Lecture 4:
RF Gun Fields and Single Particle Dynamics

High Brightness Electron Injectors for Light Sources

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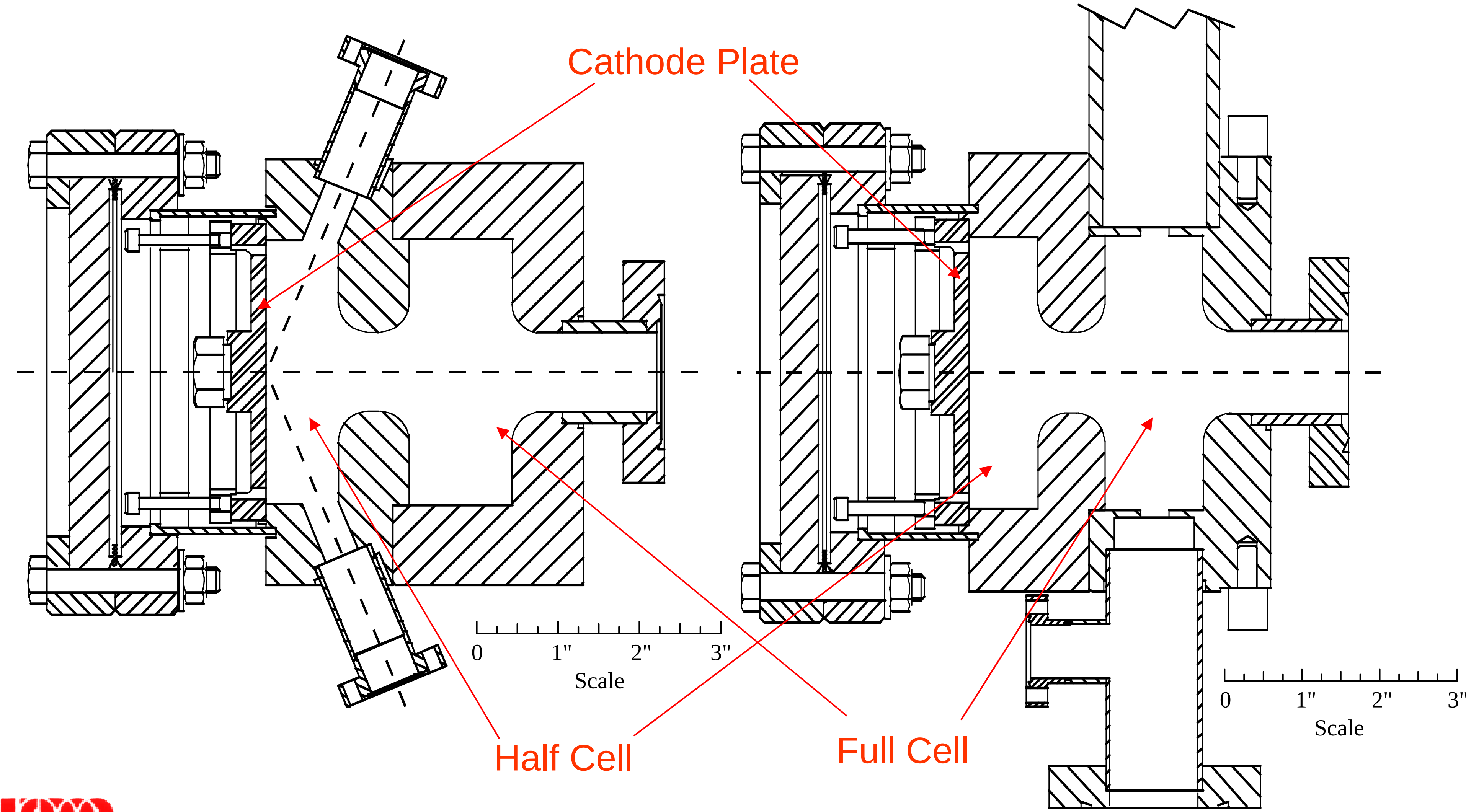
Outline

- RF Gun Basics
  - Geometry
  - Standing Wave vs Traveling Wave
  - Simulations
  - Pillbox Comparison
- RF Properties
  - Cold Test Measurements
    - Modes
    - Model
  - Time Domain
- Energy Gain
  - Energy
  - Energy Spread

