Lecture 14

Ferrite Materials

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Ferrite Devices

We now study a wave propagation through ferrimagnetic materials, and the design of practical ferrite devices such as isolators, circulators, phase shifter, and gyrators. These are non-reciprocal devices because the ferrimagnetic compound materials (ferrites) are anisotropic.

Ferrites are polycrystalline magnetic oxides that can be described by the general chemical formula

$$X\,O \cdot Fe_2\,O_3$$

In which X is a divalent ion such CO^{2+} or Mn^{2+}. Since these oxides have a much lower conductivity than metals, we can easily pass microwave signals through them.

Most practical materials exhibiting anisotropy are ferromagnetic compounds such as YIG (yttrium iron garnet), as well as the iron oxides.

Magnetic Materials

From an E.M. fields viewpoint, the macro (averaging over thousands or millions of molecules) magnetic response of a material can be expressed by the relative permeability μ_r, which is defined as

$$\mu_r = \frac{\mu}{\mu_0}$$

where

μ = permeability of the material (Henries/m

μ_0 = permeability of vacuum = $4\pi\times10^{-7}$ (H/m)

μ_r = relative permeability (dimensionless)

The magnetic flux density B is related to the magnetic field intensity H by

Magnetic Materials

$$\overline{B} = \mu \overline{H} = \mu_0 \mu_r \overline{H}$$

Where in Sytem Internationale (S.I.) system of units,

$\overline{B}$ = magnetic flux density (Tesla) → (Gauss) ← cgs units

$\overline{H}$ = magnetic field intensity density (Amps/m) → (Oersted) ← cgs units

We think of B as magnetic flux density response of a material to an applied magnetic force or cause H.

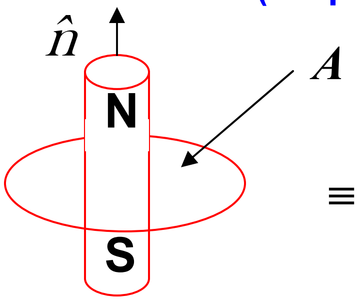
Depending on their magnetic behavior, materials can be classified as:

- dimagnetic
- paramagnetic
- Ferromagnetic
- anti-ferromagnetic
- ferrimagnetic

Magnetic Materials

Material	Group Type	Relative Permeability
silver	diamagnetic	0.99998
lead	diamagnetic	0.999983
copper	diamagnetic	0.999991
water	diamagnetic	0.999991
vacuum	non-magnetic	1.000000000000000
air	paramagnetic	1.0000004
aluminum	paramagnetic	1.00002
palladium	paramagnetic	1.0008
cobalt	ferromagnetic	250
nickel	ferromagnetic	600
mild steel	ferromagnetic	2,000
purified iron	ferromagnetic	200,000

The magnetic behavior of materials is due to electron orbital motion, electron spin, and to nuclear spin. All three of these can be modeled as tiny equivalent atomic currents flowing in circular loops, having magnetic moment IA, where I is the current (Amps) and A is the loop area (m^2):



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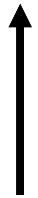


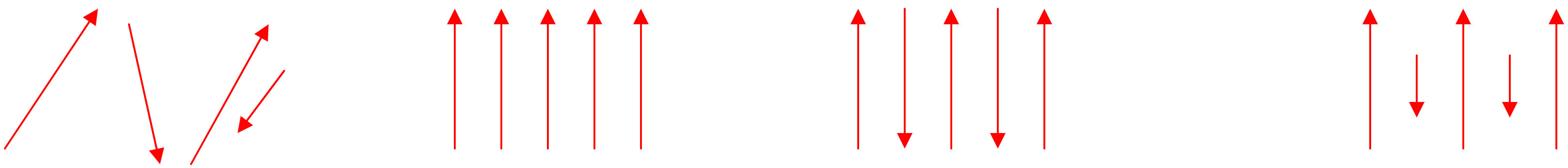
$$\overline{m} = \hat{n} I A$$

tiny bar current = current loop = magnetic moment

The magnetic dipole moment of an electron due to its spin is

$$m = \frac{q\hbar}{2m_e} = 9.27 \times 10^{-24}\left(Am^2\right)$$

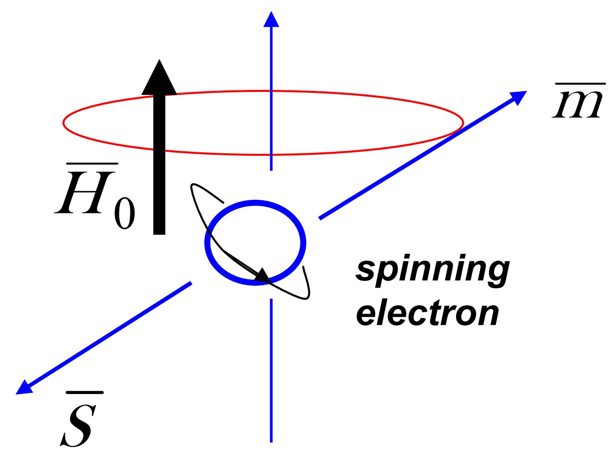
Materials are magnetically classified by their net (volume average) magnetic moments:



