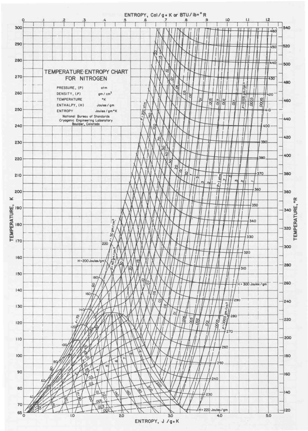
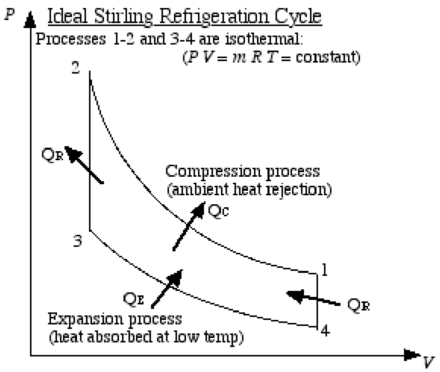
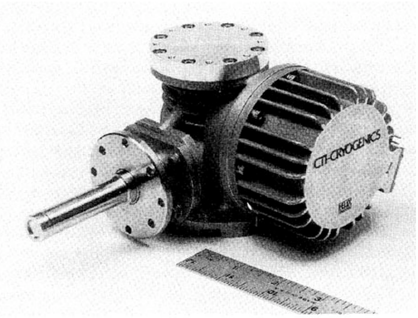
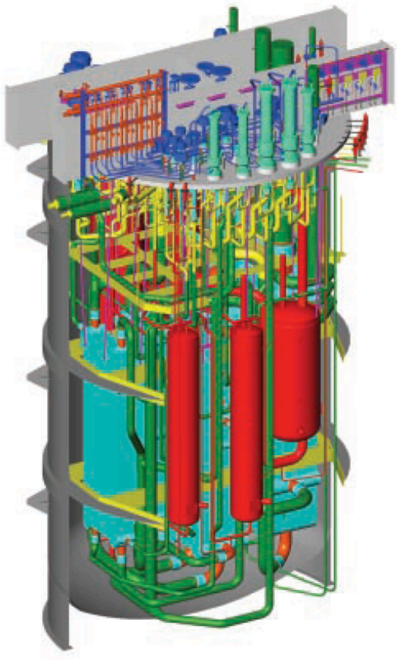








Refrigeration & Liquefaction

J.M Pfotenhauer

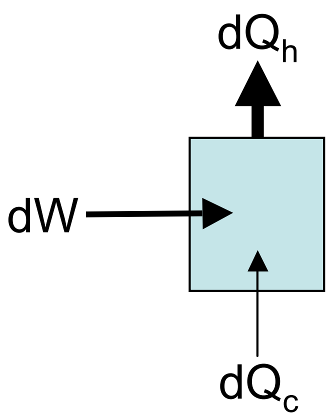
University of Wisconsin - Madison

Outline

- Recuperative systems
  - Ideal refrigeration / liquefaction
  - Joule Thomson expansion
  - System analyses: 1st and 2nd law applied to:
    - Simple Linde-Hampson cycle
    - Variations and improved performance cycles
    - Claude and Collins cycles
  - Introduction to EES
- Regenerative systems
  - Overview of regenerative coolers
  - Stirling Cryocoolers
  - Gifford-McMahon Cryocoolers
  - Pulse tube cryocoolers

Ideal Refrigeration/Liquefaction

- 'Moving' heat from a cold reservoir to a warm reservoir requires energy



The amount of heat moved is associated with an amount of entropy by the relationship:

$$dQ = TdS$$

• In an ideal process, the entropy associated with the two heat flows is the same, that is:

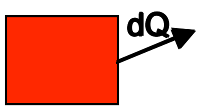
$$dS = \frac{dQ_c}{T_c} = \frac{dQ_h}{T_h}$$

- In an ideal process the amount of work (energy) required to 'move' the heat is

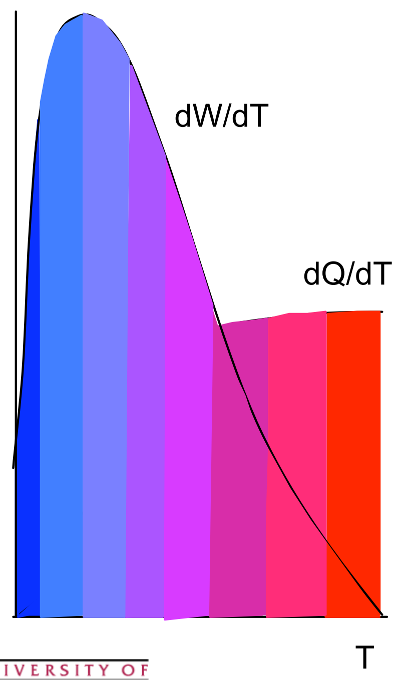
$$dW = dQ_h - dQ_c$$

Ideal Cool Down

- Extracting an amount of heat to lower the temperature of (whatever) by dT, and releasing the heat at T_h:



$$dQ = mc_p dT, \quad dW = dQ_h \quad dQ = mc_p \; \frac{T_h}{T} \; 1 \; dT$$



Including the temperature dependence of the specific heat, the ideal cool down work becomes:

$$(\; W = \int_{T_c}^{T_h} mc_p(T) \; \frac{T_h}{T} \; 1 \; dT$$

Compare this to the amount of energy required to warm up the same mass:

$$E = \int_{T_c}^{T_h} mc_p(T) dT$$

Ideal Liquefaction

- To cool down a parcel of gas, <u>and</u> convert it from saturated vapor to saturated liquid at its normal boiling temperature:

Temperature dependent specific heat

$$(W = \int_{T_{nbp}}^{T_h} \overbrace{mc_p(T)} \frac{T_h}{T} \quad 1 \; dT + mh_{fg} \; \frac{T_h}{T_{nbp}} \quad 1$$

Work to extract sensible heat   Work to extract latent heat

- Re-arranging terms we have:

$$W = mT_h \quad \frac{c_p(T)}{T} dT + \frac{h_{fg}}{T_{nbp}} (\;)\; m \quad c_p dT + h_{fg}$$

$$W = mT_h \;\; s \quad m \;\; h$$

- Or, in the 'rate' form:

$$\dot{W} = \dot{m} T_h \;\; s \quad \dot{m} \;\; h$$

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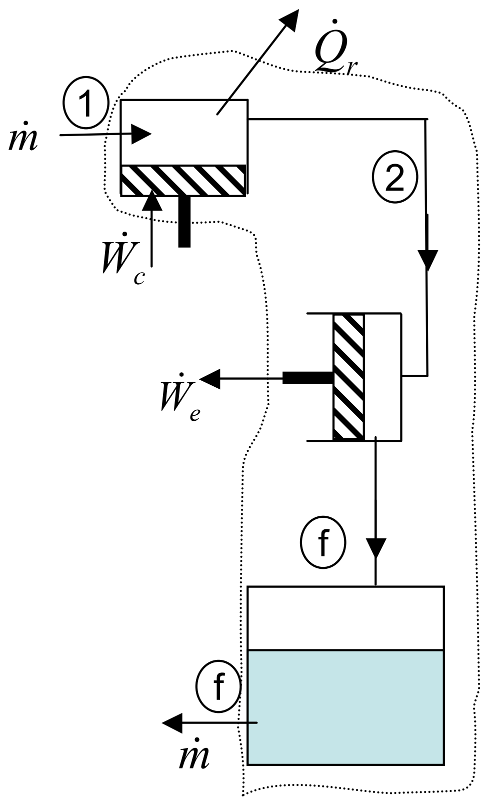
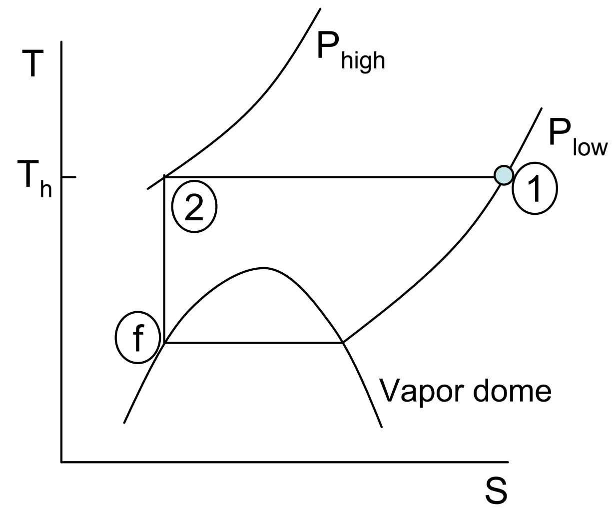
Ideal Liquefaction



A 1st-law, 2nd-law analysis around an ideal cycle reveals the same expression



1st law: Energy balance around system:
In steady state, the sum of the energies into and out of the system = 0

$$\dot{W}_c + \dot{m}h_1 = \dot{W}_e + \dot{Q}_r + \dot{m}h_f \quad or \quad \dot{W}_{net} = \dot{Q}_r \quad \dot{m}\left(h_1 \quad h_f\right)$$

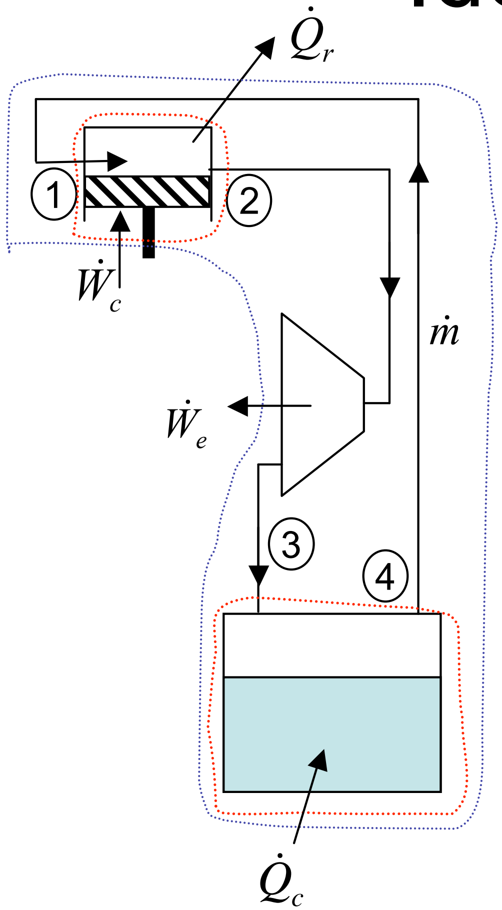
2nd law: Entropy balance around system:
In steady state, the sum of the entropies into and out of the system = 0

$$\dot{m}s_1 = \dot{m}s_f + \frac{\dot{Q}_r}{T_1} + \nearrow^{0} \quad or \quad \dot{Q}_r = T_1\dot{m}\left(s_1 \quad s_f\right)$$

Combining,we have: $$\dot{W}_{net} = T_1\dot{m}\left(s_1 \quad s_f\right) \quad \dot{m}\left(h_1 \quad h_f\right)$$

Note the SI units of h(kJ/kg) and s(kJ/kg-K)

Ideal Refrigeration



- In steady state, the 1st law around the whole system gives:

$$\dot{W}_c \quad \dot{W}_e = \dot{Q}_r \quad \dot{Q}_c \quad or \quad \dot{W}_{net} = \dot{Q}_r \quad \dot{Q}_c$$

- The 2nd law around the compressor gives:

$$\dot{Q}_r = T_H \dot{m}\left(s_1 \quad s_2\right)$$

- The 2nd law around the evaporator gives:

$$\dot{Q}_c = T_c \dot{m}\left(s_4 \quad s_3\right)$$

- Combining, and noting that s_1=s_4 and s_2 =s_3 we have:

$$\frac{\dot{W}_{net}}{\dot{m}} = \left(T_H \quad T_c\right)\left(s_4 \quad s_3\right) \quad = \frac{S}{\dot{m}}\left(T_H \quad T_c\right) \quad = \frac{\dot{Q}_c}{\dot{m}} \frac{T_H}{T_C} \quad 1($$

- The coefficient of performance (COP) for the refrigerator is then

$$COP_{ideal} \quad \frac{\dot{Q}_c}{\dot{W}_{net}} = \frac{T_H}{T_C} \quad 1^{\quad 1} = \frac{T_C}{T_H \quad T_C}$$

Ideal Liquefaction / Refrigeration

Table 3.1. Ideal-work requirements for liquefaction of gases beginning at 300 K (80°F) and 101.3 kPa (14.7 psia)