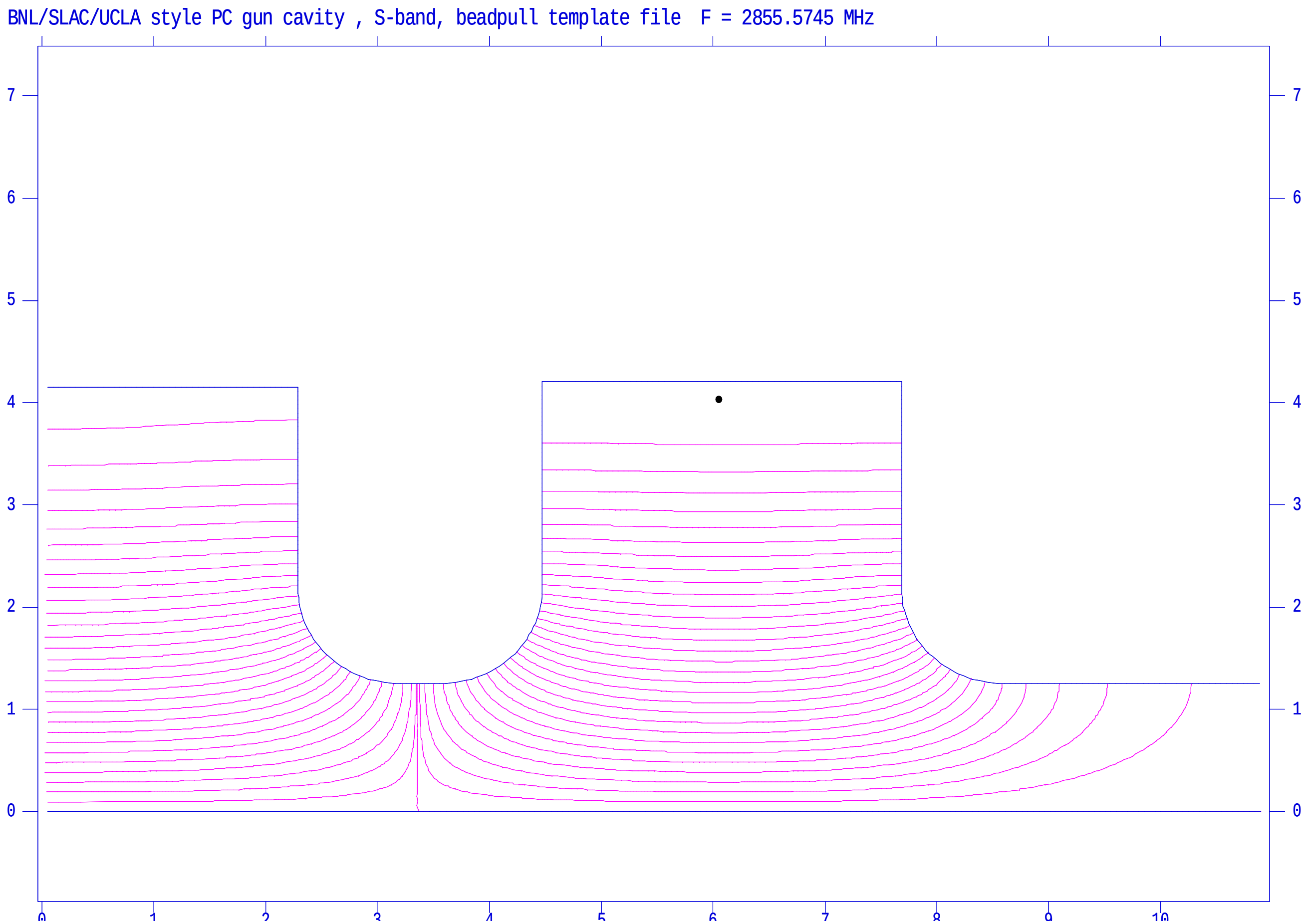
S-Band, 1.6 cell, RF Gun



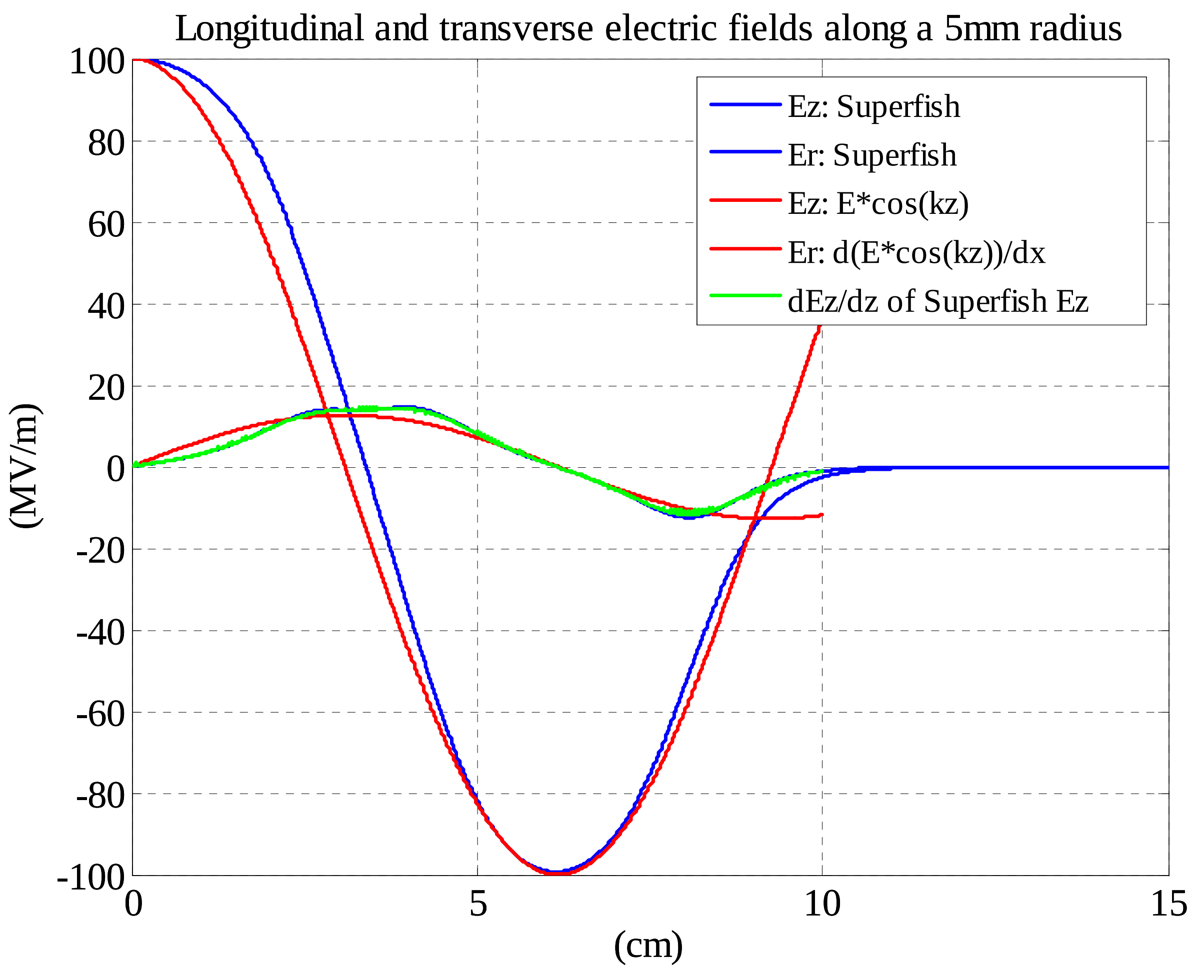
Standing Wave vs. Traveling Wave

- Standing Wave
  - Single Input/Output
  - Allows for possibility of cathode plate
  - Buildup Time
  - Reduced voltage gain (transit time factor)
- Traveling Wave
  - Single Input and Single Output
  - Would require RF coupler in cathode cell
  - Filling Time
  - Maximum voltage gain (phase velocity chosen to match beam velocity)

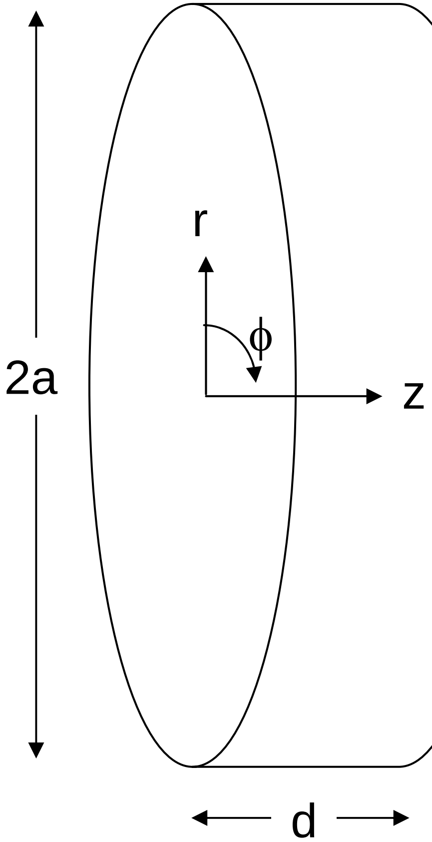
SUPERFISH Simulation I



SUPERFISH Simulation 2



Pill Box Cavity



$$TM_{011} \quad \text{mode} \quad \text{where } p_{01} = 2.405$$

$$E_z = E_0 J_0\left(\frac{p_{01} r}{a}\right)\cos\left(\frac{\pi z}{d}\right)\sin(\omega t)$$

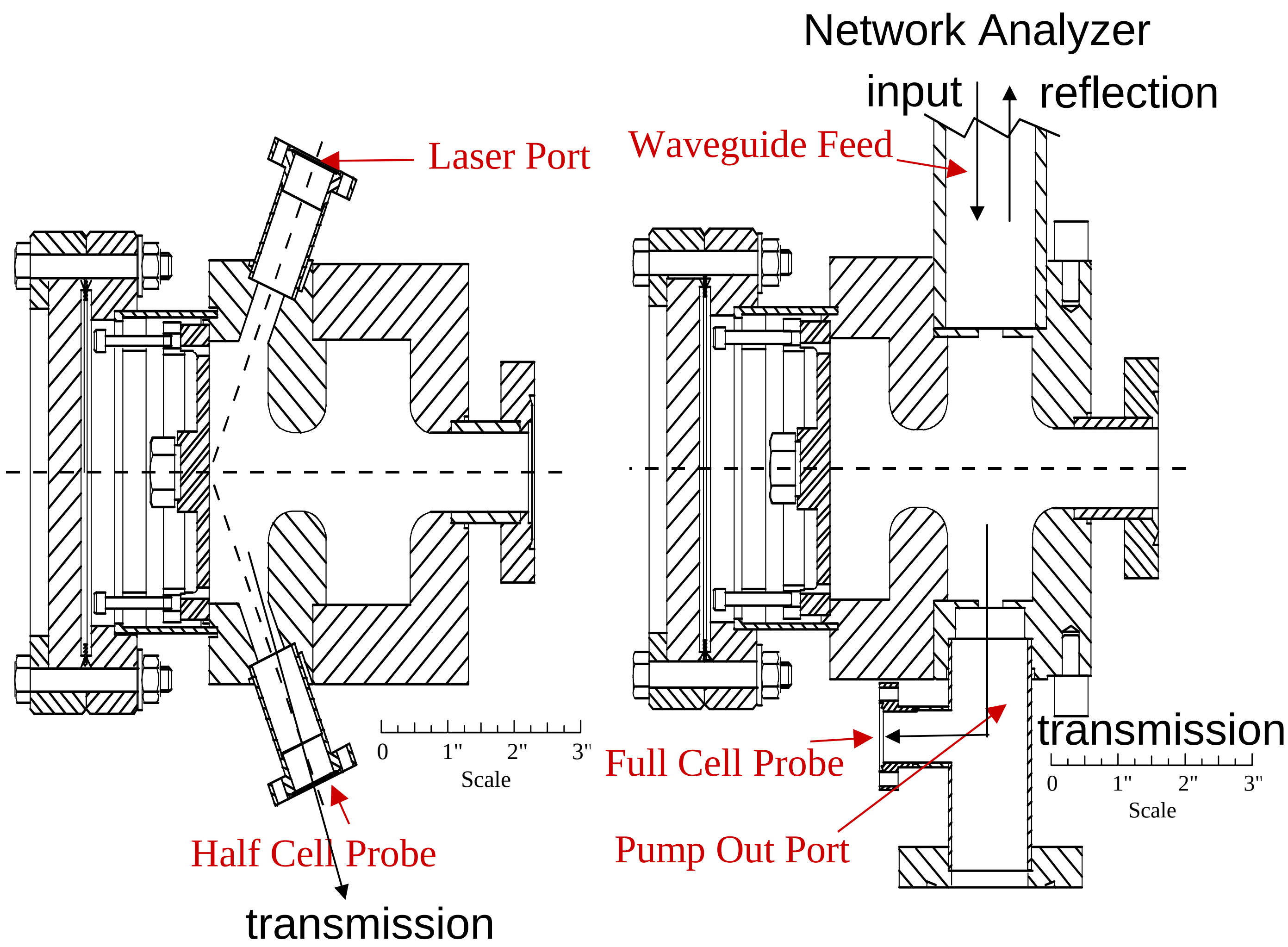
$$E_r = \frac{-\pi a}{p_{01} d} E_0 J_1\left(\frac{p_{01} r}{a}\right)\sin\left(\frac{\pi z}{d}\right)\sin(\omega t)$$