paramagnetic **ferromagnetic** **anti-ferromagnetic** **ferrimagnetic**

When we apply an external magnetic bias field (from a permanent magnet, for example), a torque will be exerted on the magnetic dipole:

$$\overline{T} = \mu_0 \overline{m} \times \overline{H}_0$$



An spinning electron has a spin angular momentum given by

$$\bar{S} = \frac{1}{2}\hbar$$

where $\hbar$ = Planck's constant/2π. We next define the gyromagnetic ratio as

$$\gamma = \frac{m}{S} = \frac{q}{m_e} = 1.759 \times 10^{11} coulombs / kg$$

Thus we can relate the magnetic moment for one spinning electron to its angular momentum

$$\overline{m} = -\gamma \bar{s}$$

Now we can write the torque exerted by the magnetic applied field on the magnetic dipole:

$$\overline{\mathrm{T}} = -\mu_0 \gamma \bar{s} \times \overline{H}$$

$$\overline{\mathrm{T}} = \frac{d\bar{s}}{dt} \Rightarrow \frac{d\bar{s}}{dt} = -\frac{1}{\gamma}\frac{d\overline{m}}{dt} = \overline{\mathrm{T}} = \mu_0 \overline{m} \times \overline{H}_0 \quad \longrightarrow \quad \frac{d\overline{m}}{dt} = -\mu_0 \overline{m} \times \overline{H}_0$$

Let $$\overline{m} = \hat{x}m_x + \hat{y}m_y + \hat{z}m_z \quad and \quad \overline{H}_0 = \hat{z}H_0$$

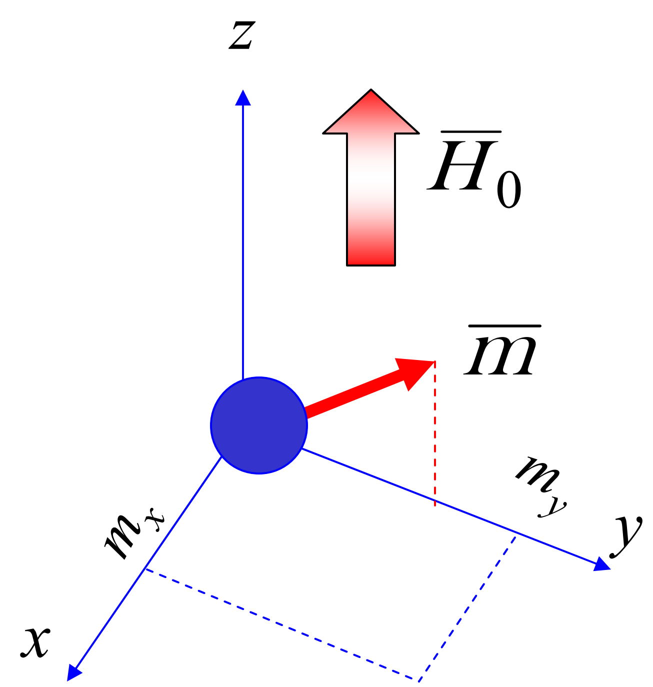
Then $$\overline{m} \times \overline{H}_0 = -\hat{y}m_x H_0 + \hat{x}m_y H_0$$

$$\left\{ \begin{aligned} \therefore \frac{dm_x}{dt} &= -\mu_0 \gamma m_y H_0 \\ \frac{dm_y}{dt} &= \mu_0 \gamma m_x H_0 \end{aligned} \right.$$

$$\frac{dm_z}{dt} = 0$$

$$\frac{d^2 m_x}{dt^2} + \omega_0^2 m_x = 0$$

$$\frac{d^2 m_y}{dt^2} + \omega_0^2 m_y = 0$$



$$\boxed{\omega_0 = m_0 \gamma H_0}$$ **Larmor frequency (precession frequency)**

These are classical S.H.O 2nd order D.Es with solutions:

$$m_x = A\cos\omega_0 t$$

$$m_y = A\sin\omega_0 t$$

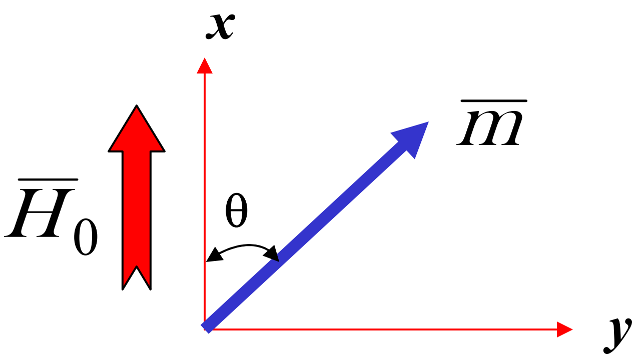
$$m_z = constant$$

The magnitude of m is a constant = 9.27×10^{-24} Am^2, thus

$$|\overline{m}|^2 = m_x^2 + m_y^2 + m_z^2 = A^2 + m_z^2$$

The precession angle θ is given by

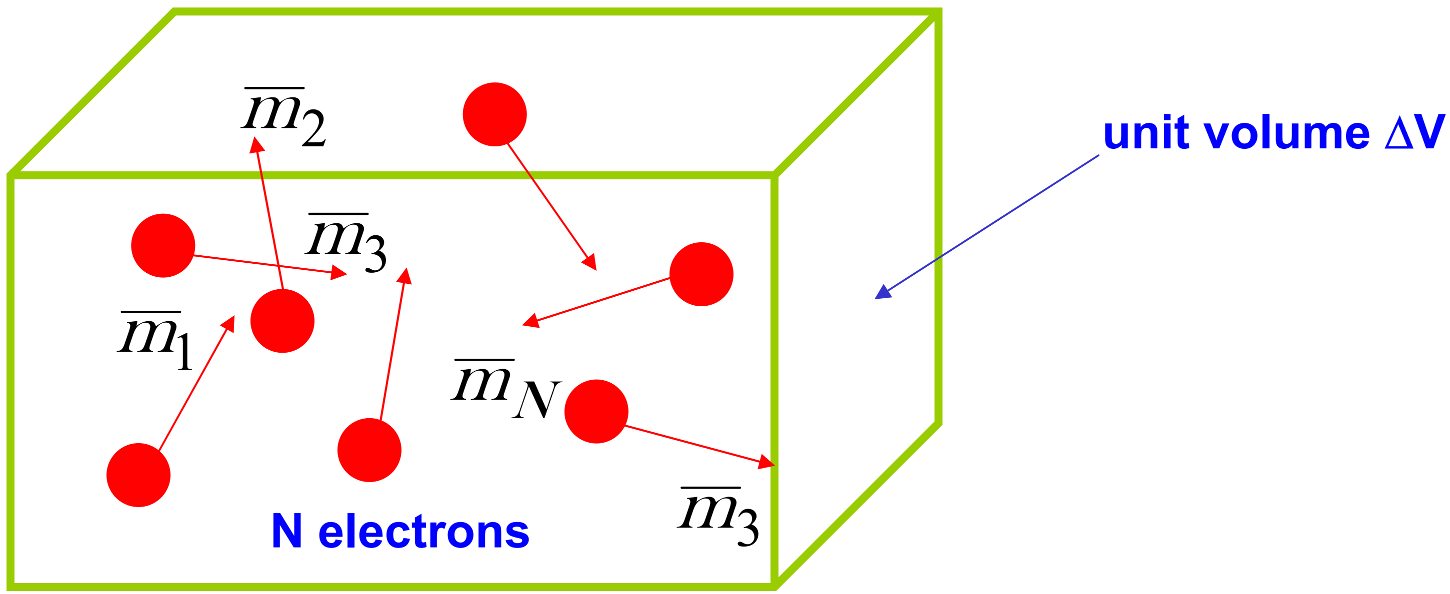
$$\sin\theta = \frac{\sqrt{m_x^2 + m_y^2}}{|\overline{m}|} = \frac{A}{|\overline{m}|}$$



The projection of $\overline{m}$ onto the x-y plane is a circular path:

If there were no damping forces, the precession angle will be constant and the single spinning electron will have a magnetic moment $\overline{m}$ at angle θ indefinitely. But in reality all materials exert a damping force so that $\overline{m}$ spirals in from its initial angle until it is aligned with H_0

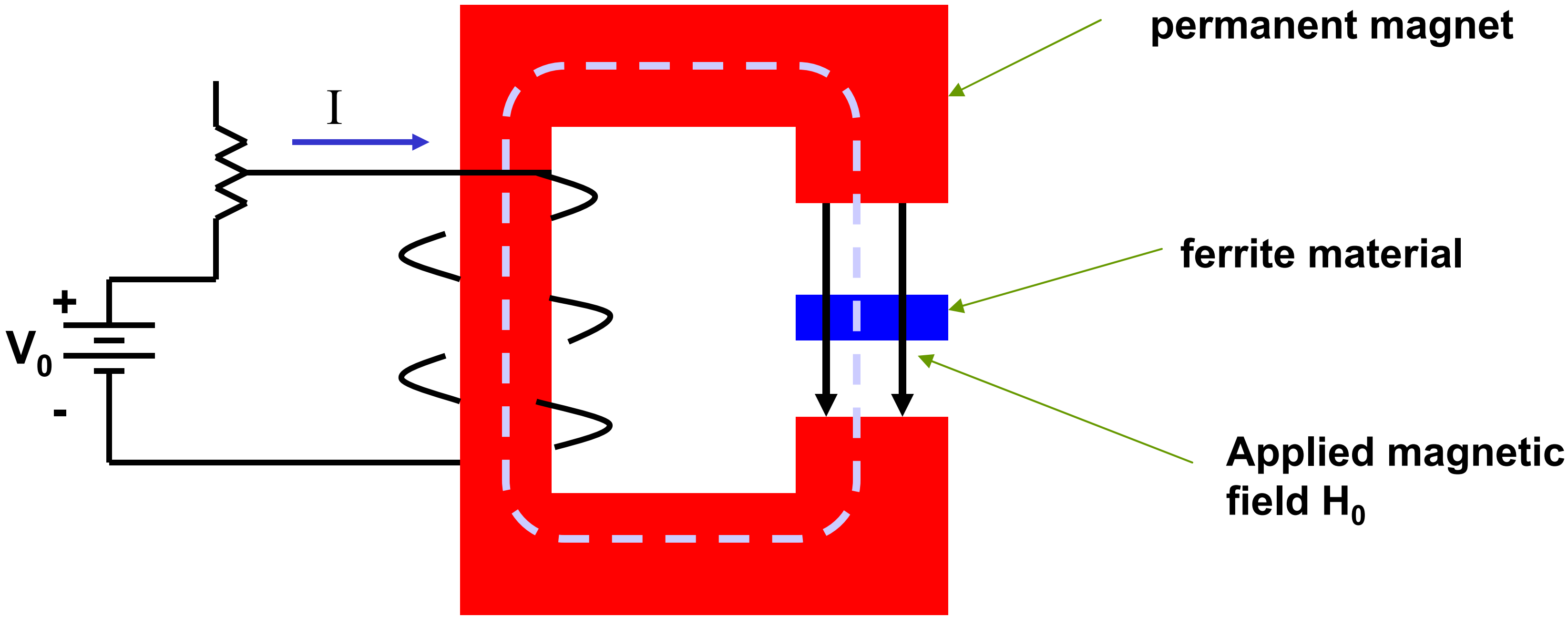
Now consider N electrons in a unit volume, each having a distinct magnetic moment direction m:



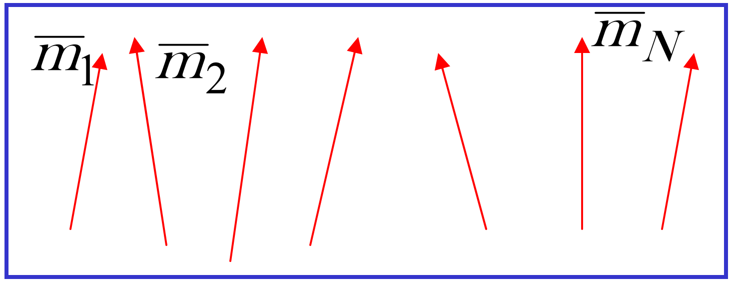
The total or net magnetization of the volume is given by

$$\overline{M} = \frac{\overline{m}_1 + \overline{m}_2 + \overline{m}_3 + \cdots + \overline{m}_N}{\Delta V}$$

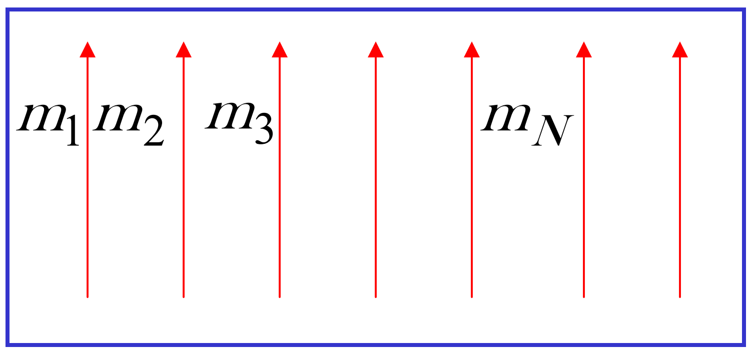
If we now assume the material is ferrimagnetic and apply an external magnetic field H_0, these magnetic moments will line up, and m_0 = m when H_0 is strong.



For a weak applied H_0 we get partial alignment of m_0:



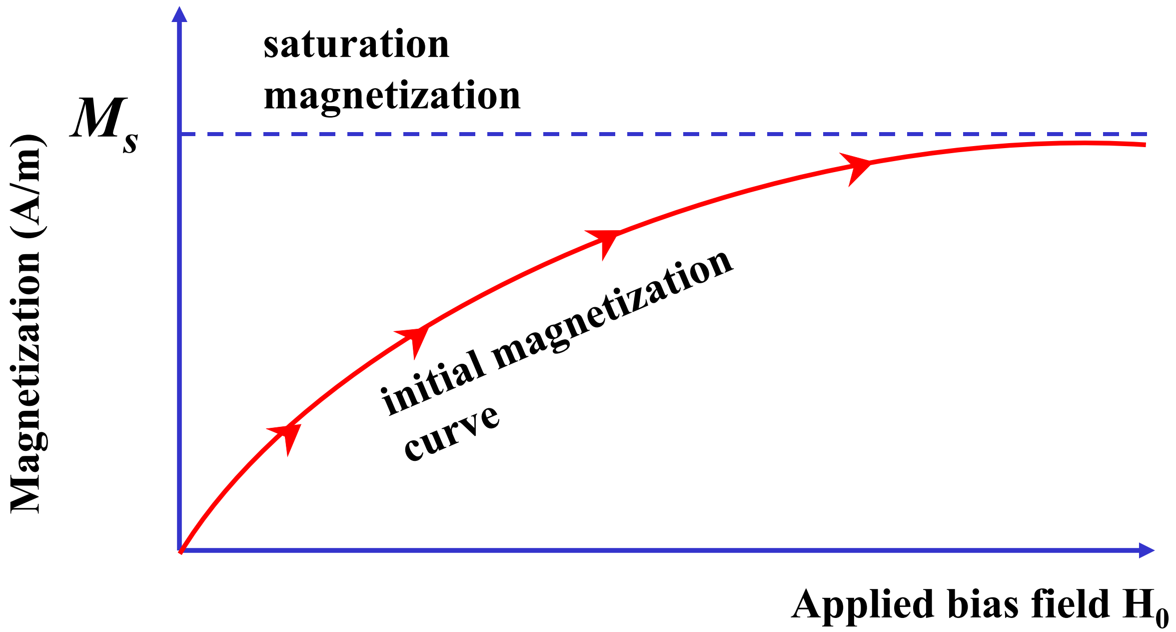
As we increase the applied magnetic field intensity H_0, all the magnetic moments line up and we reach the saturation magnetization M_s:



$$\overline{M} = N\overline{m}$$

Equation of motion: $$\frac{d\overline{M}}{dt} = -\mu_0 \gamma \overline{M} \times \overline{H}$$

applied magnetic field



If we start with a sample that is initially un-magnetized, with no applied bias field, the initial magnetization is M_0, As we increase the applied bias field H_0, the sample becomes increasingly magnetized until it reaches a saturation level M_s, beyond which no further magnetization is possible.

The magnetic flux density B in the ferromagnetic or ferrimagnetic material is given by

$$\overline{B} = \mu_0\left(\overline{H} + \overline{M}\right)$$

where

$\overline{B} =$ magnetic flux density (Tesla)

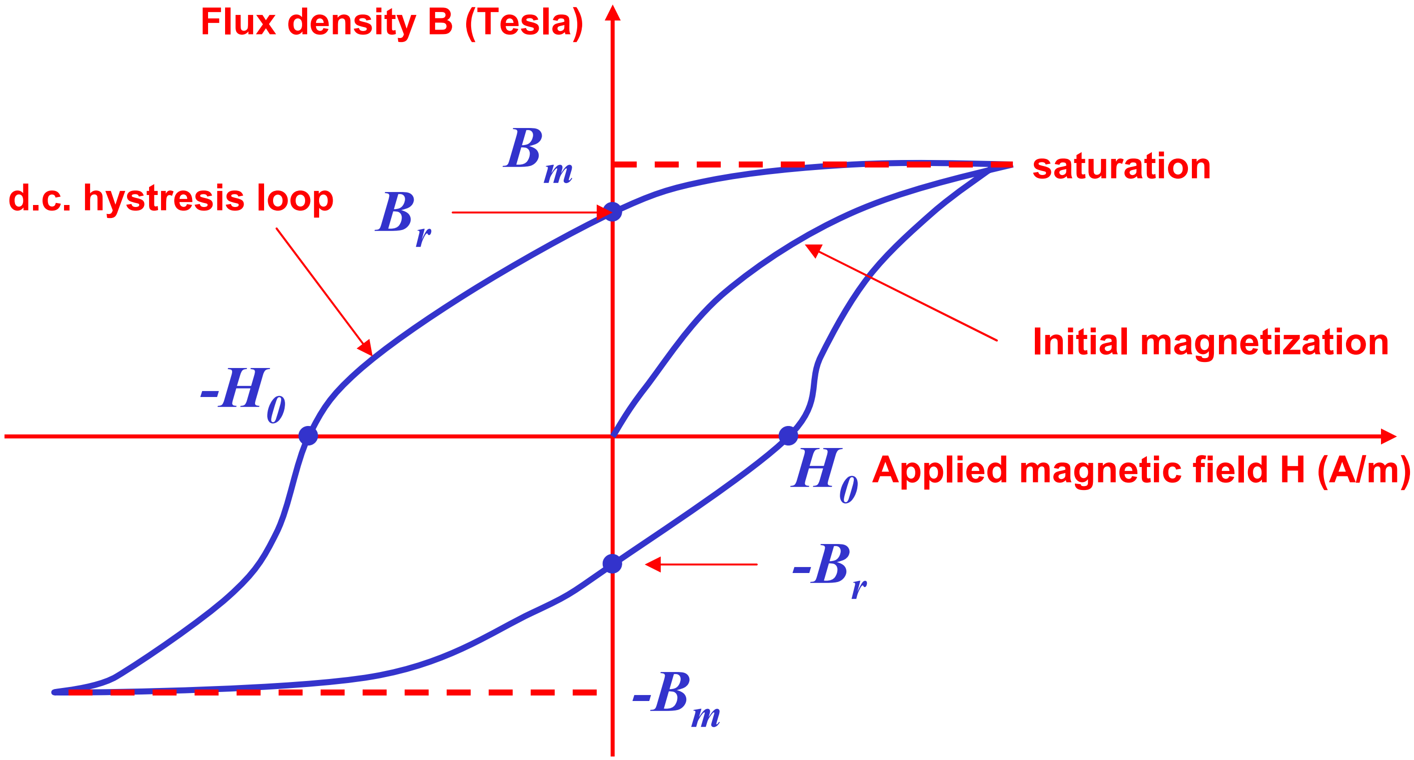
$\mu_0 =$ permeability of free space = $4\pi\times10^{-7}$(H/m)