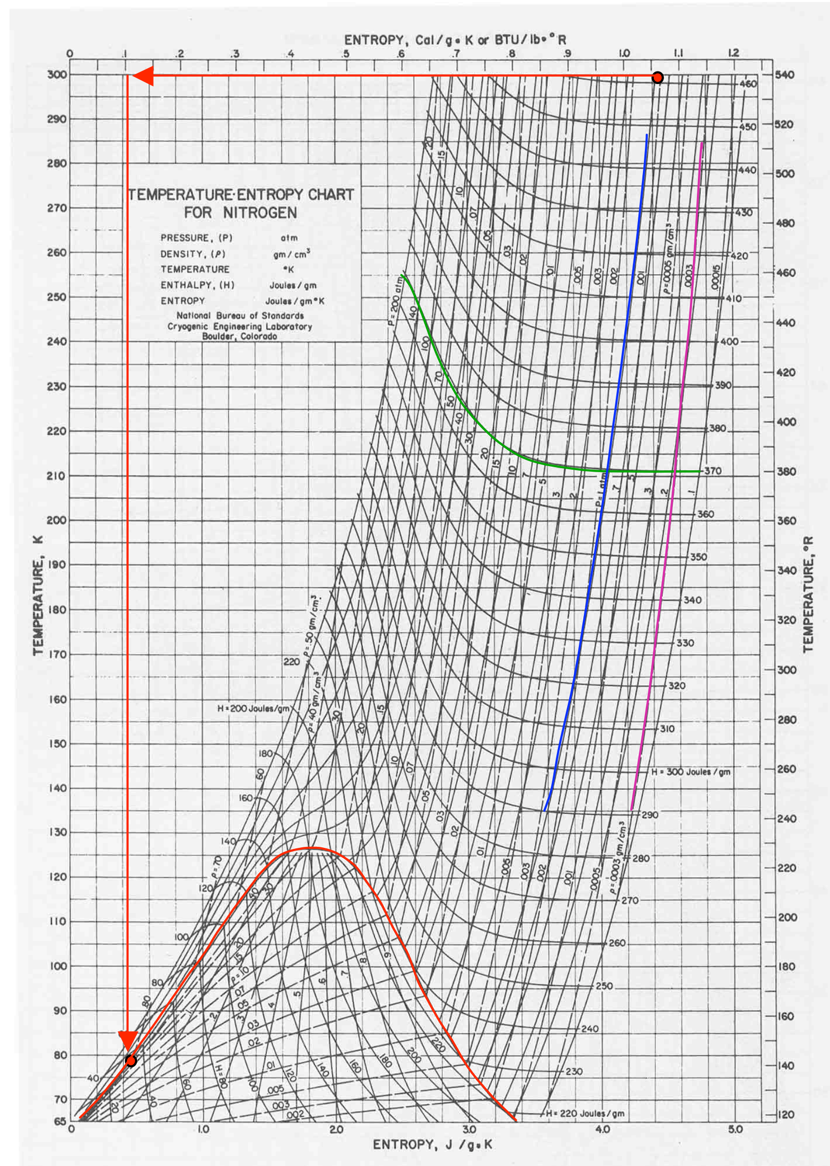
Gas	Normal Boiling Point		Ideal Work of Liquefaction, $-\dot{W}_i/\dot{m}_f$	
	K	°R	kJ/kg	Btu/lb$_m$
Helium-3	3.19	5.74	8 178	3 516
Helium-4	4.21	7.58	6 819	2 931
Hydrogen, H_2	20.27	36.5	12 019	5 167
Neon, Ne	27.09	48.8	1 335	574
Nitrogen, N_2	77.36	139.2	768.1	330.2
Air	78.8	142	738.9	317.7
Carbon monoxide, CO	81.6	146.9	768.6	330.4
Argon, A	87.28	157.1	478.6	205.7
Oxygen, O_2	90.18	162.3	635.6	273.3
Methane, CH_4	111.7	201.1	1 091	469
Ethane, C_2H_6	184.5	332.1	353.1	151.8
Propane, C_3H_6	231.1	416.0	140.4	60.4
Ammonia, NH_3	239.8	431.6	359.1	154.4

- Ideal liquefaction work for cryogens (from Barron)
- Comparison with ideal performance defined by Figure of Merit (FOM), for refrigeration sometimes referred to as "% of Carnot."

$$FOM_{liquefier} = \frac{\left(\dot{W}_{net}/\dot{m}_f\right)_{ideal}}{\left(\dot{W}_{net}/\dot{m}_f\right)_{actual}} \qquad FOM_{refrigerator} = \frac{COP_{actual}}{COP_{ideal}}$$

Practical Limitations

- Not possible to achieve ideal-scenario pressure
  - Inspect T-S diagram: find lines of constant pressure, constant enthalpy, constant density, vapor dome
  - Estimate required pressure for 'ideal' liquefaction of nitrogen
- Isentropic expansion is very difficult to achieve.
  - Isenthalpic (or throttle) expansion is very easy to achieve
  - Cooling associated with throttle process exploits 'real-gas' properties. Note that at high T, low P, h is independent of pressure, but elsewhere it is not.



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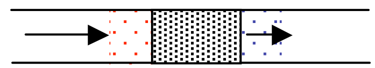
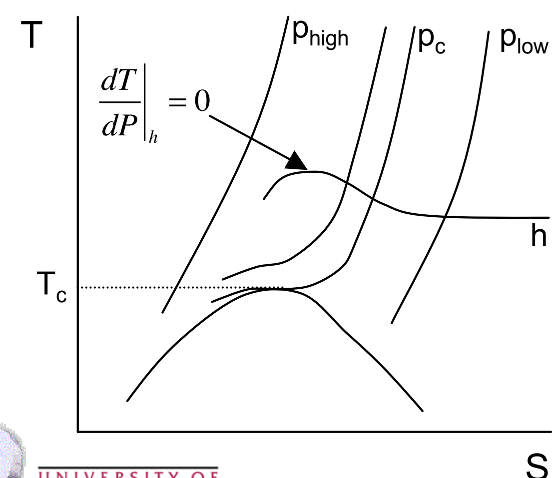
Joule-Thomson Coefficient

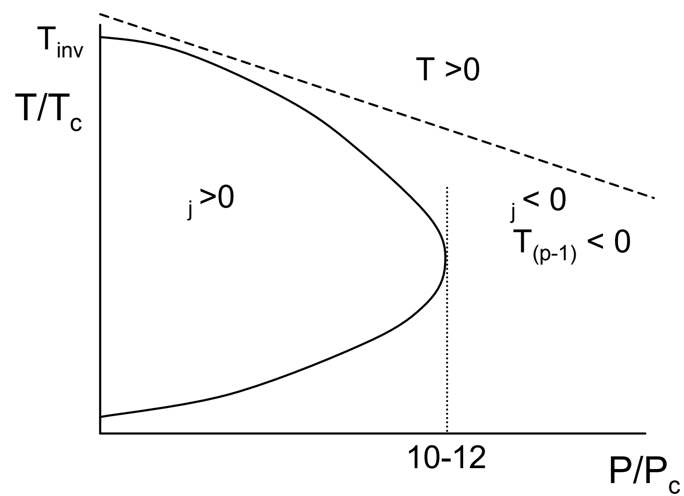
- 1885 - Joule & _ Thomson (Lord Kelvin) confirm that a gas flow through a restriction experiences a temperature drop along with the pressure drop.



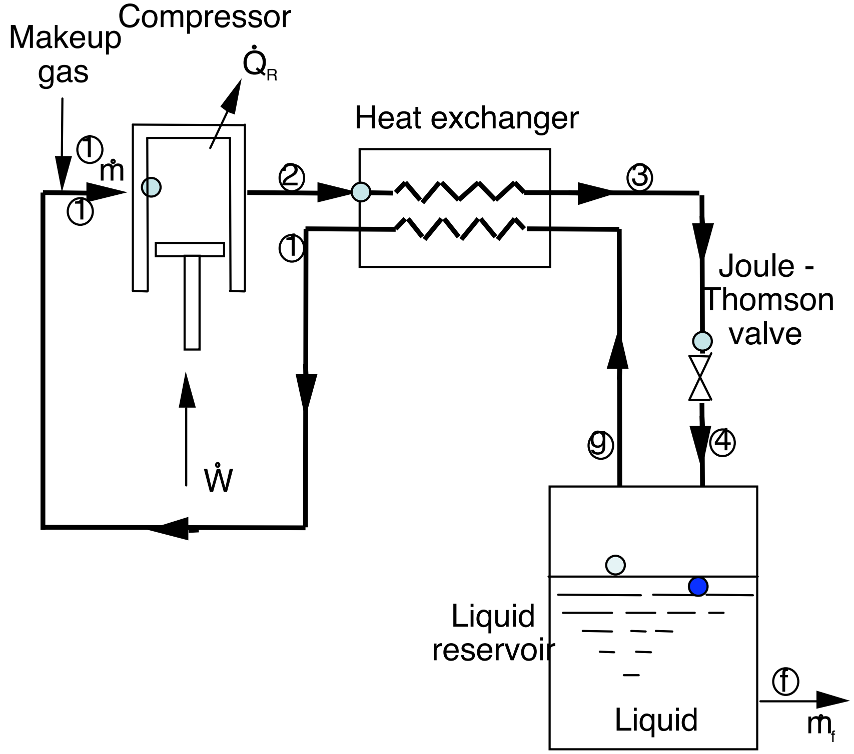
- The Joule-Thomson coefficient: $_j = \left.\frac{dT}{dP}\right|_h$ characterizes the phenomenon.
- When $_j$>0, cooling accompanies a pressure drop.
- Regions of positive and negative $_j$ are reflected in T-S diagrams and inversion curves:

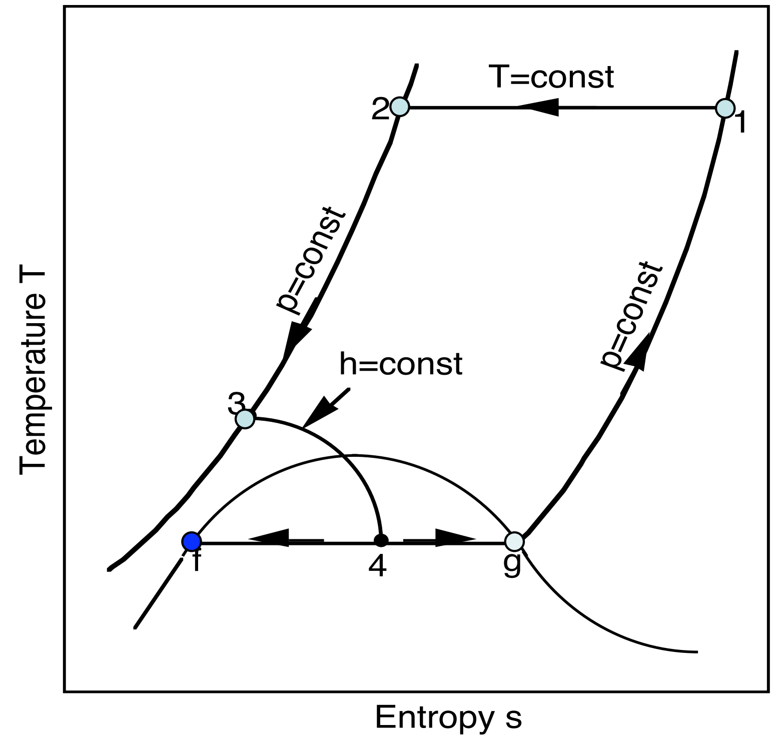


- Above the inversion temperature, $_j \leq 0$ for all pressures.
- Pre-cooling required for helium, hydrogen, and neon.



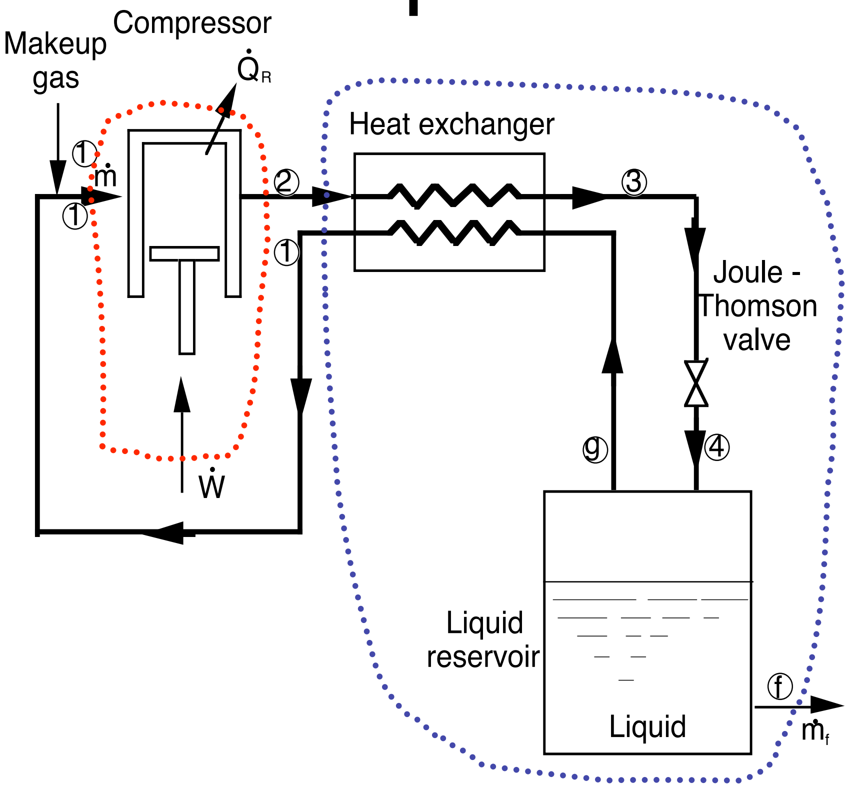
Simple Linde-Hampson Cycle





- Inversion temperature must be above compression temperature, or pre-cooling via a higher temperature refrigerant liquid is required.
- Recuperative heat exchanger pre-cools high pressure stream.
- Liquefier requires source of make-up gas.
- Refrigerator absorbs heat converting liquid to vapor at saturation temperature of low pressure.

Simple Linde-Hampson Cycle



- In steady state conditions, the 1st law around the compressor gives:

$$\dot{W}_c - \dot{Q}_r + \dot{m}\left(h_1 - h_2\right) = 0$$

- The 2nd law around the compressor gives:

$$\dot{m}s_1 = \dot{m}s_2 + \frac{\dot{Q}_r}{T_1} \quad or \quad \dot{Q}_r = \dot{m}T_1\left(s_1 - s_2\right)$$

(Note the assumption of isothermal compression)

- Combining, we have:

$$\frac{\dot{W}_c}{\dot{m}} = T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)$$

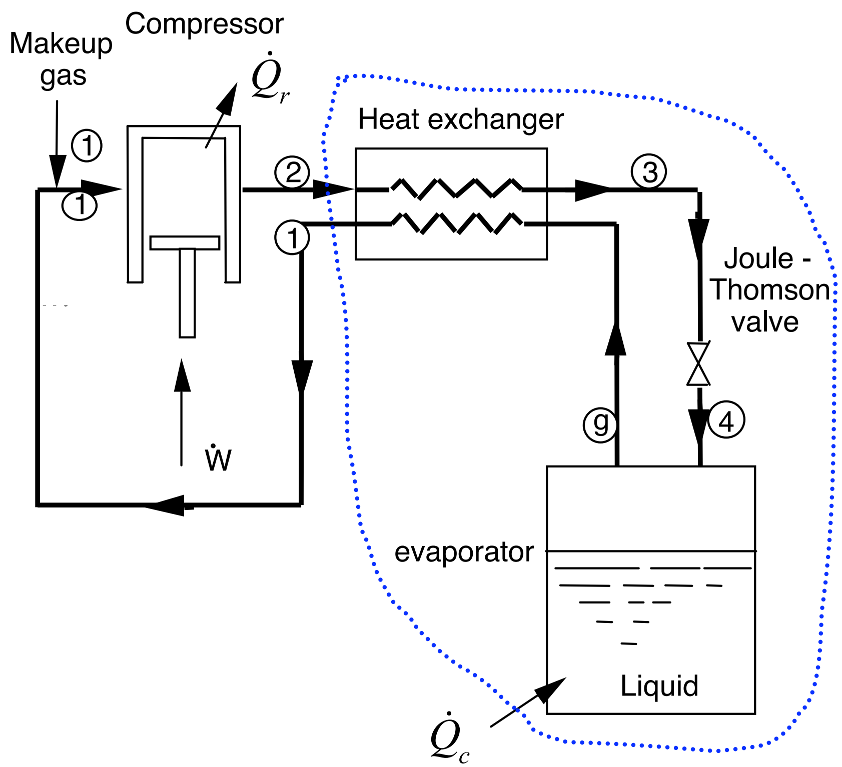
- Applying the 1st law around everything except the compressor gives:

$$\dot{m}h_2 - \left(\dot{m} - \dot{m}_f\right)h_1 - \dot{m}_f h_f = 0 \quad or \quad \dot{m}\left(h_1 - h_2\right) = \dot{m}_f\left(h_1 - h_f\right)$$