$$B_\phi = \frac{-1}{\omega}\left(\frac{p_{01}}{a} - \frac{\pi^2 a}{p_{01} d^2}\right) E_0 J_1\left(\frac{p_{01} r}{a}\right)\cos\left(\frac{\pi z}{d}\right)\cos(\omega t)$$

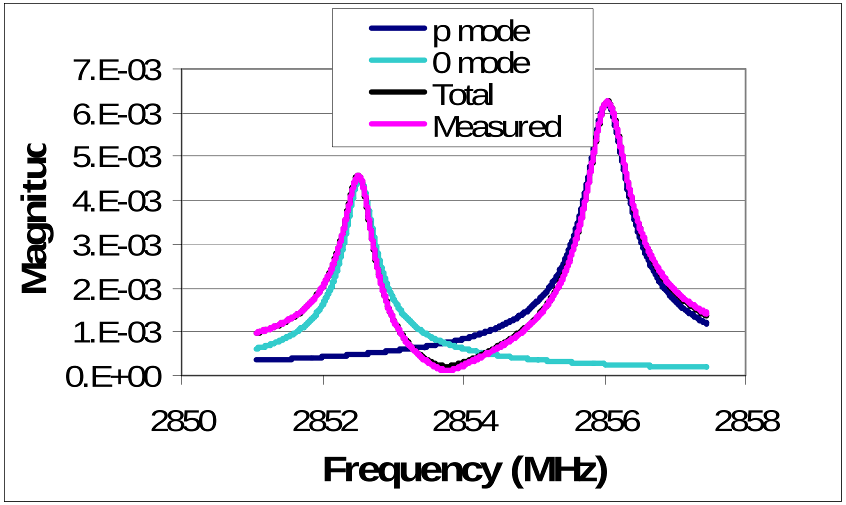
Modes

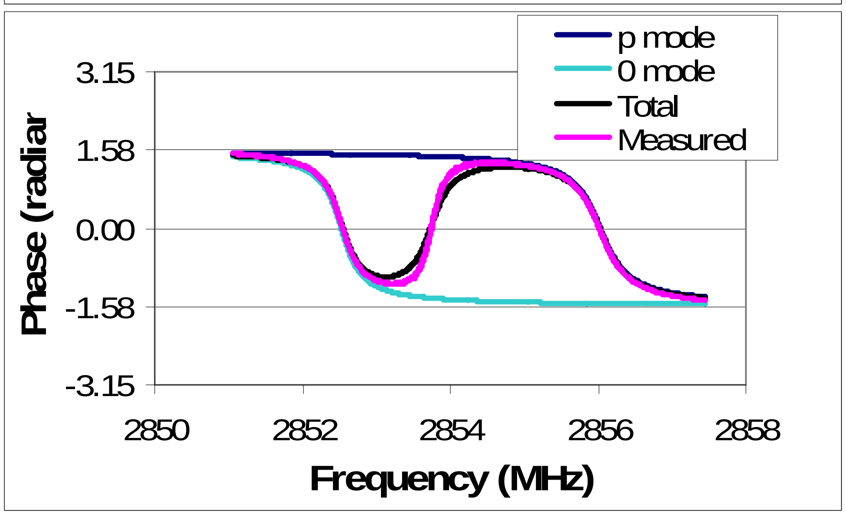
- Gun has two cells and thus at least two resonances or modes
- The field for the accelerating mode has a 180° phase difference between the two cells. This mode is called the π mode.
- This structure also supports a mode with 0° phase difference between cells. However, the 0 mode does not accelerate electrons since the field has the wrong polarity in the full cell at the time the electron arrives at the full cell.
- In addition there are undoubtedly additional modes at much higher resonant frequencies.