$\overline{H} =$ applied magnetic bias field (A/m)

$\overline{M} =$ magnetization (A/m)

If we increase the bias field $\overline{H}$ to the point where we reach saturation, and then decrease $\overline{H}$, the flux density $\overline{B}$ decreases, but no as rapidly as shown by the initial magnetization. When $\overline{H}$ reaches zero, there is a residual $\overline{B}$ density (called the remanance).

In order to reduce $\overline{B}$ to zero, we must actually reverse the applied magnetic field.



Interaction of RF Signals with Ferrites

Consider an RF wave propagating through a very large region of ferrimagnetic material with a D.C. bias field $\hat{z}H_0$.

The RF field is:

$$\overline{H}_{rf} = \hat{x}H_x + \hat{y}H_y + \hat{z}H_z$$

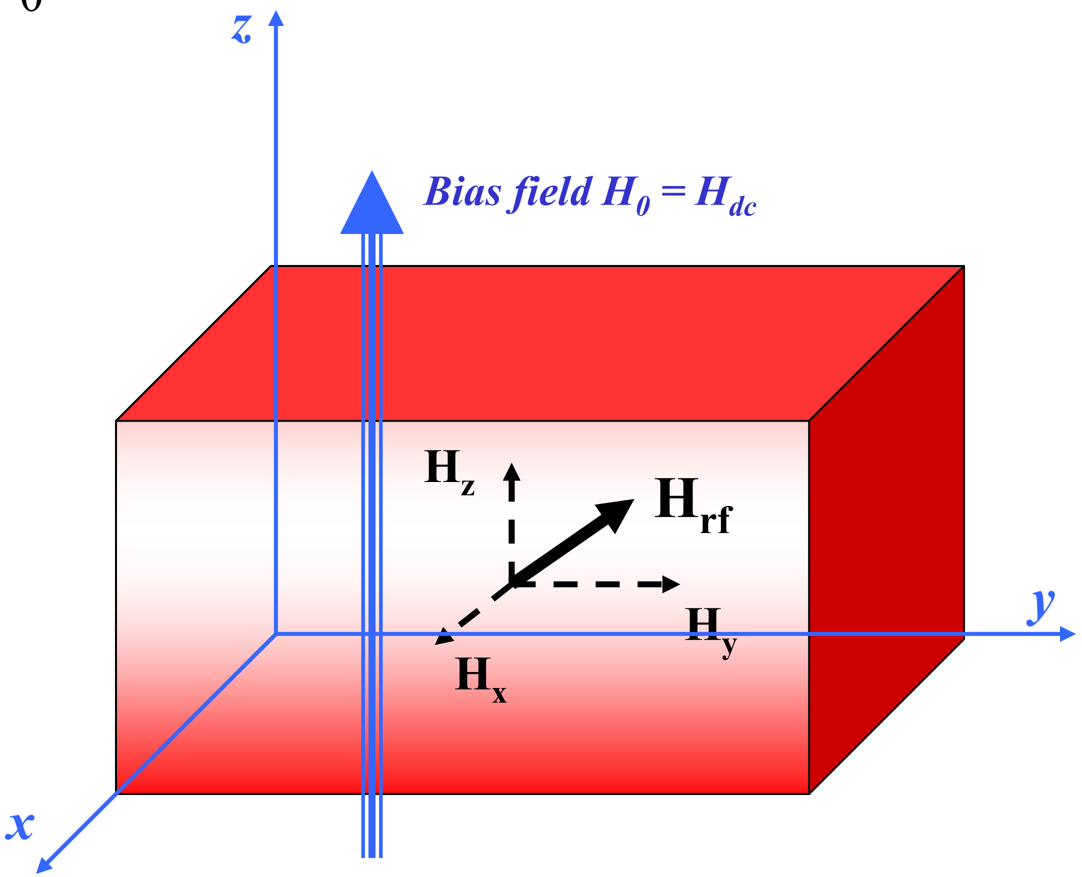
The DC field is: $\overline{H}_{dc} = \hat{z}H_0$

The total field is:

$$\overline{H}_{total} = \overline{H}_{rf} + \overline{H}_{dc}$$

This field produces material magnetization:

$$\overline{M}_t = \overline{M}_{rf} + \hat{z}\overline{M}_s$$



The equation of motion becomes:

$$\frac{d\overline{M}_t}{dt} = -\mu_0 \gamma M_t \times H_t$$

$$\begin{cases} \dfrac{dM_x}{dt} = -\omega_0 M_y + \omega_m H_y \\ \dfrac{dM_y}{dt} = \omega_0 M_x - \omega_m H_x \\ \dfrac{dM_z}{dt} = 0 \end{cases}$$