- Defining yield, $Y = \frac{\dot{m}_f}{\dot{m}} = \frac{h_1 - h_2}{h_1 - h_f}$ and combining with compression work gives:

$$\frac{\dot{W}_c}{\dot{m}_f} = \frac{\dot{W}_c}{\dot{m}Y} = \left[T_1\left(s_1 - s_2\right) - \left(h_1 - h_2\right)\right]\left(\frac{h_1 - h_f}{h_1 - h_2}\right)$$

Simple Linde-Hampson (JT) Refrigerator



- Applying 1st law (energy balance) to everything except the compressor gives:

$$\dot{Q}_c = \dot{m}\left(h_1 \quad h_2\right) = \dot{m}y \quad h_{fg}$$

- Combining with the expression for the compressor work provides an equation for the COP:

$$COP = \frac{\dot{Q}_c}{\dot{W}} = \frac{\left(h_1 \quad h_2\right)}{T_1\left(s_1 \quad s_2\right) \quad \left(h_1 \quad h_2\right)}$$

- Comparing with the Carnot COP gives the FOM (or % of Carnot):

$$FOM = \frac{\left(h_1 \quad h_2\right)\left(T_1 \quad T_c\right)}{T_1\left(s_1 \quad s_2\right) \quad \left(h_1 \quad h_2\right) \quad T_c}$$

Linde-Hampson Performance

- Optimum theoretical performance realized by minimizing h_2 (P_2 such that h is on the inversion curve)
- P_2 is typically ~ 100 atm.
- Theoretical performance with P_2 = 20 atm.(from Barron):

Table 3.3. Performance of the Linde-Hampson system using different fluids. p_1 = 101.3 kPa (14.7 psia); p_2 = 20.265 MPa (200 atm); T_1 = T_2 = 300 K (80°F); heat-exchanger effectiveness = 100 percent; compressor overall efficiency = 100 percent

	Normal Boiling Point		Liquid Yield $y =$	Work per Unit Mass Compressed		Work per Unit Mass Liquefied		Figure of Merit FOM =
Fluid	K	°R	$\dot{m}_f/\dot{m}$	kJ/kg	Btu/lb$_m$	kJ/kg	Btu/lb$_m$	$\dot{W}_i/\dot{W}$
N_2	77.36	139.3	0.0708	472.5	203.2	6673	2869	0.1151
Air	78.8	142	0.0808	454.1	195.2	5621	2416	0.1313
CO	81.6	146.9	0.0871	468.9	201.6	5381	2313	0.1428
A	87.28	157.1	0.1183	325.3	139.8	2750	1182	0.1741
O_2	90.18	162.3	0.1065	405.0	174.1	3804	1636	0.1671
CH_4	111.7	201.1	0.1977	782.4	336.4	3957	1701	0.2758
C_2H_6	184.5	332.1	0.5257	320.9	138.0	611	262	0.5882
C_3H_8	231.1	416.0	0.6769	159.0	68.4	235.0	101.0	0.5976
NH_3	239.8	431.6	0.8079	363.1	156.1	449.4	193.2	0.7991

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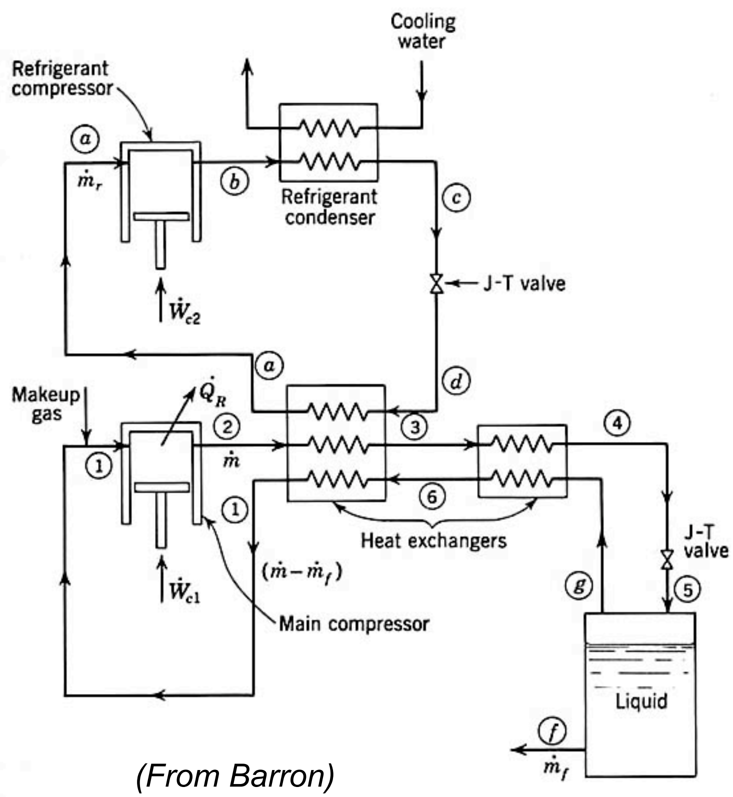
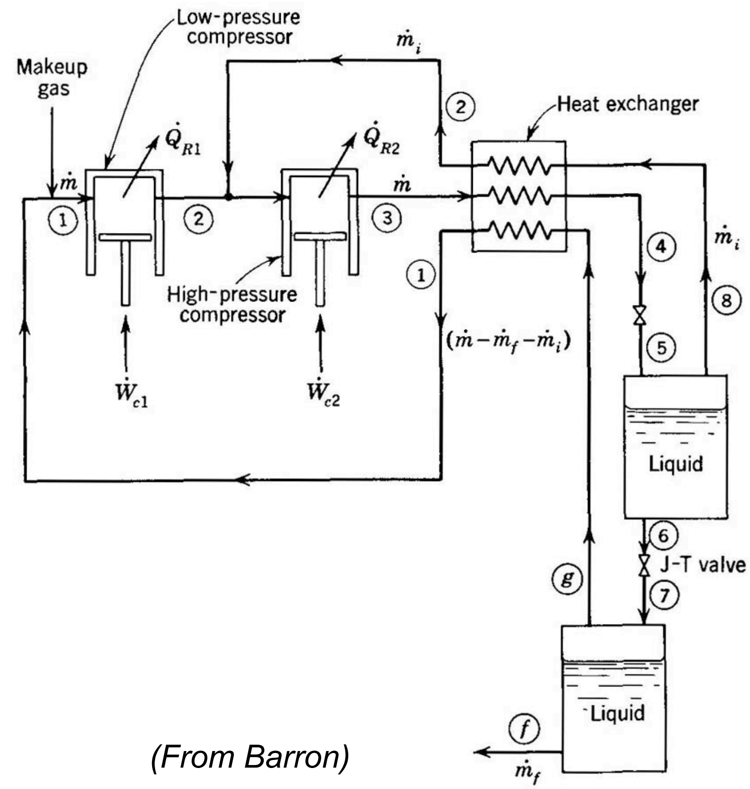
Linde-Hampson Cycle Enhancements



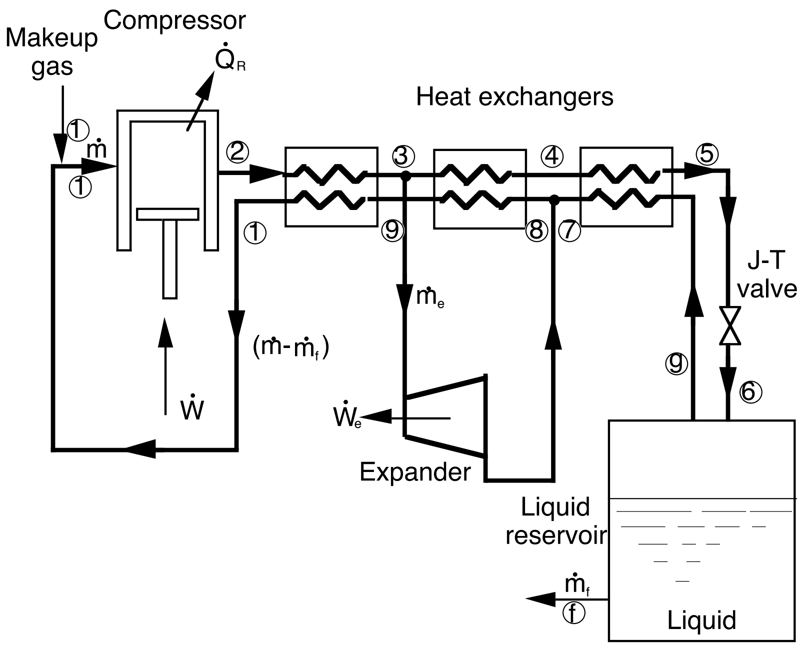
(From Barron)

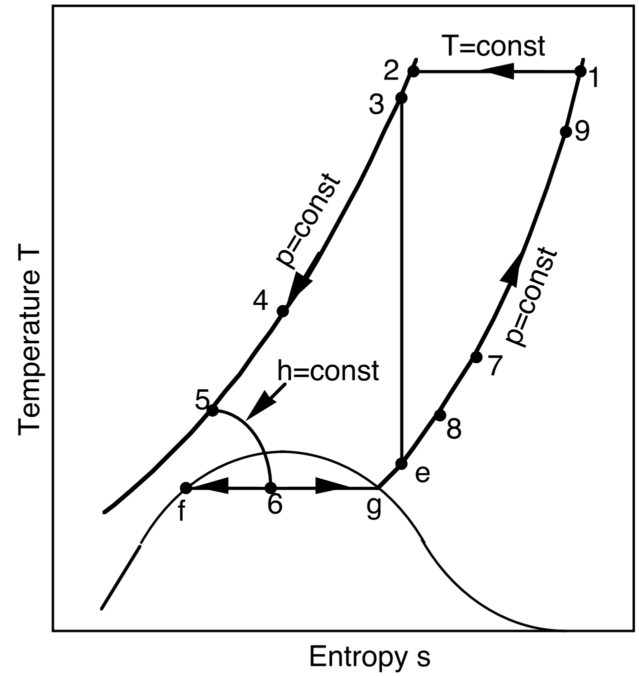


(From Barron)

- Pre-cooled L-H cycle
  - Optimize performance via pressure, pre-cooling temperature and mass flow ratio
  - FOM increased by ~ factor of 2
- Dual-pressure L-H cycle
  - Optimize performance via two pressures and fractional mass flow ratio
  - FOM increased by ~ factor of 1.9

Claude Cycle: isentropic expansion





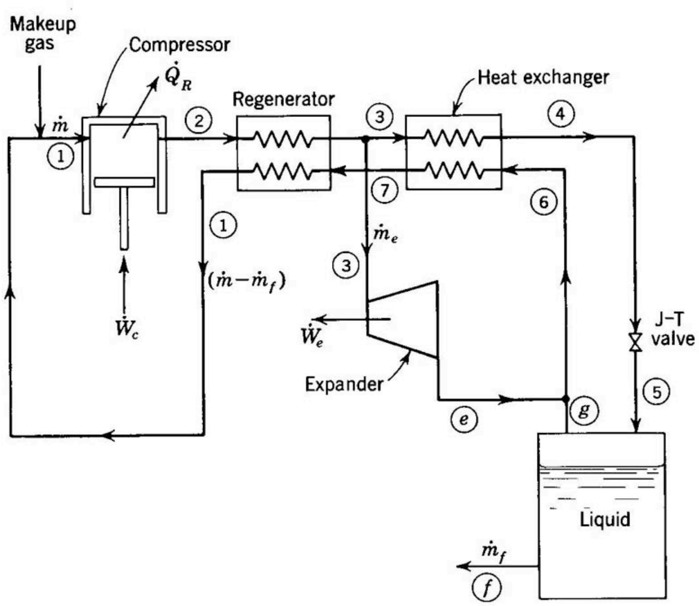
- Isentropic expansion, characterized by $_s$=dT/dP$_s$ (always >0) results in larger temperature drop for a given pressure drop than with isenthalpic expansion
- 1st and 2nd law analyses give:

$$y = \frac{h_1 \quad h_2}{h_1 \quad h_f} + x\frac{h_3 \quad h_e}{h_1 \quad h_f} \; ; \quad x = \frac{\dot{m}_e}{\dot{m}}; \quad x + y < 1$$

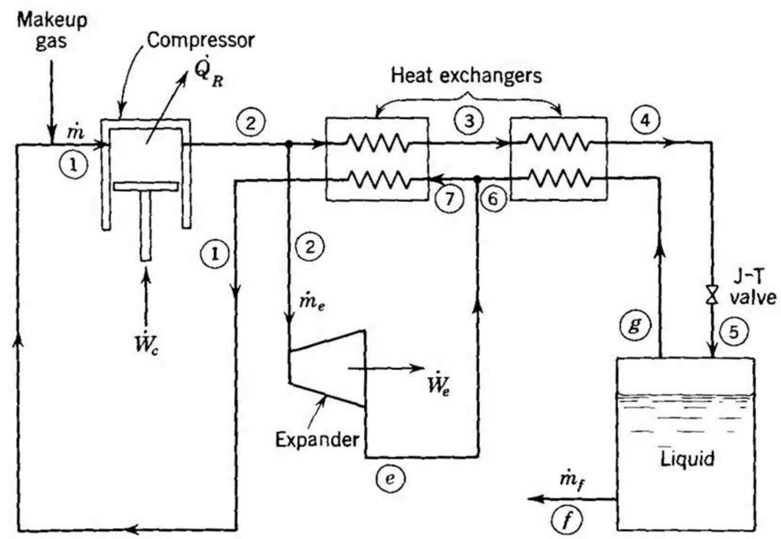
$$\frac{\dot{W}_{net}}{\dot{m}_f} = \frac{T_1(s_1 \quad s_2) \quad (h_1 \quad h_2) \quad x(h_3 \quad h_e) \quad (h_1 \quad h_f)}{(h_1 \quad h_2) + x(h_3 \quad h_e)}$$

Optimize performance by varying P_2, T_3, and x.