Measurement of Modes

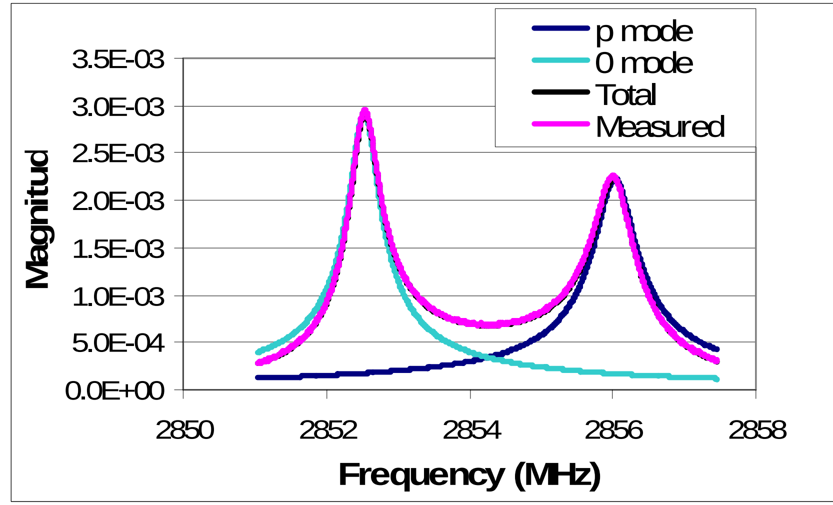


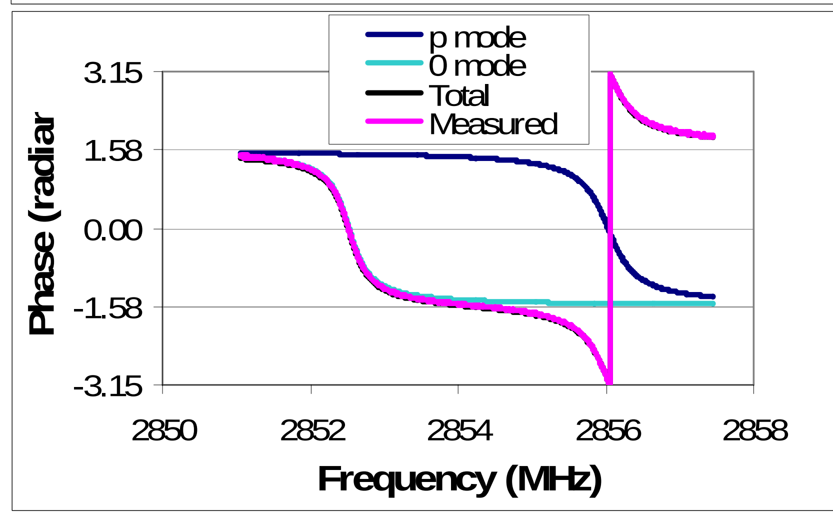
Results





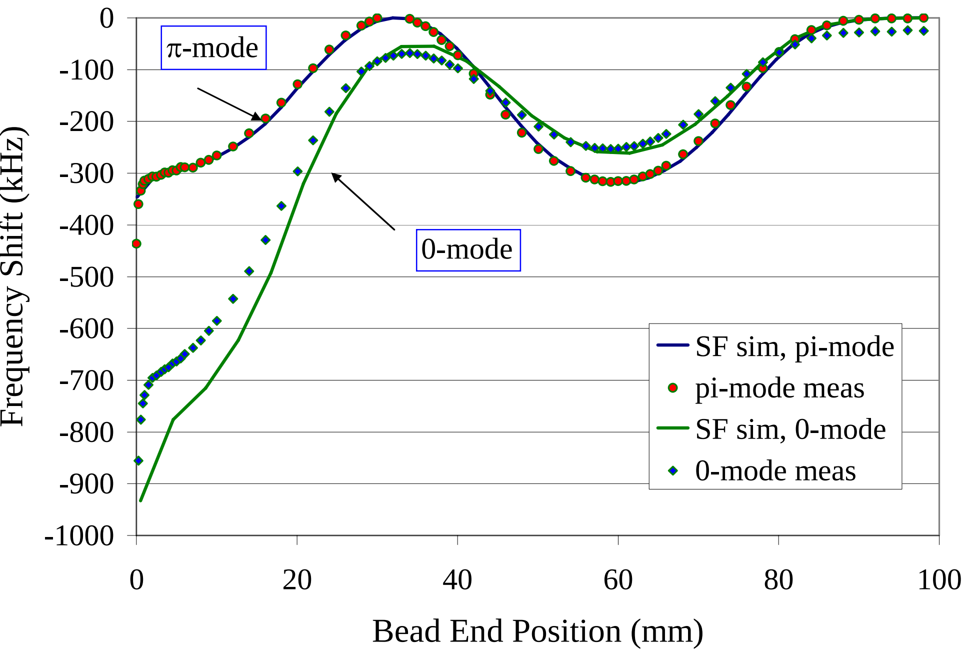
Full Cell Transmission



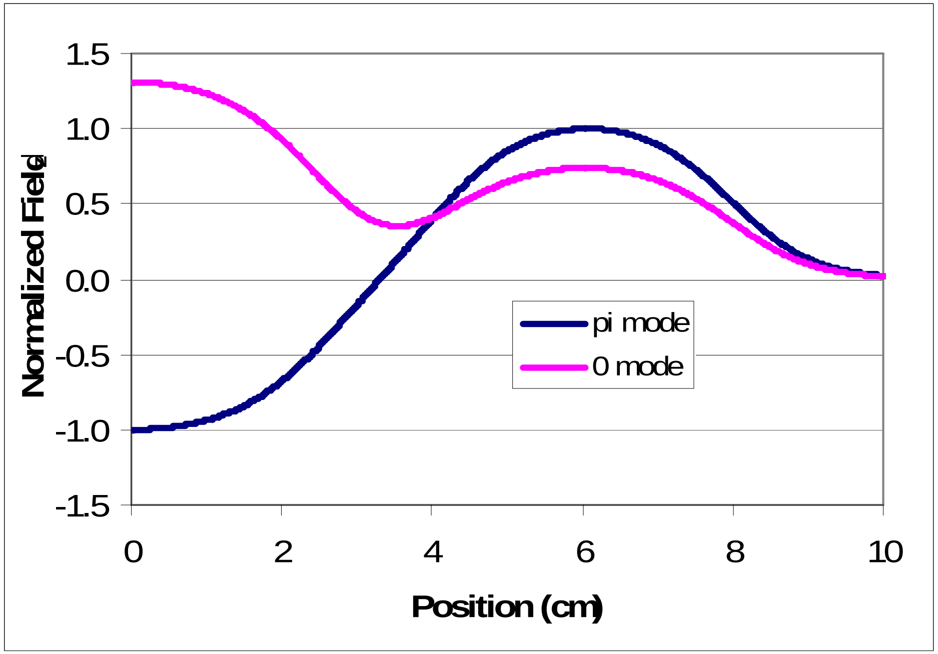


Half Cell Transmission

Field vs Position



Bead Drop Measurement



SUPERFISH simulation

Model Parameters

Parameter	π mode	0 mode
A (full cell)	+1	+0.72
A (half cell)	-1	+1.31
f (MHz)	2856	2852.5
Q (loaded)	5160	6979
β	1.34	0.68
τ (ns)	576	779

Mathematical Model

- Equivalent to band pass filter (parallel RLC circuit)

$$T(\omega,z)=\frac{jA_\pi(z)\dfrac{\omega_\pi}{Q_\pi}\omega}{\omega_\pi^2-\omega^2+j\dfrac{\omega_\pi}{Q_\pi}\omega}+\frac{jA_0(z)\dfrac{\omega_0}{Q_0}\omega}{\omega_0^2-\omega^2+j\dfrac{\omega_0}{Q_0}\omega}$$

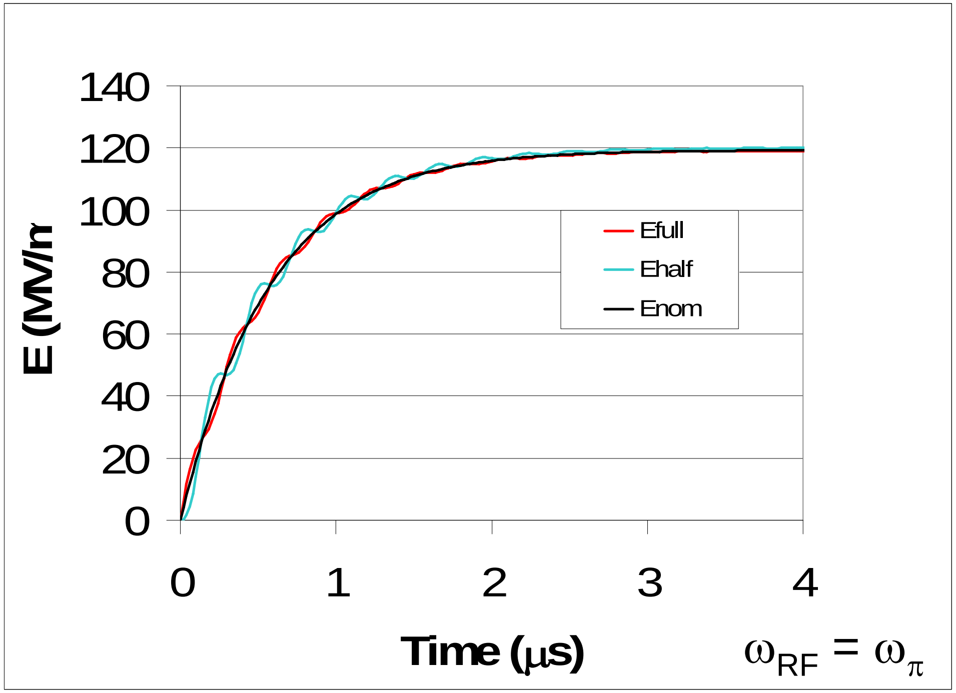
$$\frac{d^4E_{out}}{dt^4}+\left(\frac{\omega_\pi}{Q_\pi}+\frac{\omega_0}{Q_0}\right)\frac{d^3E_{out}}{dt^3}+\left(\omega_\pi^2+\omega_0^2+\frac{\omega_\pi\omega_0}{Q_\pi Q_0}\right)\frac{d^2E_{out}}{dt^2}+\left(\frac{\omega_0^2\omega_\pi}{Q_\pi}+\frac{\omega_\pi^2\omega_0}{Q_0}\right)\frac{dE_{out}}{dt}+\omega_\pi^2\omega_0^2E_{out}$$

$$=\left(\frac{A_\pi\omega_\pi}{Q_\pi}+\frac{A_0\omega_0}{Q_0}\right)\frac{d^3E_{in}}{dt^3}+\frac{\omega_\pi\omega_0}{Q_\pi Q_0}\frac{d^2E_{in}}{dt^2}+\left(\frac{A_\pi\omega_\pi\omega_0^2}{Q_\pi}+\frac{A_0\omega_0\omega_\pi^2}{Q_0}\right)\frac{dE_{in}}{dt}$$

Step Function Excitation

$$\frac{E_{out}(t)}{E_{in}} = E_\pi \cos(\omega_{RF} t + \theta_\pi) + E_0 \cos(\omega_{RF} t + \theta_0) + T_\pi e^{-\frac{t}{\tau_\pi}} \cos(\omega_\pi t + \phi_\pi) + T_0 e^{-\frac{t}{\tau_0}} \cos(\omega_0 t + \phi_0)$$

$$\frac{E_{steady\ state}(t)}{E_{in}} = E_\pi \cos(\omega_{RF} t + \theta_\pi) + E_0 \cos(\omega_{RF} t + \theta_0) = E_{ss} \cos(\omega_{RF} t + \theta_{ss})$$