For the time harmonic ($e^{j\omega t}$) r.f. fields we obtain:

$$M_x = \chi_{xx} H_x + \chi_{xy} H_y + 0 H_z$$

$$M_y = \chi_{yx} H_x + \chi_{yy} H_y + 0 H_z$$

$$M_z = 0 H_x + 0 H_y + 0 H_z$$

$$\left.\begin{aligned} \chi_{xx} &= \chi_{yy} = \frac{\omega_0 \omega_m}{\omega_0^2 - \omega_m^2} \\ \chi_{xy} &= -\chi_{yx} = \frac{-j\omega\omega_m}{\omega_0^2 - \omega^2} \end{aligned}\right\} \text{Magnetic susceptibility}$$

We can write this in matrix (tensor) form:

$$\underbrace{\begin{bmatrix} M_x \\ M_y \\ M_z \end{bmatrix}}_{\text{magnetization response}} = \underbrace{\begin{bmatrix} \chi_{xx} & \chi_{xy} & 0 \\ \chi_{yx} & \chi_{yy} & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\text{susceptibility tensor}} \underbrace{\begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}}_{\text{Applied RF field}} \Longrightarrow \boxed{[M] = [\chi][H]}$$

We now calculate the magnetic flux density in the ferromagnetic material, due to rf field and the d.c. bias field:

$$\overline{B} = \mu_0\left(\overline{M} + \overline{H}\right) = [\mu]\overline{H}$$

For isotropic materials, $\overline{B} = \mu\overline{H}$

$$\overline{B} = \mu[\chi]\overline{H} + \mu_0 u\overline{H}$$

$$u = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore

$$[\mu]=\mu_0\{[u]+[\chi]\}=\begin{bmatrix} \mu & j\kappa & 0 \\ -j\kappa & \mu & 0 \\ 0 & 0 & \mu_0 \end{bmatrix}$$

$$\mu=\mu_0(1+\chi_{xx})=\mu_0(1+\chi_{yy})$$

$$\left.\begin{aligned} \mu(\omega)&=\mu_0\left[1+\frac{\omega_0\omega_m}{\omega_0^2-\omega^2}\right] \\ \kappa(\omega)&=-j\mu_0\chi_{xy}=\mu_0\frac{\omega\omega_m}{\omega_0^2-\omega^2} \end{aligned}\right\}$$

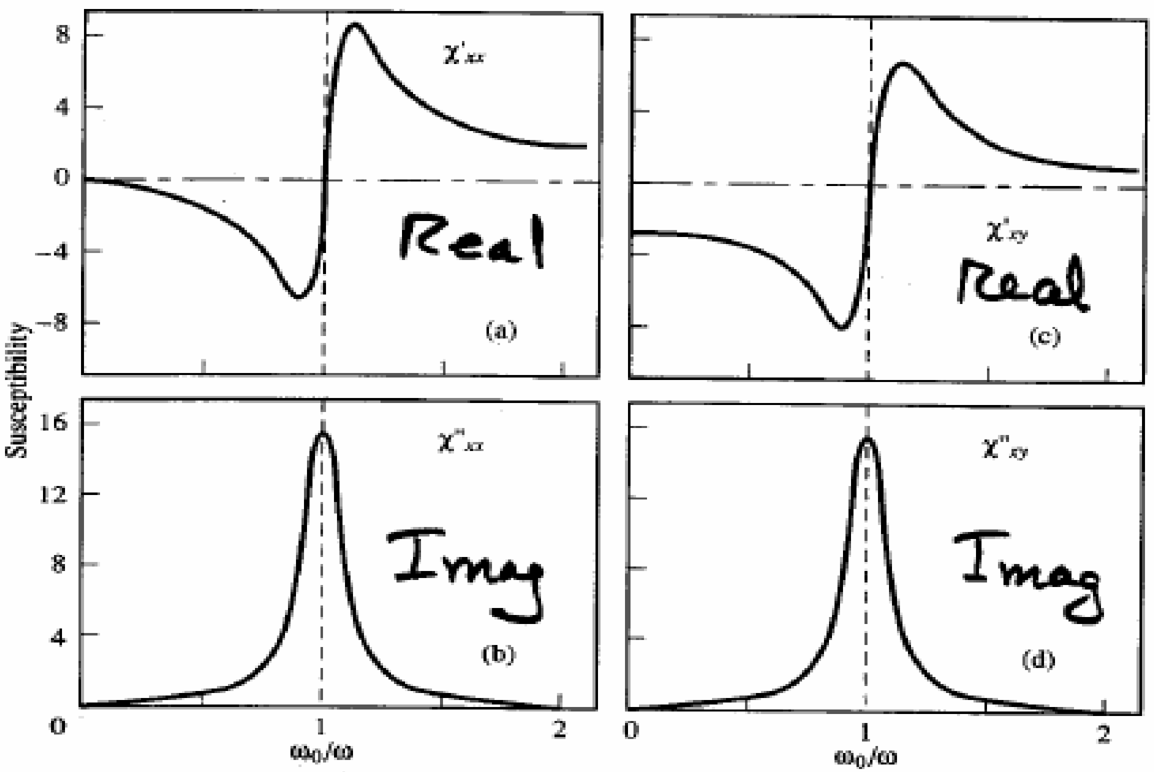
Depends on H_0,M_s and frequency

NOTE: **this assumes a z-directed bias field and that the material is magnetically lossless. In this case, both μ and κ are real-valued.**