Claude Cycle: Variations



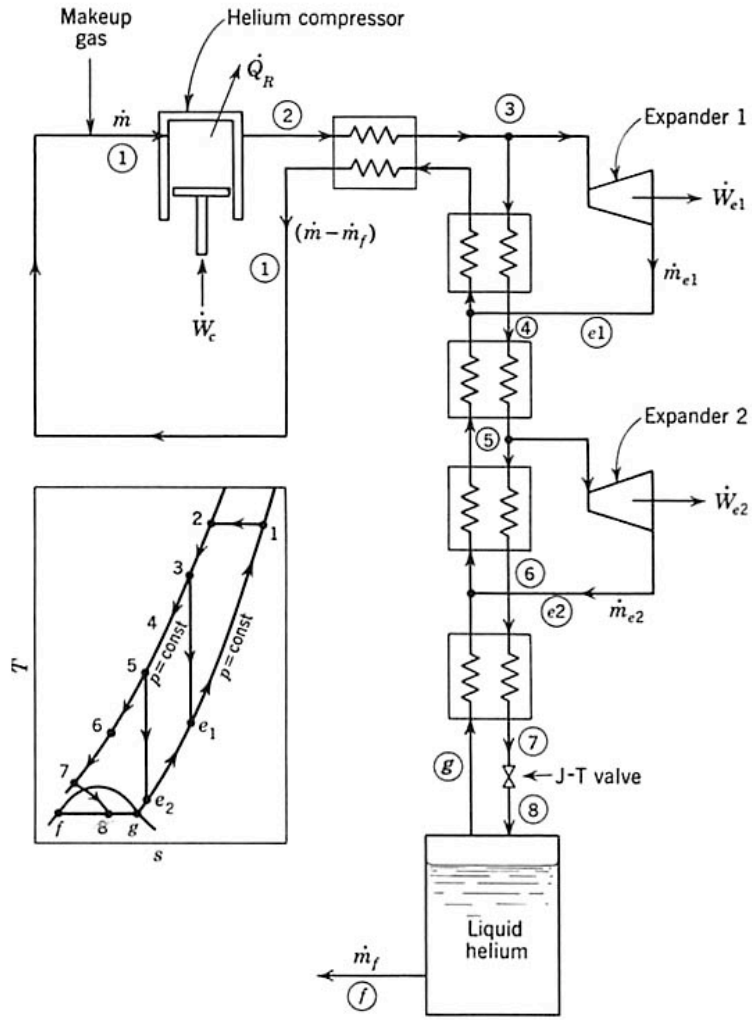
(From Barron)



(From Barron)

- Kapitza cycle
  - Low pressure (7 atm) production of liquid air
  - Regenerative heat exchanger
- Heylandt cycle
  - High pressure (200 atm) air liquefaction
  - Room temperature expander

Collins Liquefier

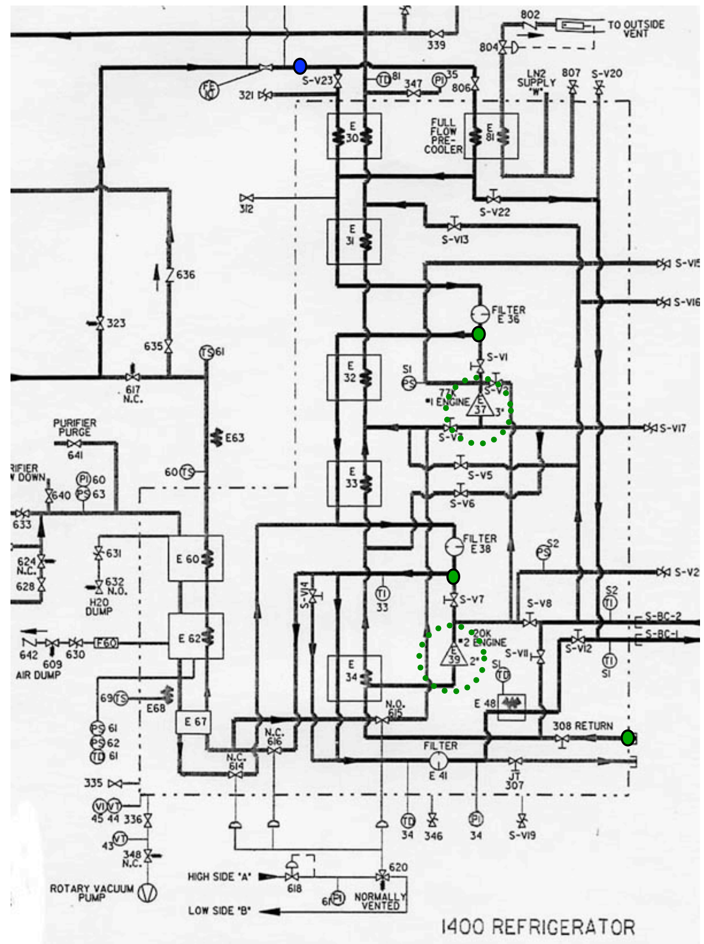


- Introduced by Sam Collins (MIT) in 1952
- Optimized performance via expander flow rates and temperatures
- LN_2 pre-cooling increases yield by factor of 3.

(From Barron)

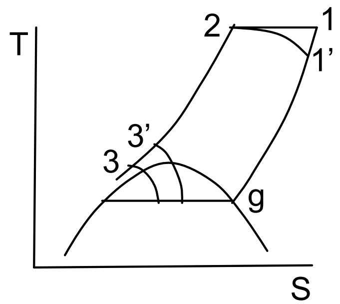
Commercial Helium Liquefier

- The dashed line encloses the ‘cold box,’ i.e. everything except the compressor.
- Find the expansion engines
- Trace the flow from LN_2 precooler through the cold box to the JT valve.



Influence of Non-Ideal Components

- A non-ideal heat exchanger will have an effectiveness less than 1.
- A non-isothermal compressor will require more work than an isothermal compressor



$$= \frac{h_{1'} \quad h_g}{h_1 \quad h_g}$$

$$_c = \frac{\left(\dot{W}/\dot{m}\right)_{isothermal}}{\left(\dot{W}/\dot{m}\right)_{actual}}$$

- The influence of these non-ideal parameters on the cooling capacity (refrigerator), liquid yield (liquefier), and compression work for a simple Linde-Hampson system is:

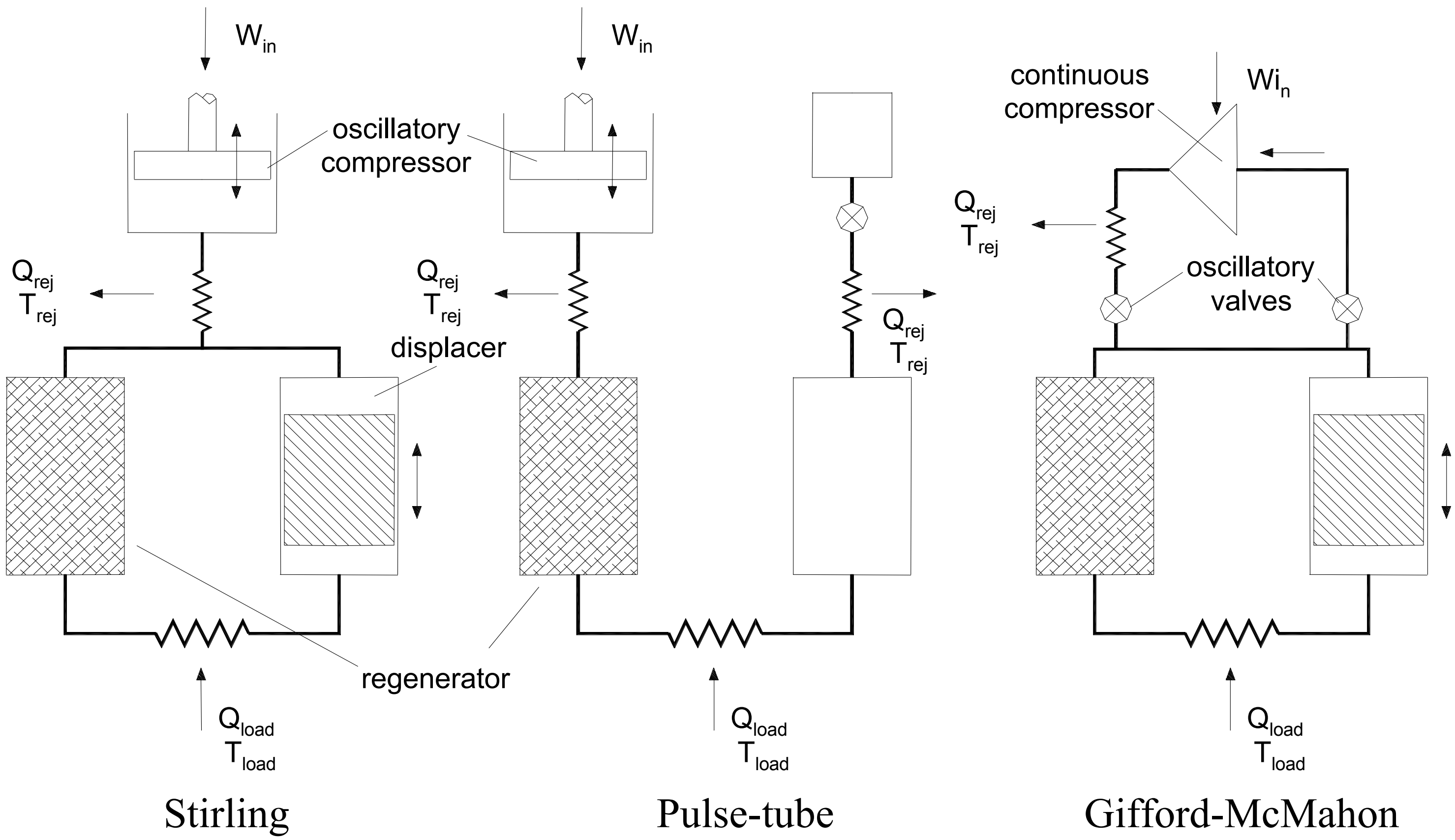
$$\frac{\dot{Q}}{\dot{m}} = \left(h_1 \quad h_2\right) \quad \left(1 \quad \right)\left(h_1 \quad h_g\right)$$

$$y = \frac{h_{1'} \quad h_2}{h_{1'} \quad h_f} = \frac{\left(h_1 \quad h_2\right) \quad \left(1 \quad \right)\left(h_1 \quad h_g\right)}{\left(h_1 \quad h_f\right) \quad \left(1 \quad \right)\left(h_1 \quad h_g\right)}$$

$$\frac{\dot{W}}{\dot{m}} = \frac{1}{_c} \quad T_1\left(s_{1'} \quad s_2\right) \quad \left(h_1 \quad h_2\right) + \left(1 \quad \right)\left(h_1 \quad h_g\right)$$

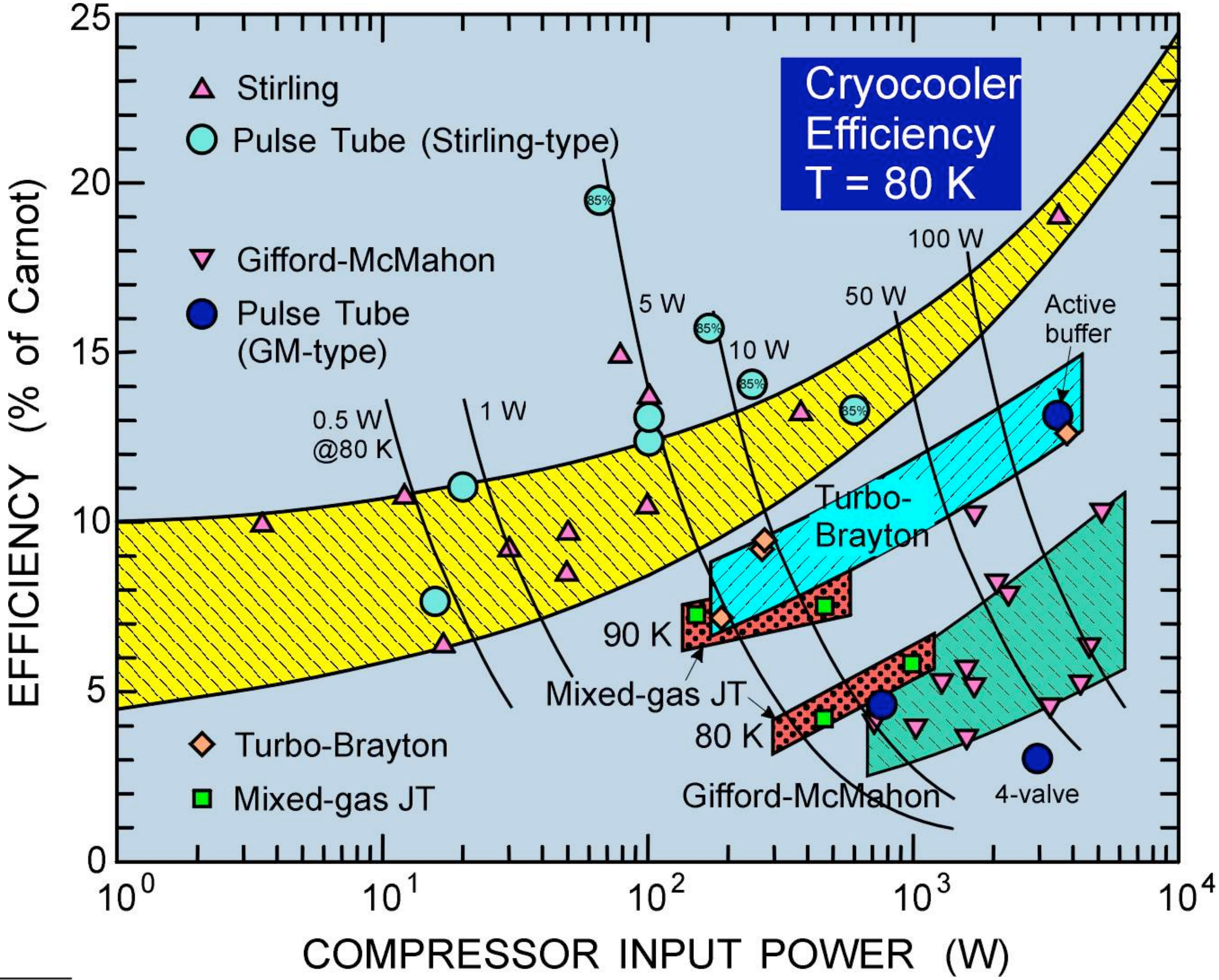
Introduction
to
Engineering Equation Solver
(EES)