Steady State Fields

Field	Full Cell	Half Cell
π mode ($E_\pi \angle \theta_\pi$)	$1\angle 0°$	$1\angle 180°$
0 mode ($E_0 \angle \theta_0$)	$0.044\angle -86.7°$	$0.076\angle -86.7°$
Total ($E_{ss} \angle \theta_{ss}$)	$1.003\angle -2.5°$	$0.998\angle 184.4°$

Coefficients

$$E_\pi e^{j\theta_\pi} = \frac{A_\pi \left(\frac{\omega_\pi \omega_{RF}}{Q_\pi}\right)^2 \left\{1 + j\frac{Q_\pi}{\omega_\pi \omega_{RF}}\left(\omega_\pi^2 - \omega_{RF}^2\right)\right\}}{\left[\left(\omega_\pi^2 - \omega_{RF}^2\right)^2 + \left(\frac{\omega_\pi \omega_{RF}}{Q_\pi}\right)^2\right]}$$

$$E_0 e^{j\theta_0} = \frac{A_0 \left(\frac{\omega_0 \omega_{RF}}{Q_0}\right)^2 \left\{1 + j\frac{Q_0}{\omega_0 \omega_{RF}}\left(\omega_0^2 - \omega_{RF}^2\right)\right\}}{\left[\left(\omega_0^2 - \omega_{RF}^2\right)^2 + \left(\frac{\omega_0 \omega_{RF}}{Q_0}\right)^2\right]}$$

$$T_\pi e^{j\phi_\pi} = \frac{A_\pi \left(\frac{\omega_\pi \omega_{RF}}{Q_\pi}\right)^2 \left\{-1 - j\left[\frac{2Q_\pi^2}{\omega_\pi^2\sqrt{4Q_\pi^2 - 1}}\left(\omega_\pi^2 - \omega_{RF}^2\right) - \frac{1}{\sqrt{4Q_\pi^2 - 1}}\right]\right\}}{\left[\left(\omega_\pi^2 - \omega_{RF}^2\right)^2 + \left(\frac{\omega_\pi \omega_{RF}}{Q_\pi}\right)^2\right]}$$

$$T_0 e^{j\phi_0} = \frac{A_0 \left(\frac{\omega_0 \omega_{RF}}{Q_0}\right)^2 \left\{-1 - j\left[\frac{2Q_0^2}{\omega_0^2\sqrt{4Q_0^2 - 1}}\left(\omega_0^2 - \omega_{RF}^2\right) - \frac{1}{\sqrt{4Q_0^2 - 1}}\right]\right\}}{\left[\left(\omega_0^2 - \omega_{RF}^2\right)^2 + \left(\frac{\omega_0 \omega_{RF}}{Q_0}\right)^2\right]}$$

Effect of Mode Separation

- 3.5 MHz mode separation (typical gun)
  - 5° difference between real field and π mode at the cathode
  - 7° additional phase shift between cells for real field compared to pure π mode fields (large correlated energy spread)
  - Beat frequency amplitude (8% in half cell, 4% in full cell)
  - Beat frequency 3.5 MHz
- 15 MHz mode separation (LCLS gun)
  - 1° difference between real field and π mode at the cathode
  - 1° additional phase shift between cells for real field compared to pure π mode fields (small correlated energy spread)
  - Beat frequency amplitude (2% in half cell, 1% in full cell)
  - Beat frequency 15 MHz

Useful Formulas

$V_{gun} = \frac{\sqrt{8\beta R_{shunt} P_{forward}}}{\beta + 1}$	Maximum Energy Gain in Gun
$\beta = \frac{Q_{internal}}{Q_{external}}$	RF Coupling Coefficient
$Q_{internal} = \frac{\omega U}{P_{wall}}$	Internal Q
$Q_{external} = \frac{\omega U}{P_{radiated}}$	External Q
$Q = \frac{Q_{internal} Q_{external}}{Q_{internal} + Q_{external}}$	Loaded Q
$\tau = \frac{2Q}{\omega}$	Filling Time
$U = \alpha E^2$	Stored Energy
$R_{shunt} = \frac{V_{gun}^2}{2P_{wall}}$	Shunt Impedance
$E = \frac{V_{gun}}{k}$	Peak on Axis Electric Field

Energy Calculation

$$F = eE(t) = \frac{dp}{dt}$$

$$eE_{applied}(z)\sin(\omega t + \phi_0) = mc\frac{d(\beta\gamma)}{dt}$$

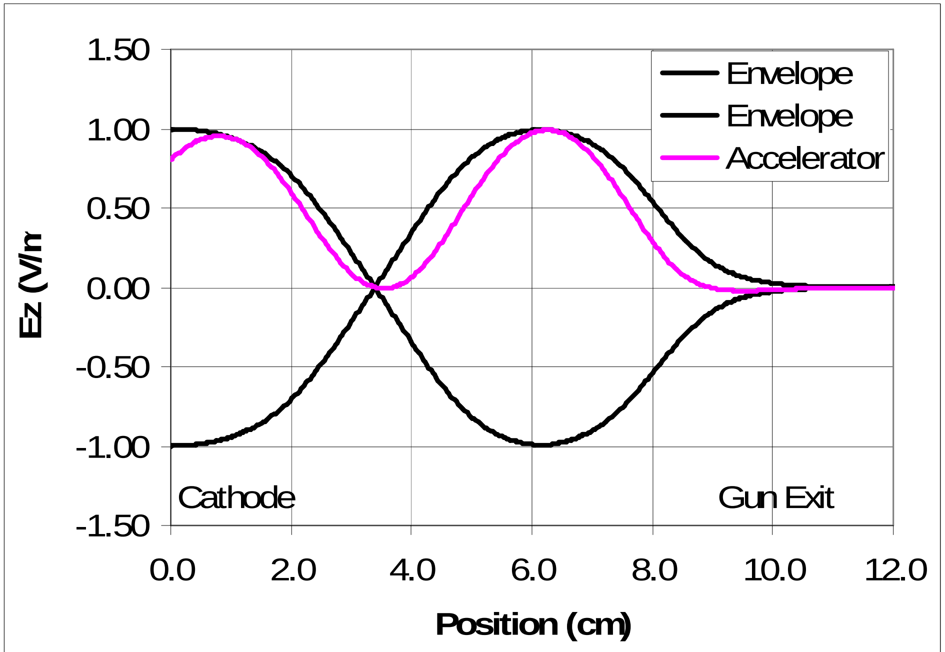
$$z = \beta ct$$

$$eE_{applied}(z)\sin(kz + \phi_0) = mc^2\frac{d\gamma}{dz} \qquad \text{for } \beta \approx 1$$

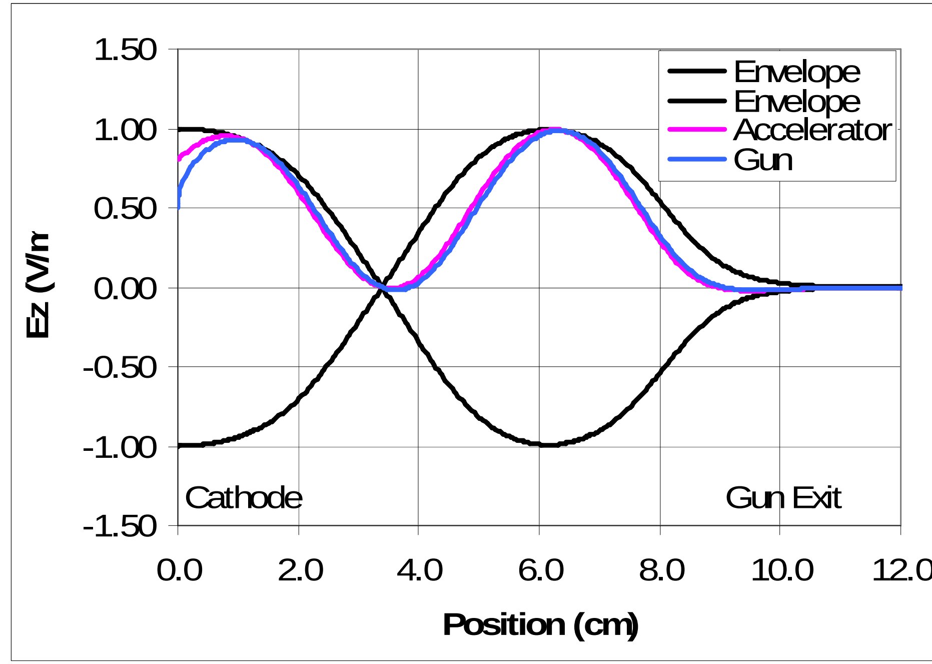
$$eE_{applied}(z)\sin\left(\frac{kz}{\beta(z)} + \phi_0\right) = mc^2\frac{d(\beta\gamma)}{dz} \qquad \text{in the gun}$$

