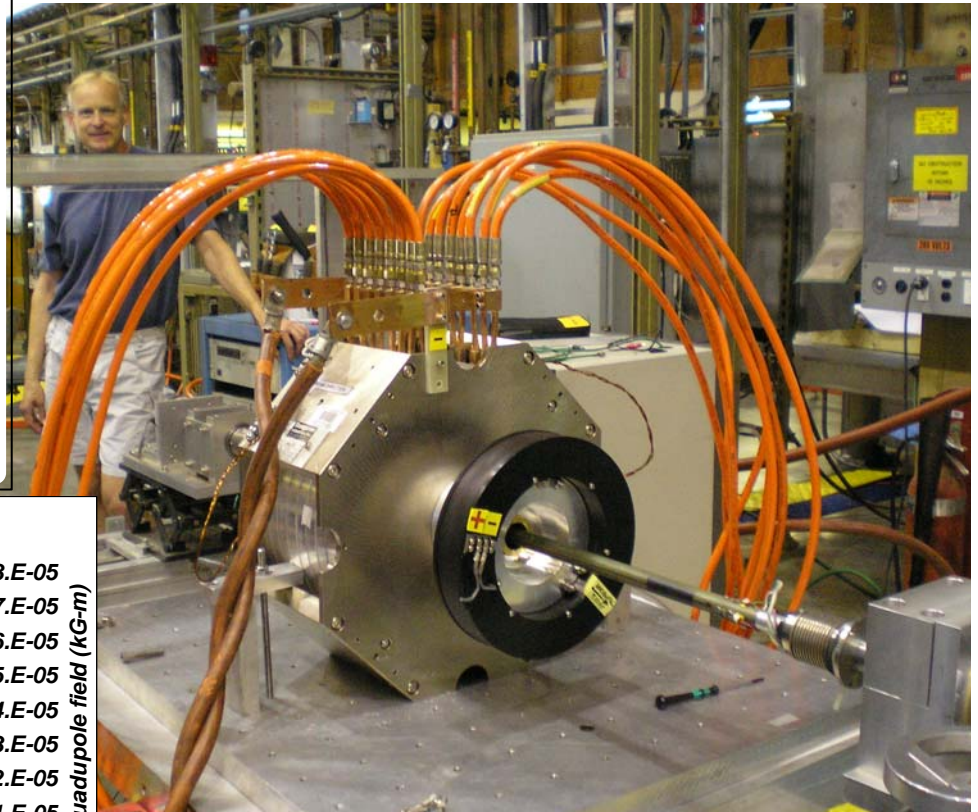
Emittance Optimization



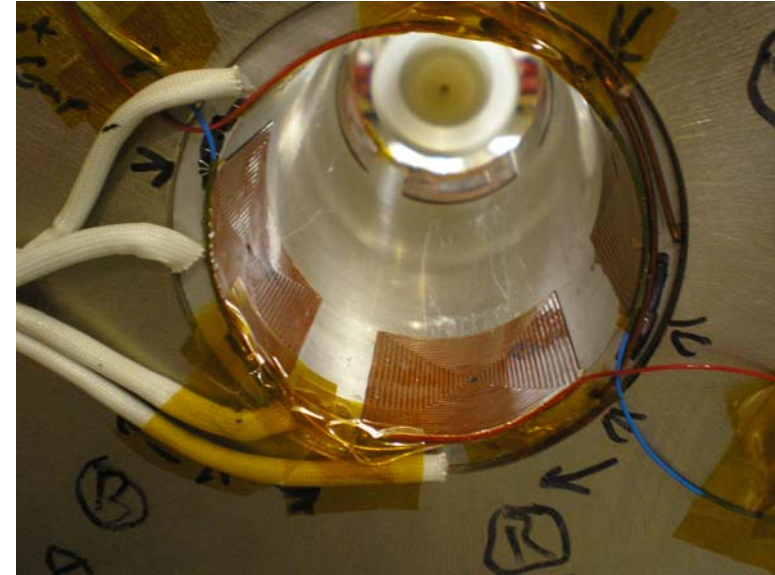
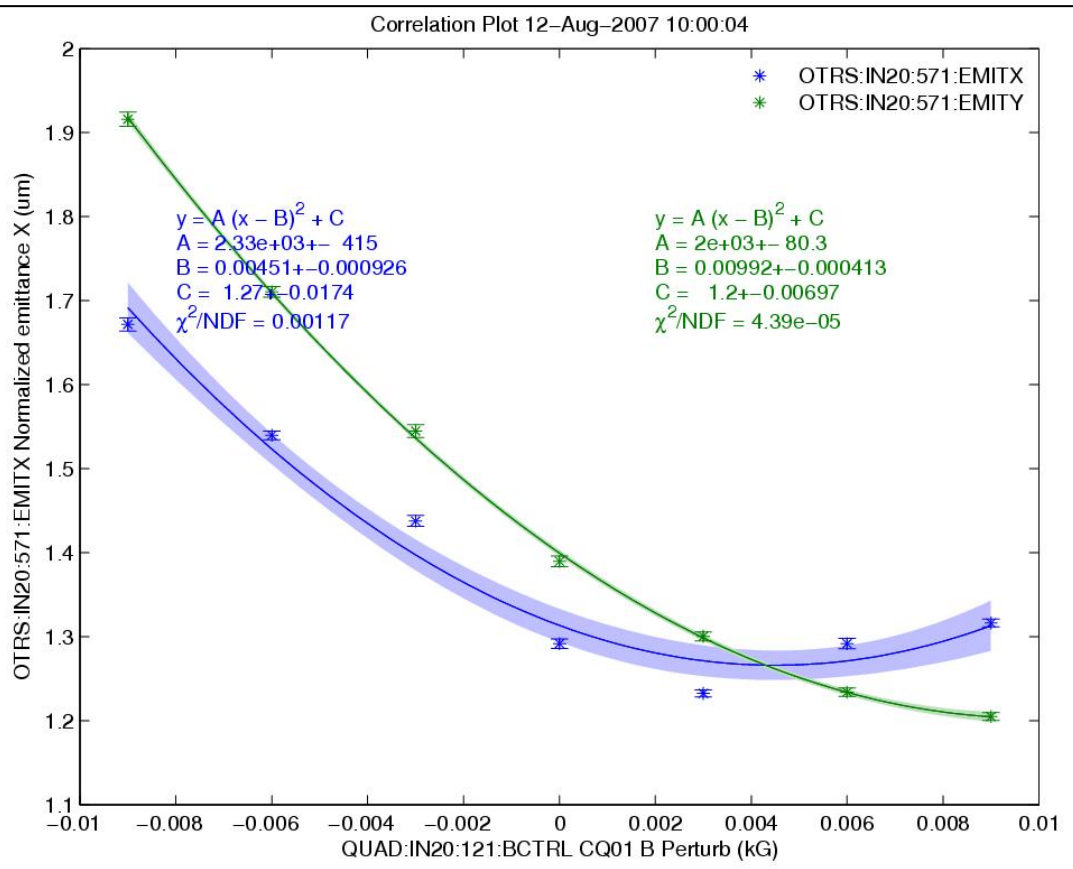
Gun Solenoid Magnetic Measurements



Gun Solenoid on Magnetic Meas. Test Stand



Quadrupole Field Correctors in Solenoid Essential to Minimize Emittance



• *PC board quads
will be used in next gun*

• *Quad wires used in present
LCLS gun*



Transverse Wakefield in X-Band Structure