Regenerative Systems



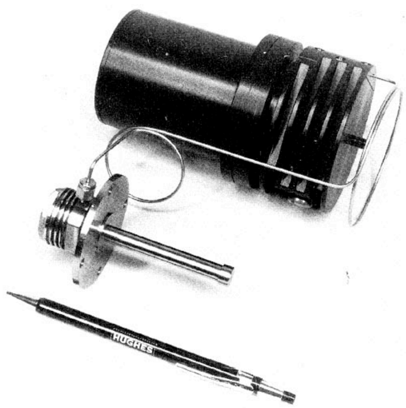
- Oscillatory flow: frequencies 1 - 100 hz
- Regenerative heat exchangers: ideal -low axial k, high transverse k, matrix specific heat much larger than gas specific heat, zero void volume, zero pressure drop
- Phase modulation (between pressure and flow waves) is crucial for performance

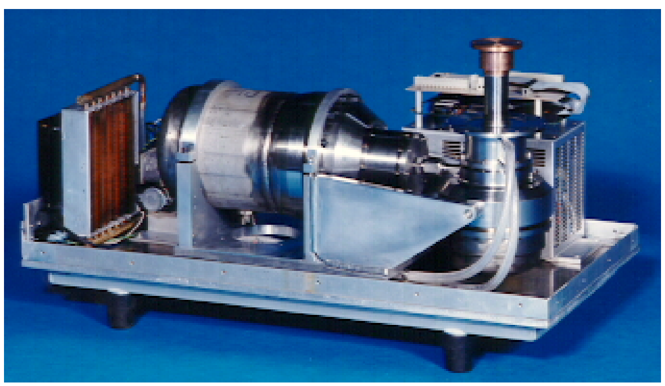
Cryocooler Actual Performance



From Ray Radebaugh (NIST) 1999

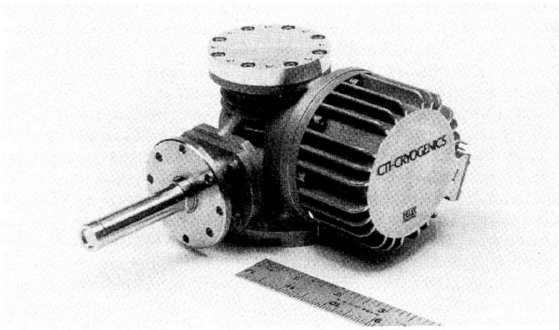
Stirling Cryocoolers

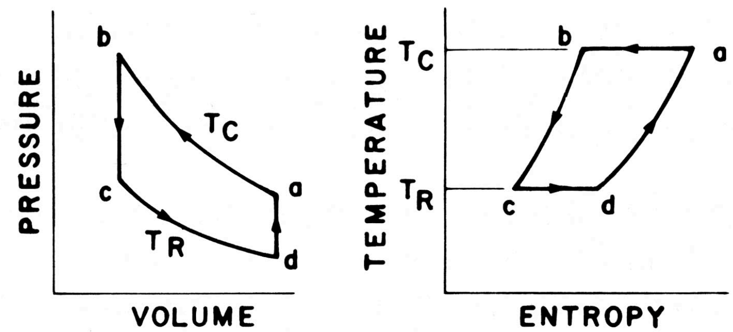


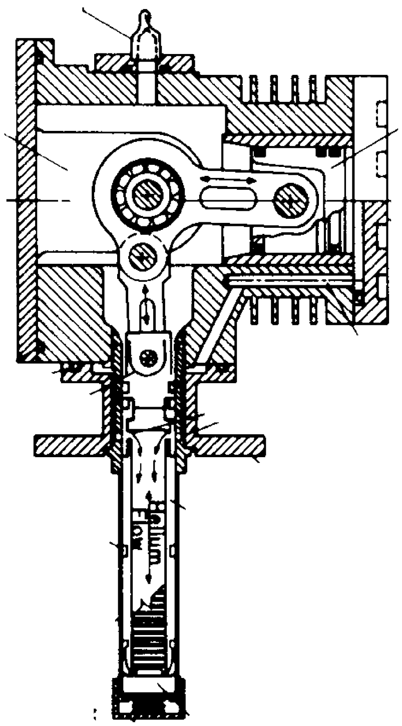


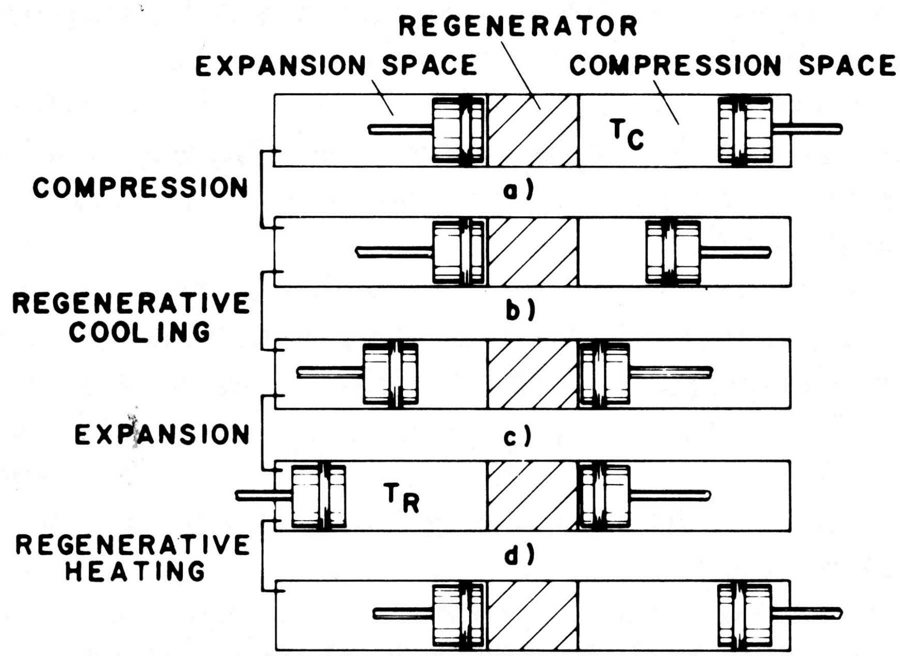
- Stirling cycle engine:
  - invented in 1815
  - 1950's bid for auto industry
  - Today: 2.5 kW generators
- Stirling cryocoolers: 1946 -
- Ideal efficiency = Carnot
  - $COP = T_c / (T_h - T_c)$
- Primary uses:
  - tactical and security IR systems
  - medical and remote- location cryogen plants
- Potential cooling for large scale HTS applications
- Commercial sources:
  - Stirling (www.stirling.nl)
  - Sunpower(www.sunpower.com)
  - Stirling Technology Company (www.stirlingtech.com)

Stirling Cryocoolers

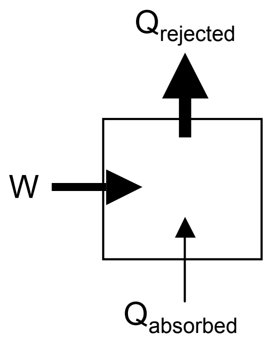




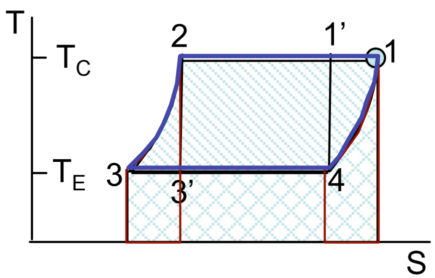




Stirling Cycle: Zero'th Order (ideal gas) Analysis



- Compare work and heat transfer for Stirling and Carnot cycles
- Use helium gas as quantitative example: T_C=300 K, T_E=100 K, P_1=1 atm., P_2=20 atm.
- Note that for an ideal gas in isothermal compression we have: $s_2 \quad s_1 = \quad R \ln \dfrac{P_2}{P_1}$



For an ideal cycle

$$W_{net} = Q_r \quad Q_a$$

$$= \bigcirc \quad Q$$

$$= \bigcirc TdS$$

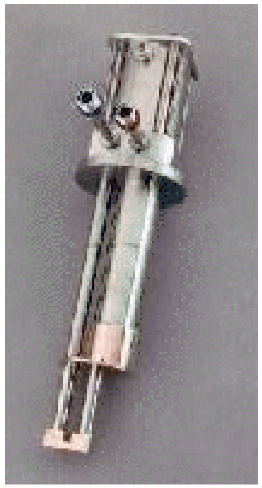
	Carnot		Stirling	
process	Q	helium (J/mol)	Q	helium (J/mol)
1(1') – 2	$T_C(s_2\text{-}s_{1'})$	-627	$T_C R\ln(P_1/P_2)$	-7472
2 – 3(3')	0	0	$C_v(T_E\text{-}T_C)$	-2500
3(3') – 4	$T_E(s_4\text{-}s_{3'})$	209	$T_E R\ln(P_3/P_4)$	2491
4 – 1(1')	0	0	$C_v(T_C\text{-}T_E)$	2500
Net	$(T_C\text{-}T_E)(s_2\text{-}s_1)$	-418	$(T_C\text{-}T_E)$x $R\ln(P_1/P_2)$	-4981
COP	$T_E/(T_C\text{-}T_E)$	0.5	$T_E/(T_C\text{-}T_E)$	0.5

Stirling cycle processes more heat than Carnot cycle, but same efficiency

Stirling Cryocoolers

- How is real machine different from ideal?
  - Harmonic motion (vs. abrupt changes)
  - Regenerator void volume
  - Regenerator ineffectiveness
  - Pressure drop through regenerator
  - Non-isothermal compression and expansion
  - Non-zero T between reservoir and heat exchangers
  - Constant temperature piston and cylinder walls
  - Isotropic pressure at all instants
- 1st order analysis (Schmidt, 1861)
  - includes
    - Harmonic motion
    - Regenerator void volume
  - Useful theoretical tool for parametric optimizations
- 2nd & 3rd order analyses – nodal simulations
  - SAGE (dgedeon@compuserve.com)

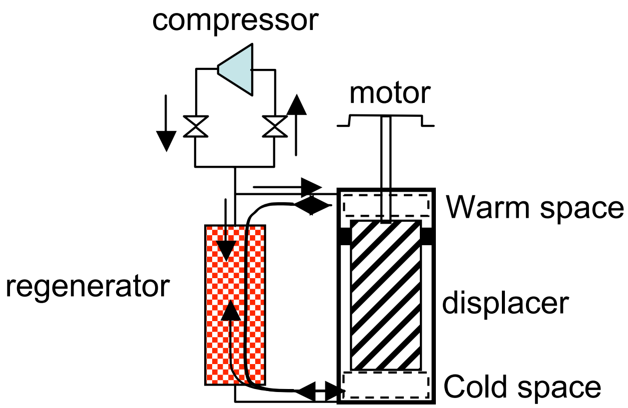
Gifford-McMahon Cryocooler

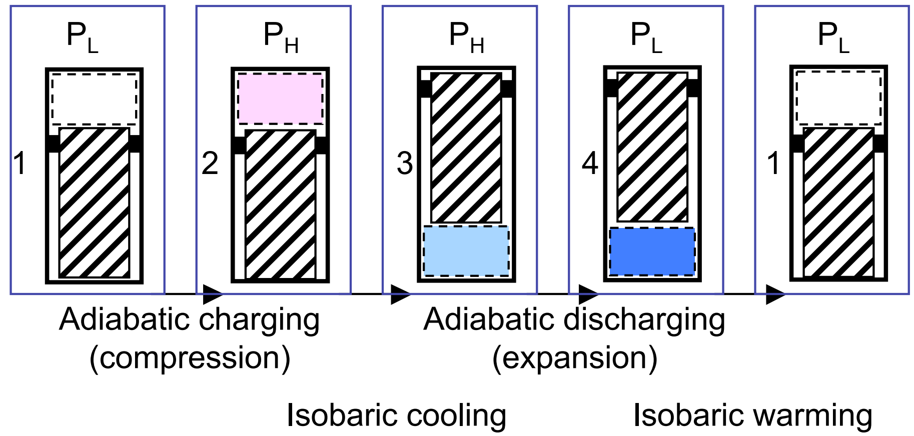


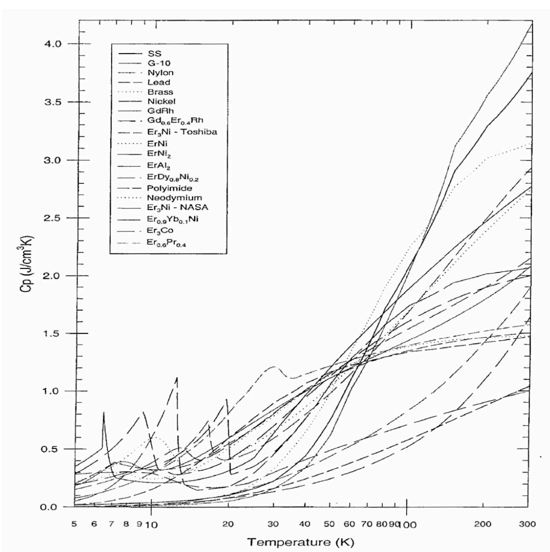
- Alternative to Stirling cryocoolers
  - Valving allows use of inexpensive compressors, and separation between cold head and compressor
  - Typical frequency 1 – 2 Hz
  - Somewhat reduced efficiency
  - Cooling power range:
    - ~ 1 watt @ 4.2 K : recondenser
    - 200 watts @ 80 K: cryo-pumps
- Primary uses:
  - Liquid nitrogen plants
  - Cryopumps
  - Conduction cooled s/c magnets - MRI, µSMES, HTS
  - Large scale HTS applications