$$\beta\gamma = \frac{\beta}{\sqrt{1-\beta^2}}$$

Energy Gain

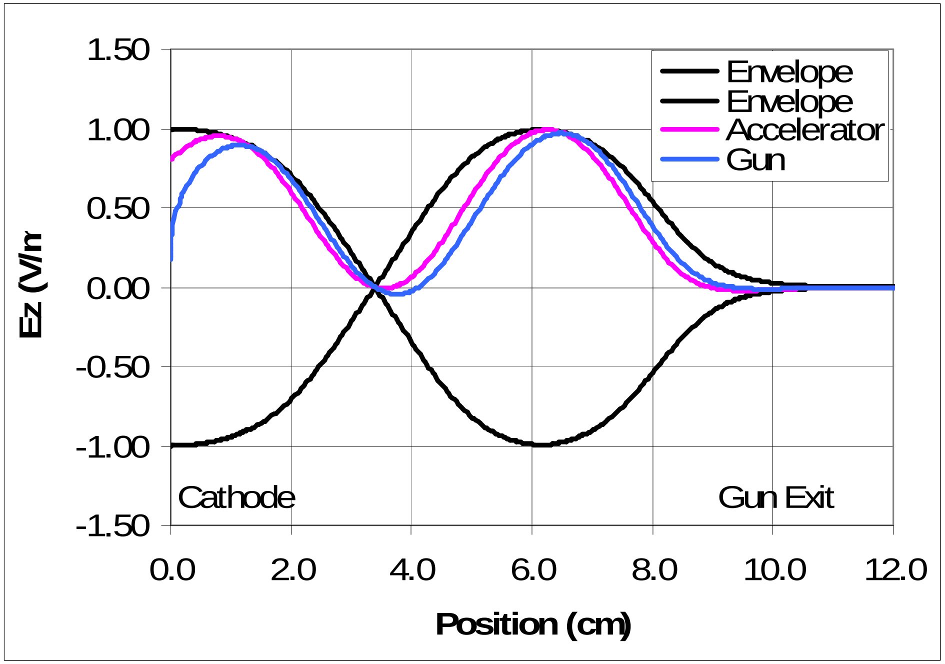


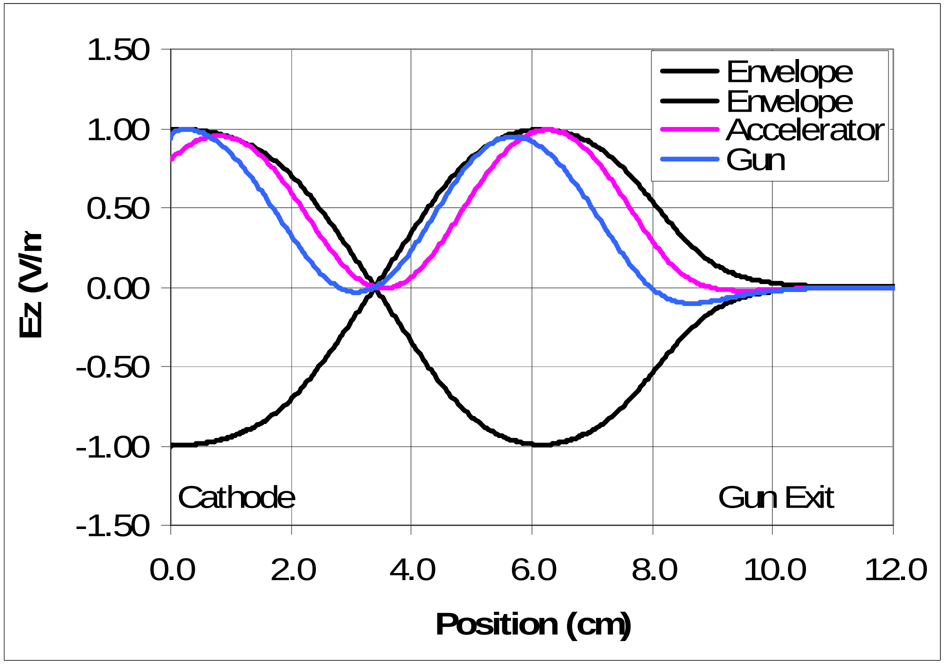
E_{peak} = 120 MV/m
$\theta_{accelerator}$ = 53°
E_{exit} = 6.26 MeV