Gifford-McMahon Cryocoolers

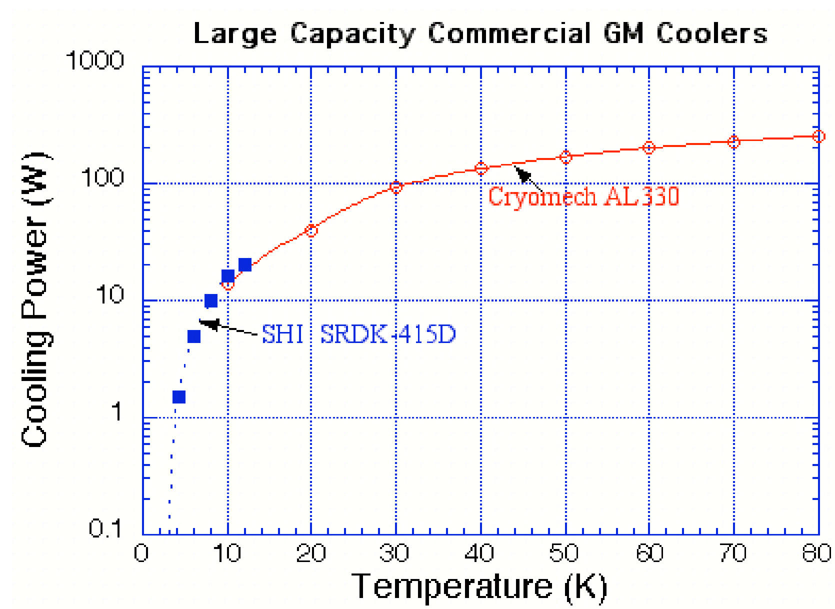
- Cycle description:







Materials research in the 80's and 90's has enabled 4 K GM machines with cooling capacity ~ 1 watt



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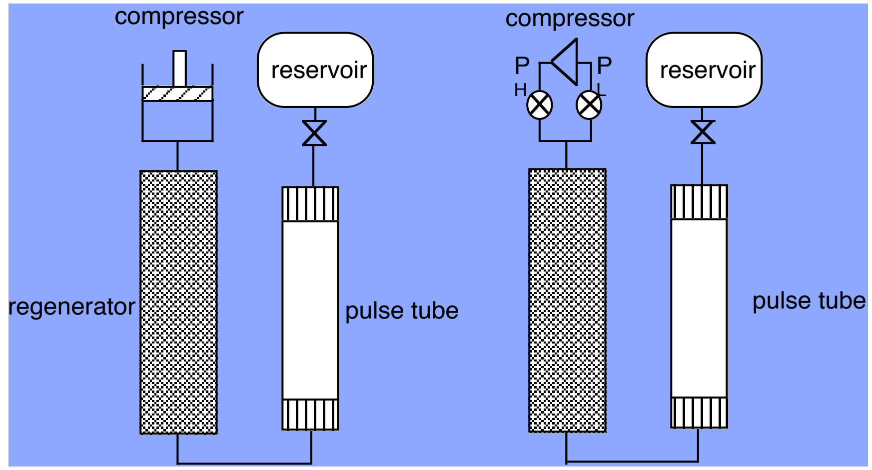
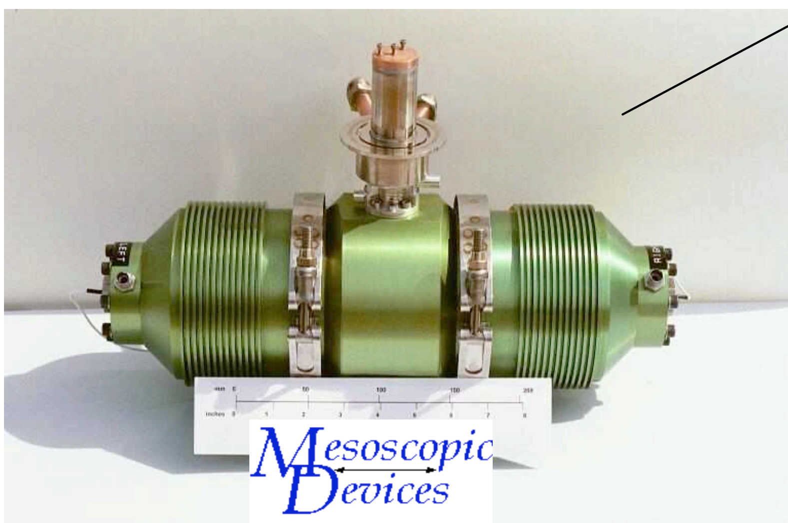
Pulse Tube Cryocoolers

- Two general types
  - Stirling type
    - High frequency ~ 60 Hz
    - High efficiency: 25% of Carnot
    - Operation down to 10 K
  - GM type
    - Low frequency ~ 1-2 Hz
    - Split design = very low vibration
    - Ideal for 4 K operation (≤ 1 watt)

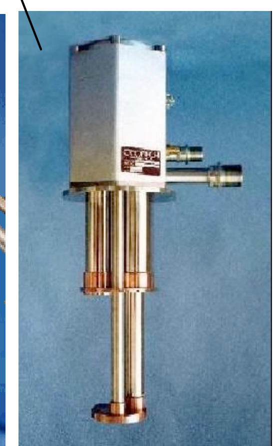


Stirling Type

GM Type



MD 200



Cryomech PT405

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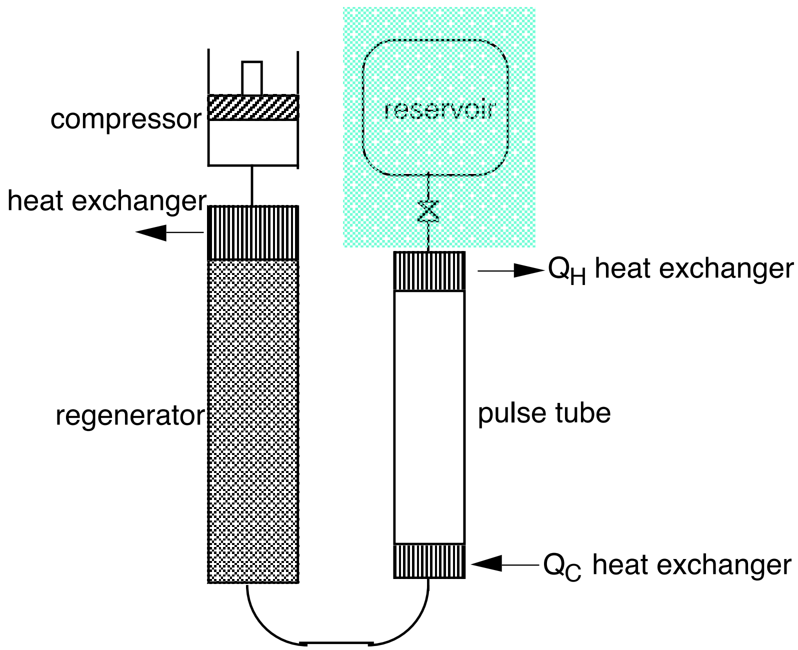
Cooling Mechanisms

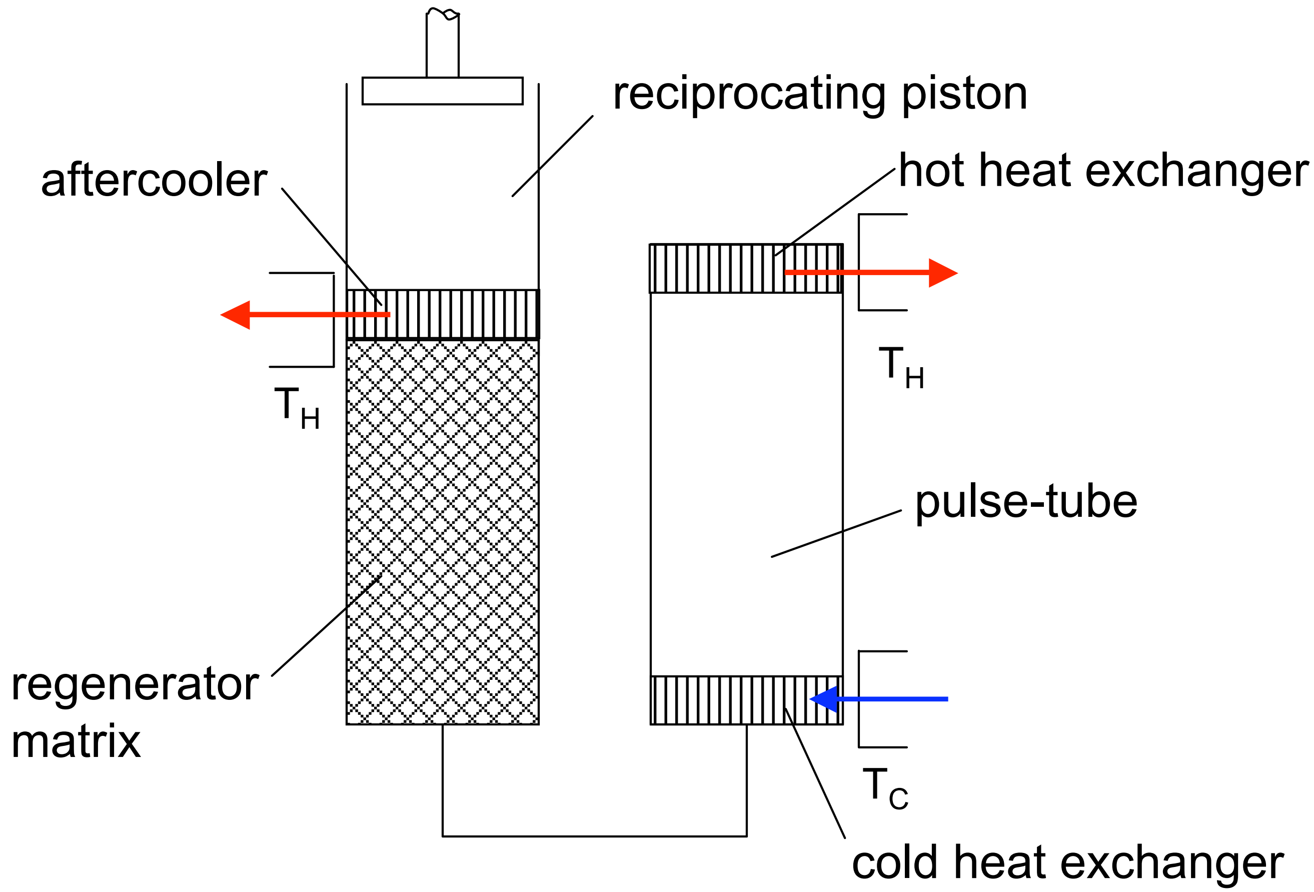
(why does this thing work?)



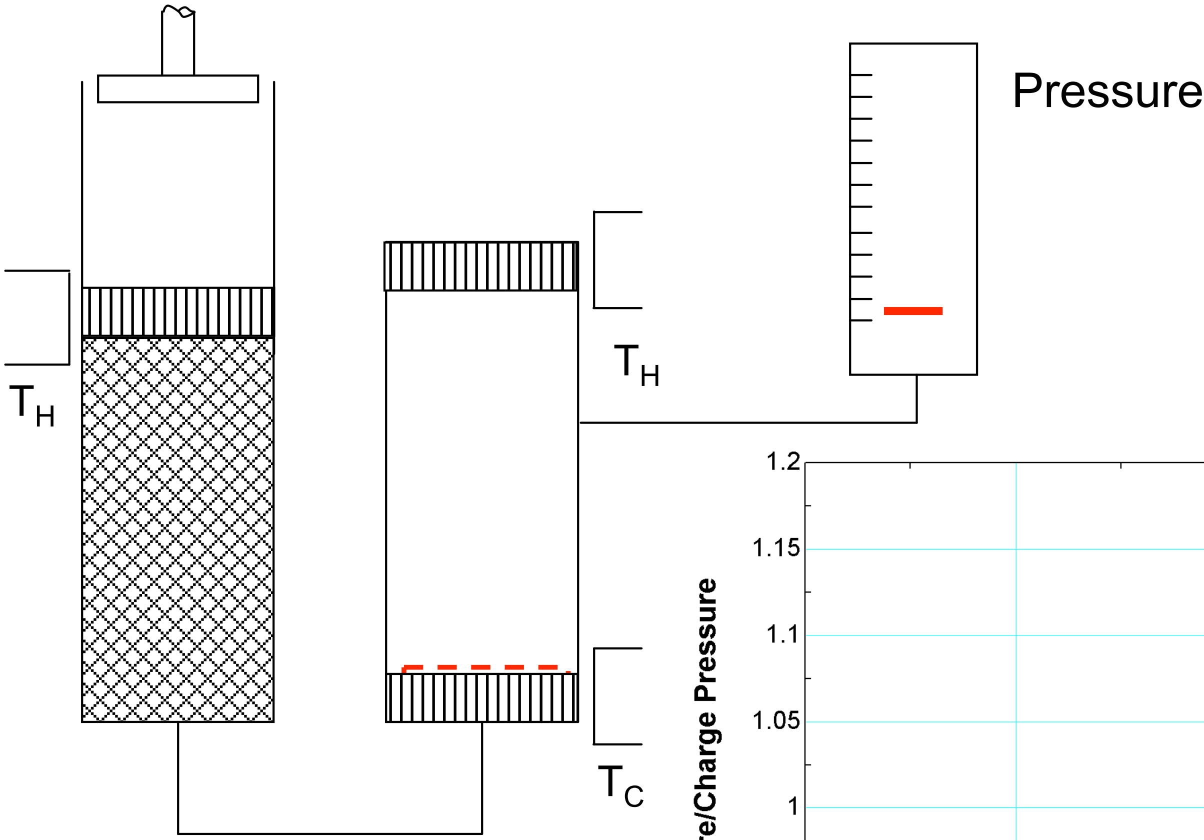
Pulse Tube Refrigerator

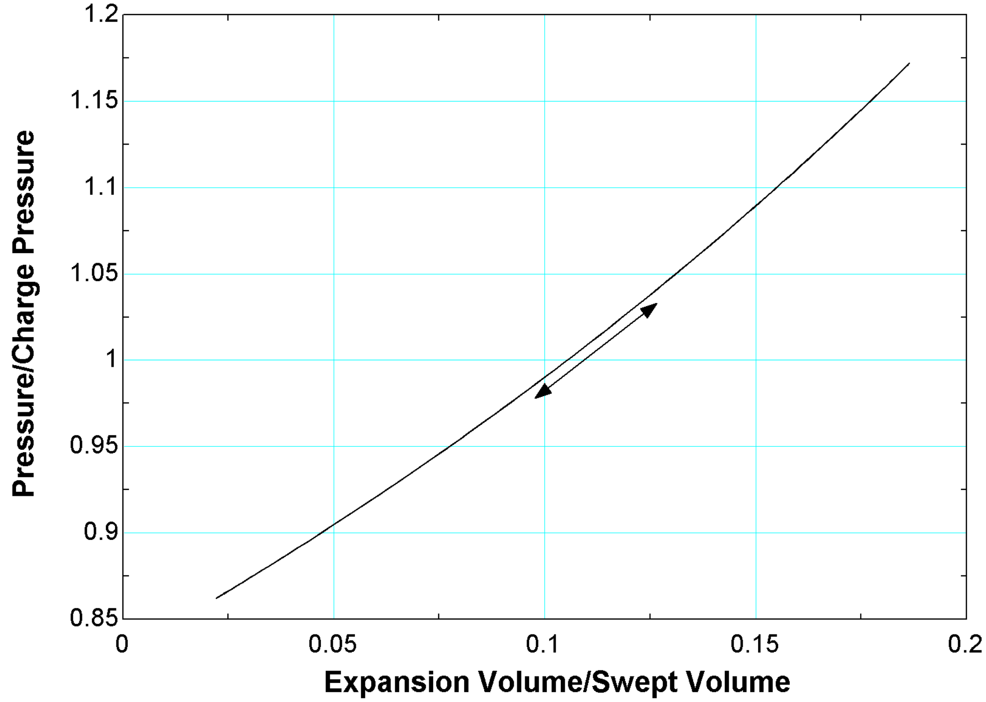
- Two mechanisms for cooling are possible
  1. Surface heat pumping
  2. Enthalpy flow
- The first mechanism is always present
- The second mechanism is not present in the basic pulse tube configuration.

Stirling-type, Basic Pulse-Tube

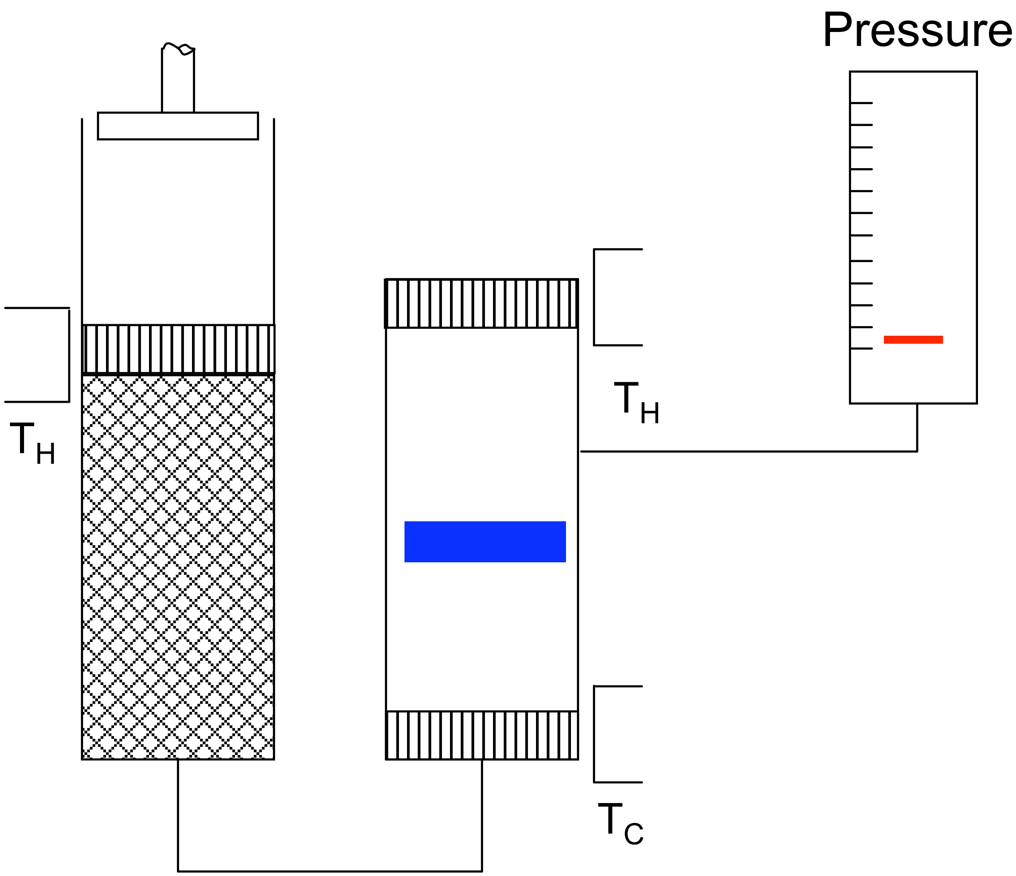


Stirling-type, Basic Pulse-Tube, P-V work

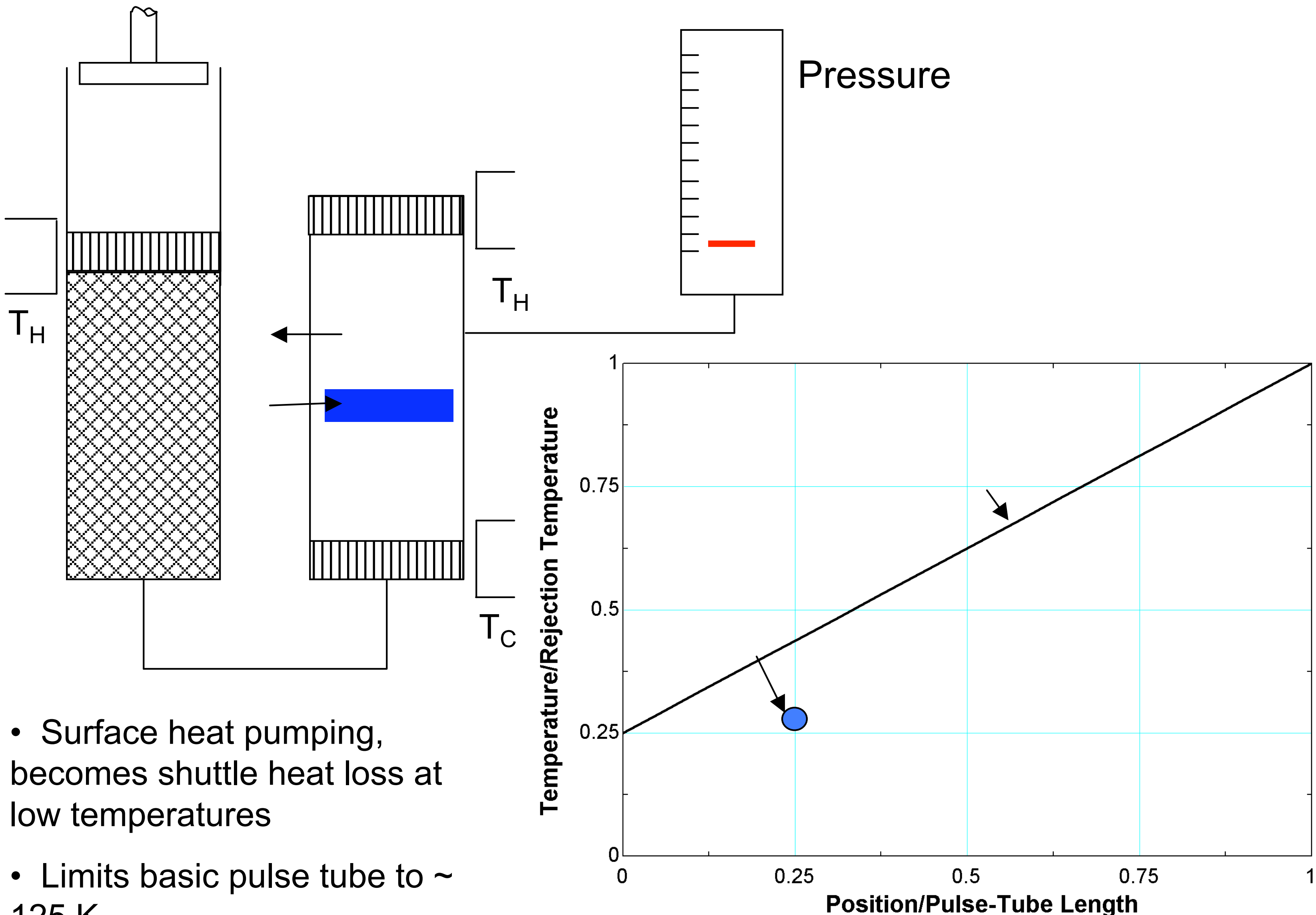




Stirling-type, Basic Pulse-Tube, Surface Heat Pumping



Stirling-type, Basic Pulse-Tube, Surface Heat Pumping



- Surface heat pumping, becomes shuttle heat loss at low temperatures
- Limits basic pulse tube to ~ 125 K

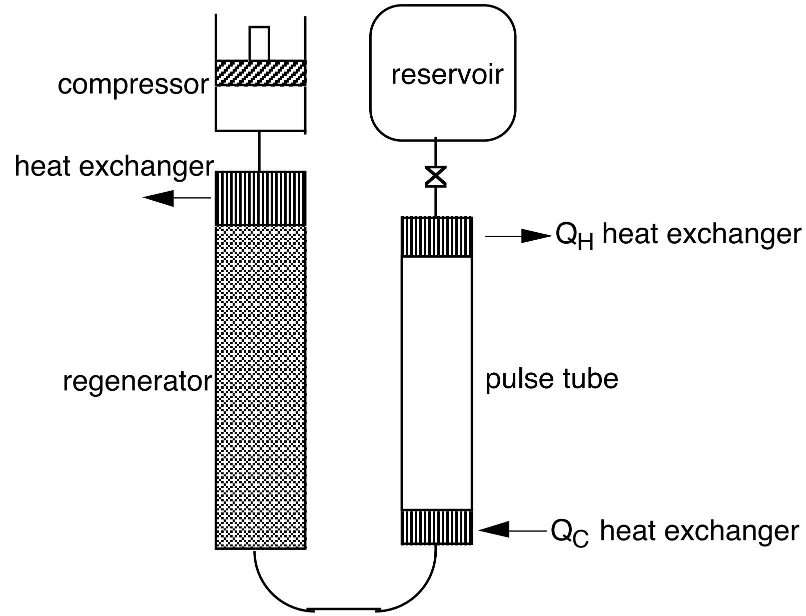
Phase Shifting & Enthalpy Flow

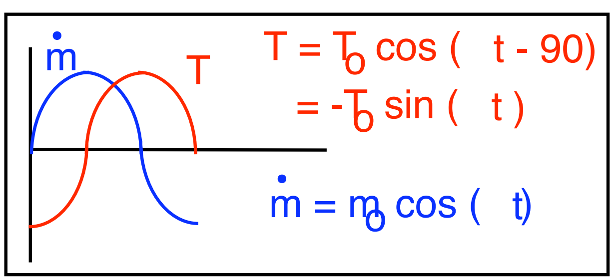
- Phase shifting orifice introduced by Mikulin 1983, Radebaugh 1984
- Cooling power understood based on enthalpy flow analysis

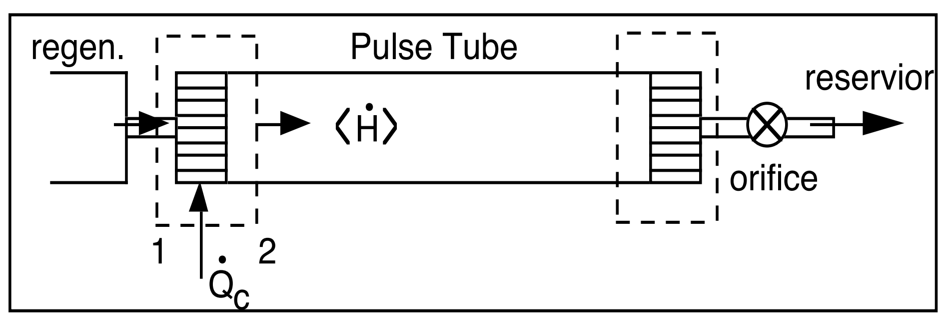
$$\dot{m}_1 h_1 \quad \dot{m}_2 h_2 + \dot{Q} \quad \dot{W} = dU/dt$$

$$Q = \langle \dot{H} \rangle = \frac{1}{\ } \circ \dot{m} c_p T dt$$

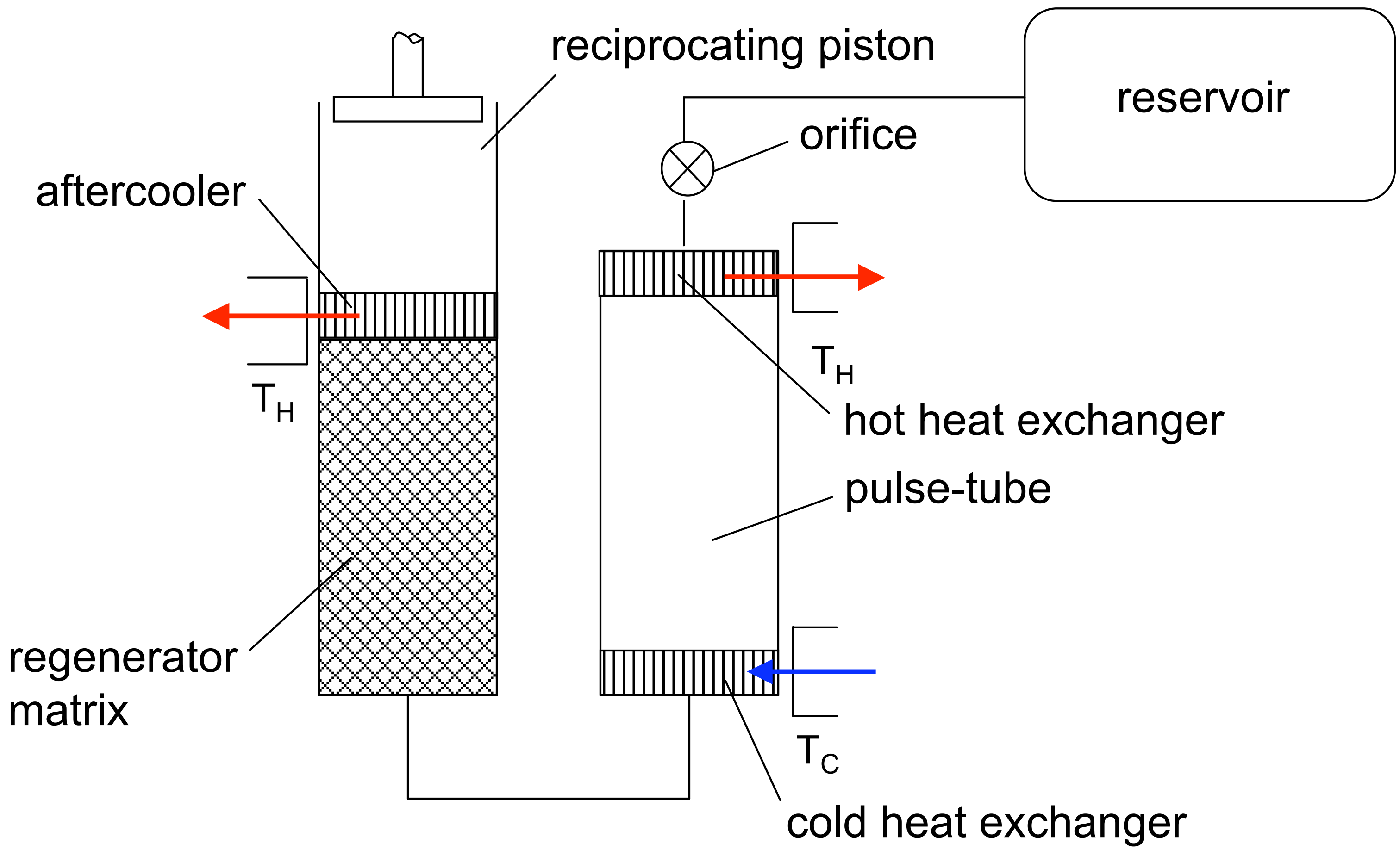
- Basic pulse tube (no orifice): <H> = 0