E_{peak} = 120 MV/m
θ_{gun} = 30°
E_{exit} = 6.17 MeV

Energy Gain

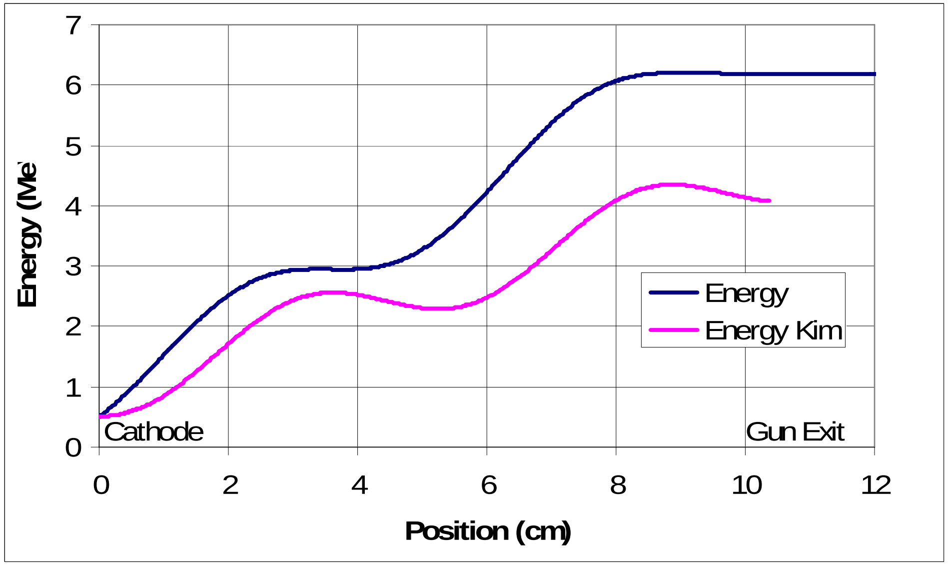


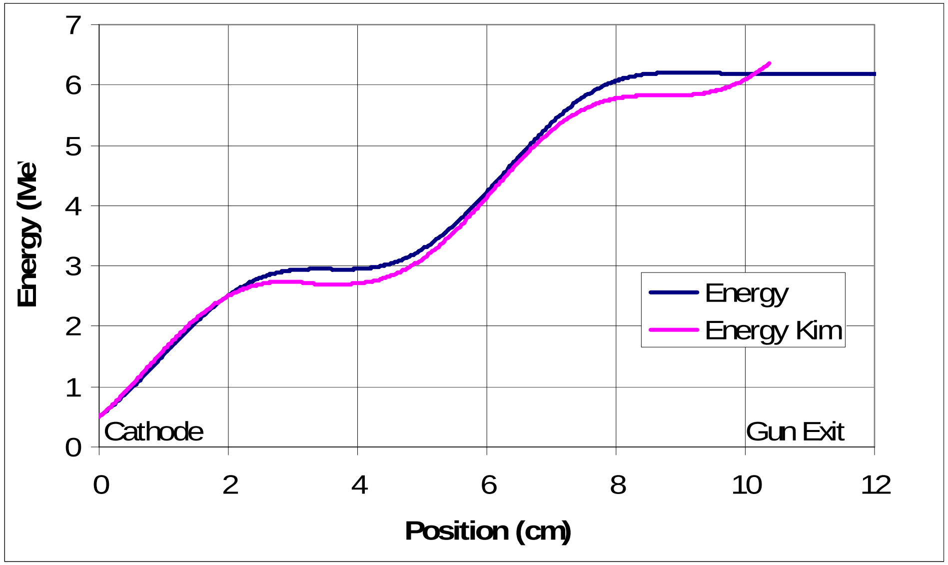


E_{peak} = 120 MV/m
θ_{gun} = 10°
E_{exit} = 6.00 MeV

E_{peak} = 120 MV/m
θ_{gun} = 70°
E_{exit} = 5.26 MeV

Energy Gain

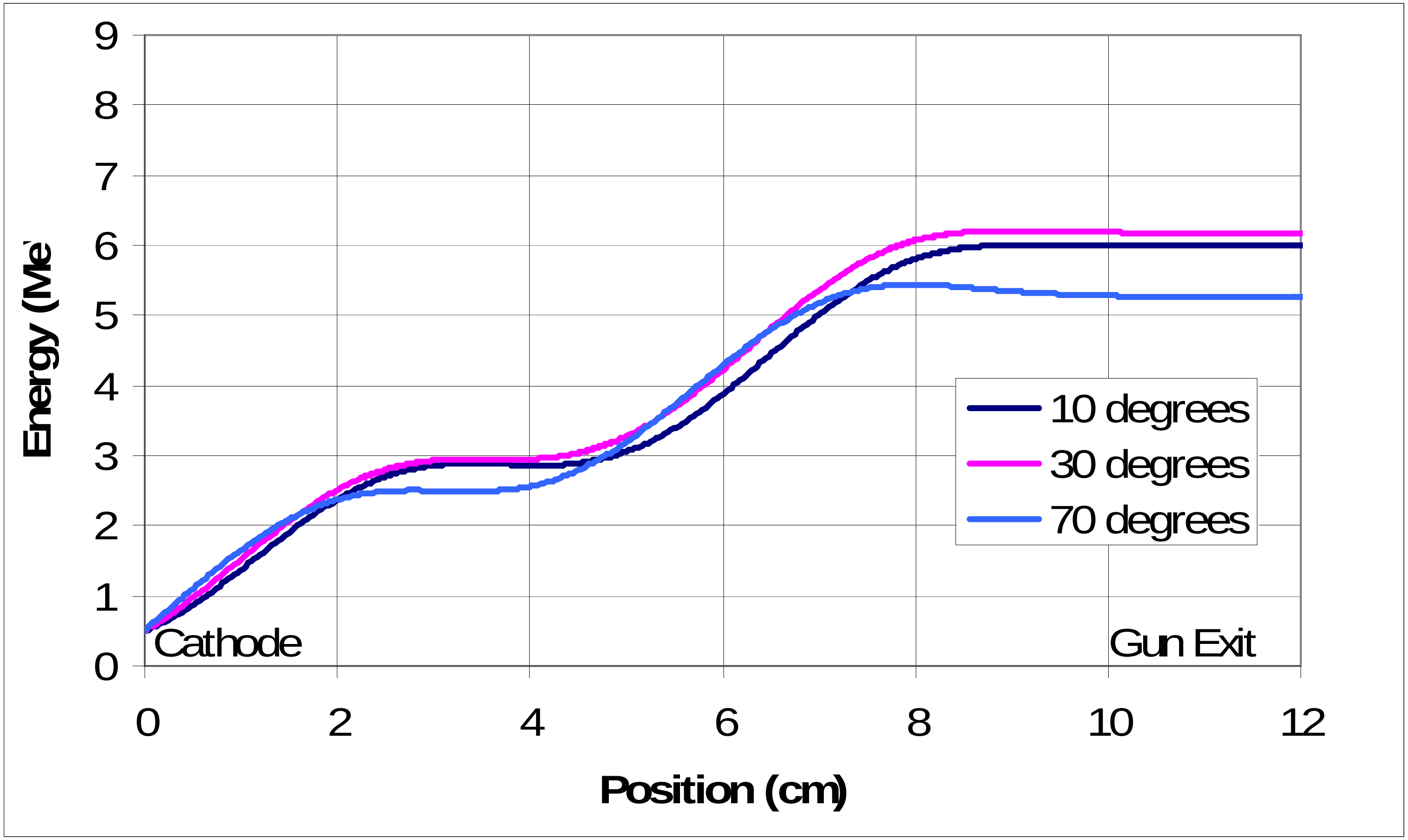




Equation for n+0.5 cell

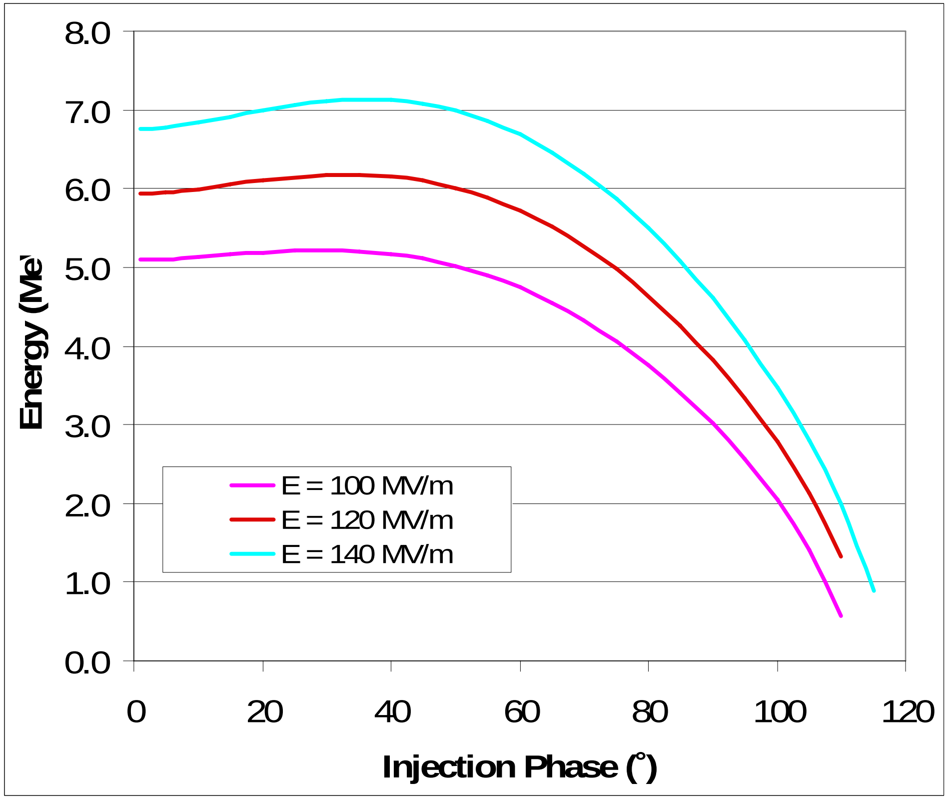
Equation for n+0.6 cell

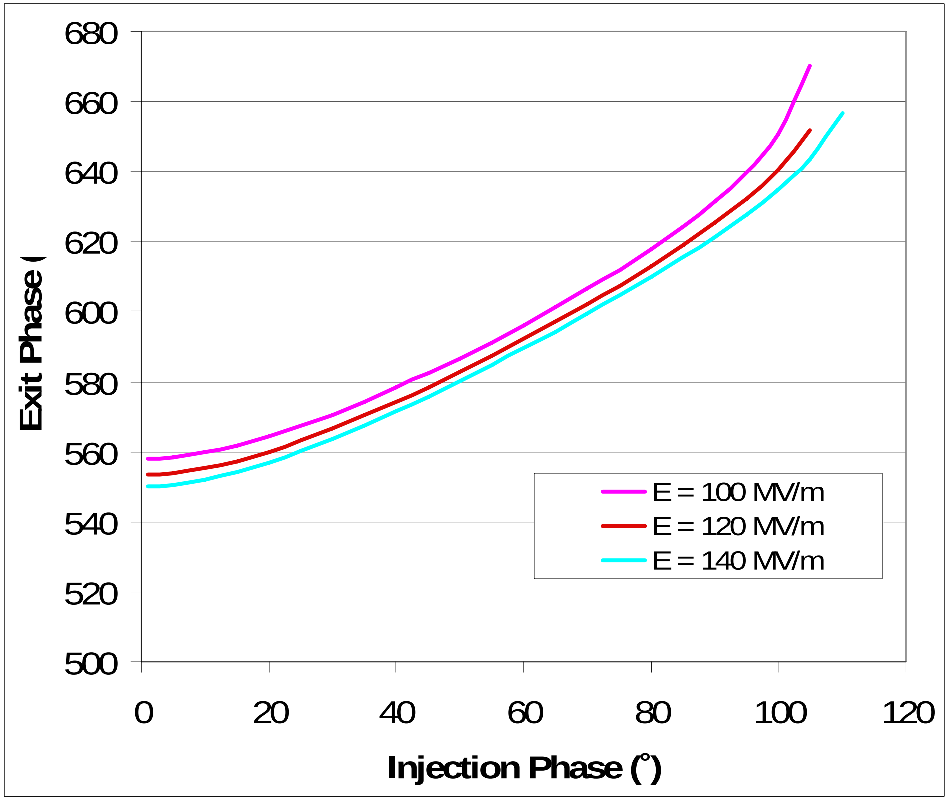
Energy Gain



9
8
7
6
5
4
3
2
1
0
Energy (Me
0
2
4
6
8
10
12
Position (cm)
Cathode
Gun Exit
10 degrees
30 degrees
70 degrees

Energy and Exit Phase vs Injection Phase





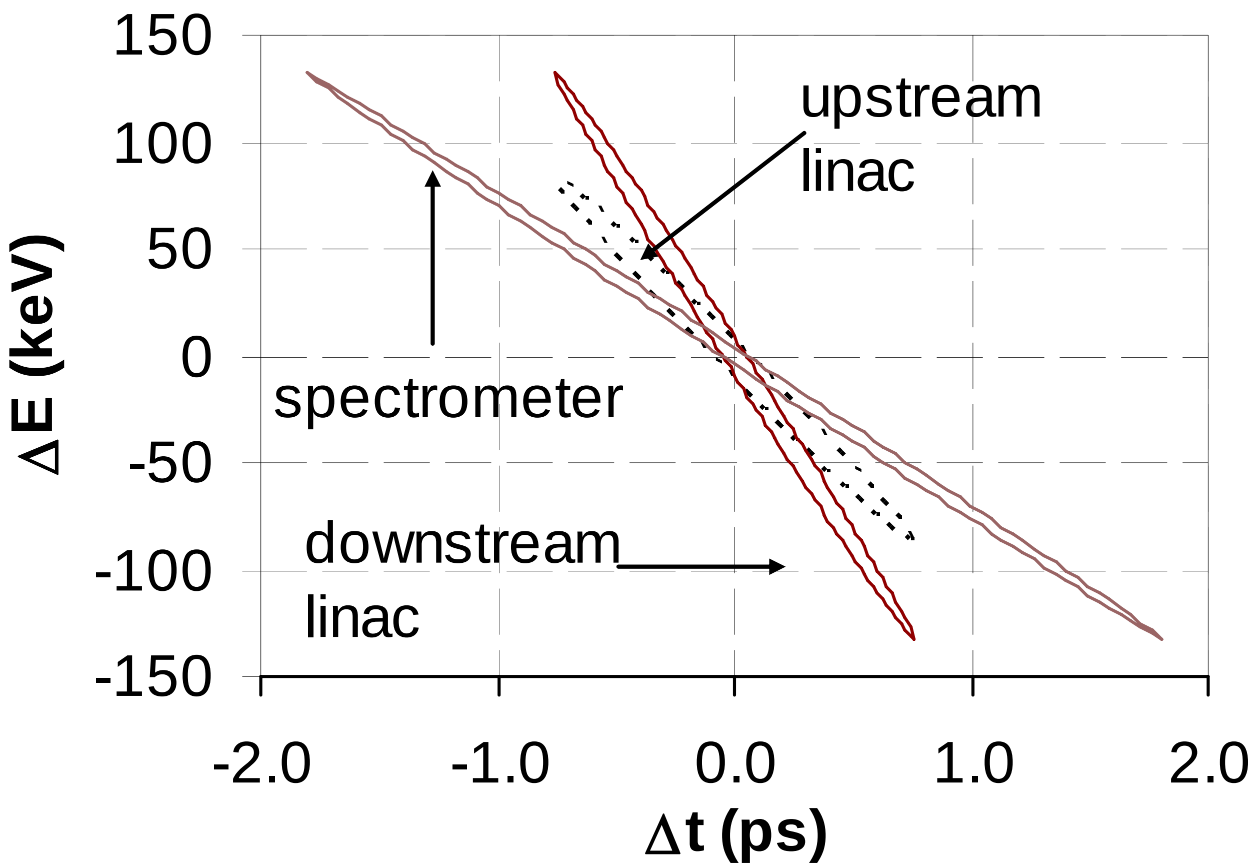
RF Pulse Compression

- Exit pulse length depends on injection phase because the transit time is a function of injection phase
- Final pulse length = $d\phi_{exit}/d\phi_{entrance}\Delta\phi_{laser}$
- Compression if $d\phi_{exit}/d\phi_{entrance}<1$
- Expansion if $d\phi_{exit}/d\phi_{entrance}>1$

Energy Spread

- Dominated by correlated energy vs time
- Energy vs time due to field temporal variation
- Energy spread estimated by $dE_{exit}/d\phi_{entrance}\Delta\phi_{laser}$

Measured Longitudinal Phase Space



Twiss Parameters
Downstream Linac

	RMS	Units
τ_{11}	0.575	ps^2
τ_{12}	-100	keV ps
τ_{22}	17629	keV^2
ε_l	6.8	keV ps
σ_E	133	keV
$\sigma_{E\ uncorr}$	9.02	keV
σ_t	0.76	ps
slope	-175	keV/ps

Home Work #1

Calculate the steady state total field amplitude and phase including the effect of the 0 mode in each cell for the three cases listed below.

- f_0 = 2841 MHz (LCLS case)
- f_0 = 2851 MHz
- f_0 = 2854 MHz