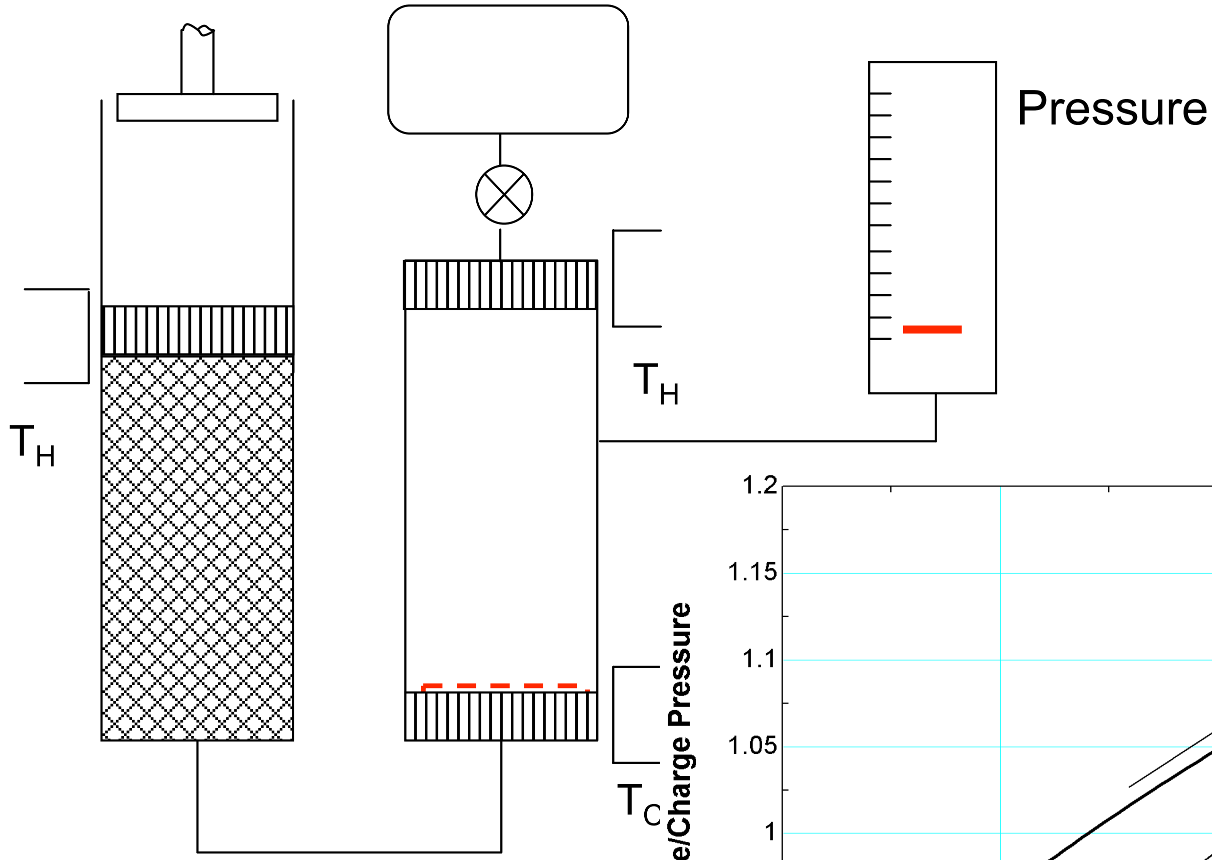


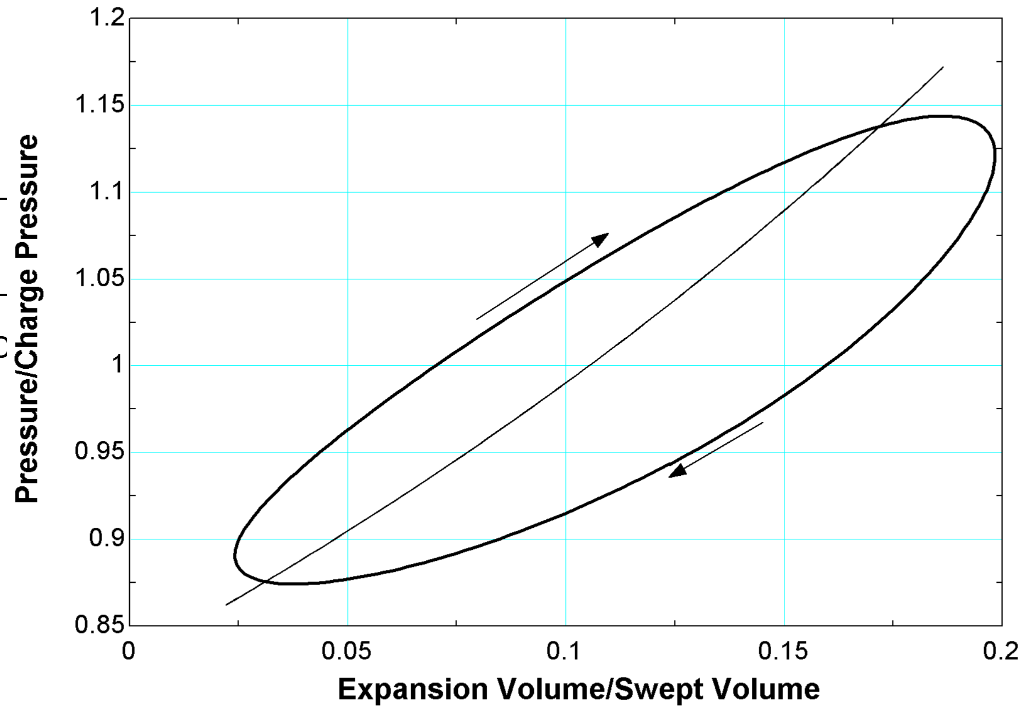


Stirling-type, Orifice Pulse-Tube

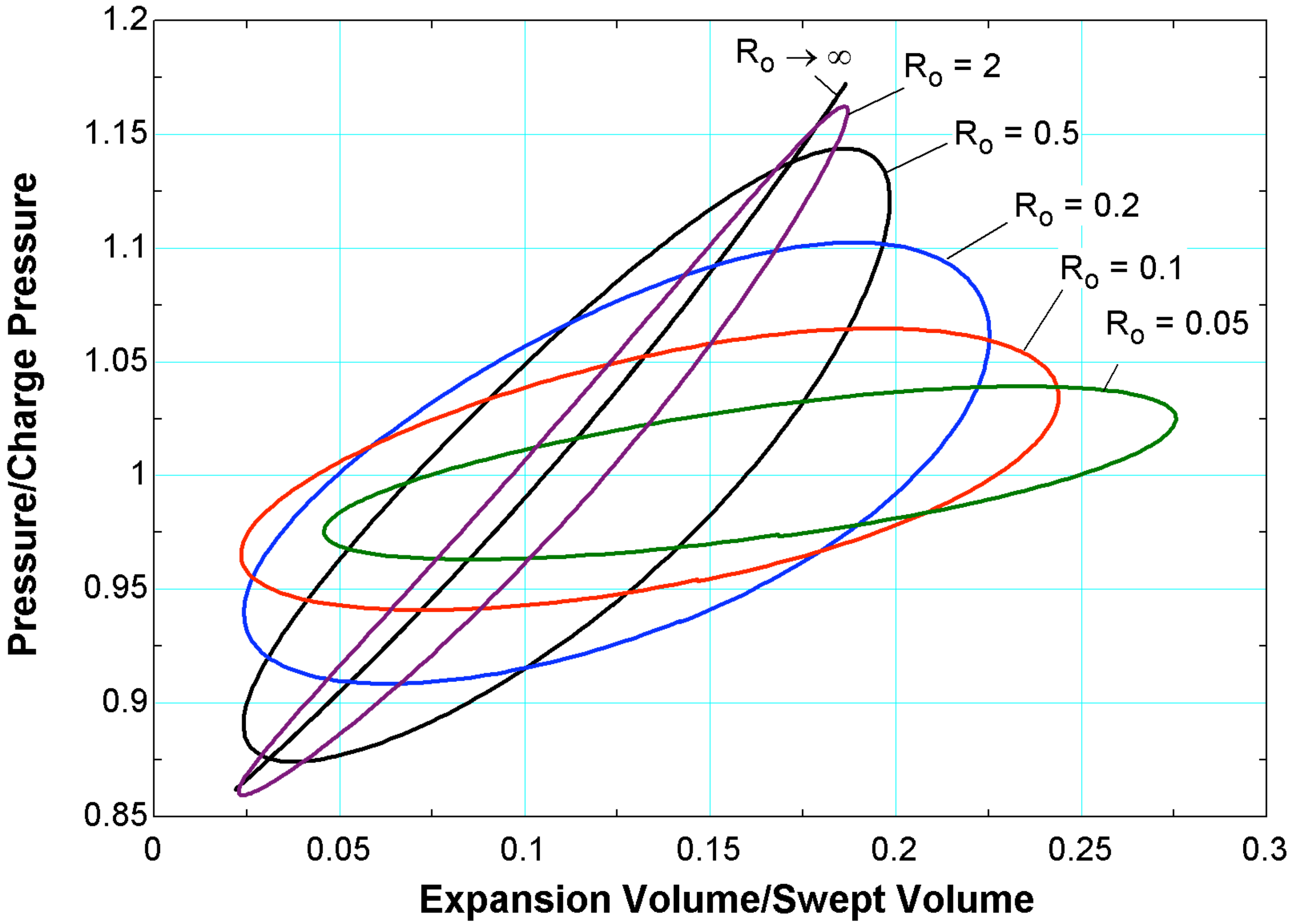


Stirling-type, Orifice Pulse-Tube, P-V work





Stirling-type, Orifice Pulse-Tube, P-V work



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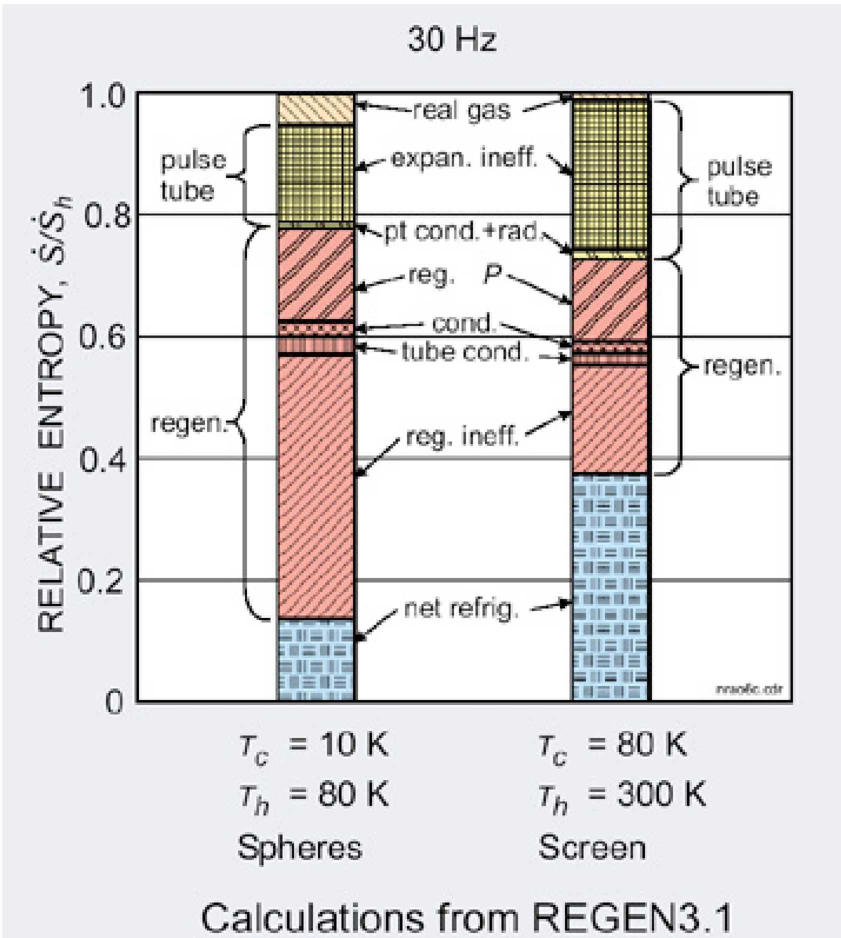
Losses in the Stirling-type Pulse tube



From Ray Radebaugh

- In the real system, entropy is generated in the regenerator and the pulse tube, reducing the amount of acoustic work that is available for enthalpy flow
- The major losses in the regenerator are proportional to the magnitude of mass flow through the regenerator.
- Optimimzed performance minimize the magnitude of the mass flow through the regenerator.

Pulse Tubes: Future Directions & Commercial Sources

- R&D:
  - Phase shifting mechanisms - inertance tubes
  - Large capacity - modeling & losses
  - Performance improvements
- Sources:
  - GM-type
    - Cryomech, SHI (Sumitomo Heavy Industry), U of Giessen,
  - Stirling type
    - Atlas Scientific, STC, Sunpower, TRW, Martin-Marietta, Praxair, Sierra-Lobo