Assume all other parameters are as given on slide 12 and the excitation frequency is 2856 MHz.

Home Work #1 Solution

Calculate the steady state total field amplitude and phase including the effect of the 0 mode in each cell for the three cases listed below.

- f_0 = 2841 MHz
- f_0 = 2851 MHz
- f_0 = 2854 MHz

$$f_0 = 2841\,MHz \quad \begin{aligned} A_{ss\,full} &= 1 + 0.00976e^{-j1.557} = 1.000e^{-j0.010} = 1.000e^{-j\pi/180\,0.559} \\ A_{ss\,half} &= -1 + 0.0178e^{-j1.557} = 1.000e^{-j3.124} = 1.000e^{-j\pi/180\,178.98} \end{aligned}$$

$$f_0 = 2851\,MHz \quad \begin{aligned} A_{ss\,full} &= 1 + 0.0293e^{-j1.530} = 1.002e^{-j0.029} = 1.000e^{-j\pi/180\,1.676} \\ A_{ss\,half} &= -1 + 0.0535e^{-j1.530} = 0.999e^{-j3.088} = 1.000e^{-j\pi/180\,175.26} \end{aligned}$$

$$f_0 = 2854\,MHz \quad \begin{aligned} A_{ss\,full} &= 1 + 0.0730e^{-j1.469} = 1.010e^{-j0.072} = 1.010e^{-j\pi/180\,4.125} \\ A_{ss\,half} &= -1 + 0.133e^{-j1.469} = 0.995e^{-j3.008} = 0.995e^{-j\pi/180\,172.35} \end{aligned}$$

Assume all other parameters are as given on slide 12 and the excitation frequency is 2856 MHz.

Home Work #2

- The exit phase from the SLAC S-band (2856 MHz) rf gun operated at a peak field of 120 MV/m can be approximated by the following polynomial over the range 0-105° of injection phase

$$\theta_{exit} = 0.0443\,\theta_{inject}^3 + 0.213\,\theta_{inject}^2 + 0.390\,\theta_{inject} + 9.63$$

- Calculate the electron bunch length at the gun exit for a 5 ps laser pulse length with a 10°, 30° and 70° injection phase.
- Calculate the range of injection phases that compress the bunch.
- Calculate the range of injection phases that lengthen the bunch.