Lossy Magnetic Materials

We consider a magnetically lossy material. Let α = loss damping factor so that $\omega_0 \rightarrow \omega_0 + j\alpha\omega$ becomes the complex resonant frequency. Then

$$\left.\begin{aligned} \chi_{xx} &= \chi'_{xx} - j\chi''_{xx} \\ \chi_{xy} &= \chi'_{xy} - j\chi''_{xy} \end{aligned}\right\} \text{complex susceptibilities}$$



typical susceptibility curves for a ferrite material with α = 0.1

For z-biased lossy ferrites, we can show that the susceptibilities are given by

$$\chi'_{xx} = \left(4\pi^2\right)\frac{f_0\, f_m\left[f_0^2 + f^2\left(1+\alpha^2\right)\right]}{D_1}$$

$$\chi''_{xx} = \left(4\pi^2\right)\frac{f_m\, f\, \alpha\left[f_0^2 + f^2\left(1+\alpha^2\right)\right]}{D_1}$$

$$\chi'_{xy} = \left(4\pi^2\right)\frac{f\, f_m\left[f_0^2 - f^2\left(1+\alpha^2\right)\right]}{D_1}$$

where $$D_1 = \left[f_0^2 - f^2\left(1+\alpha^2\right)\right] + 4 f_0^2 f^2 \alpha^2$$

For a given ferrite, we can experimentally determine H_0 vs. χ_{xx} and thus can measure the line width ΔH.

$$\alpha = \frac{\Delta H}{2H_0^r}$$ **(attenuation factor)**

H_0^r **= resonant value of applied field H_0**

$$\Delta H = \frac{2\alpha\omega}{\mu_0\gamma}$$

Plane Wave Propagation in Ferrite Media

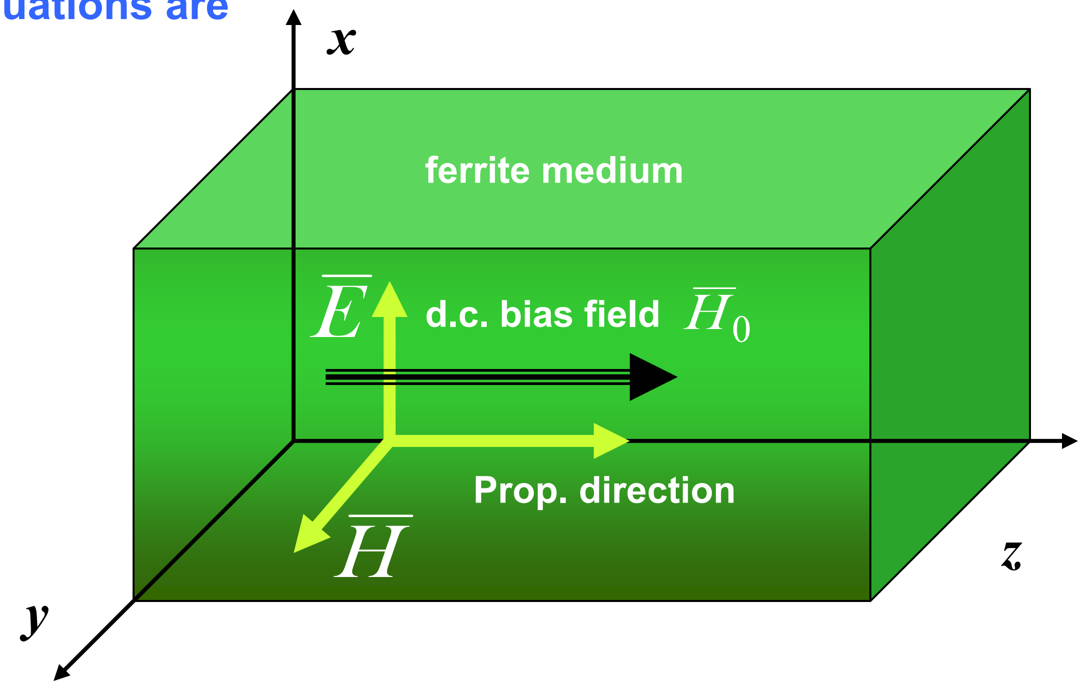
Propagation parallel to bias field. Assume an infinite ferrite medium with a d.c. bias field H_{dc} =zH_0, and an rf field (E,H). There are no free charges or conduction currents in this medium. Thus, Maxwell's equations are

$$\overline{\nabla} \times \overline{E} = -j\omega[\mu]\overline{H}$$

$$\overline{\nabla} \times \vec{H} = j\omega\varepsilon\overline{E}$$

$$\overline{\nabla} \cdot \overline{D} = 0$$

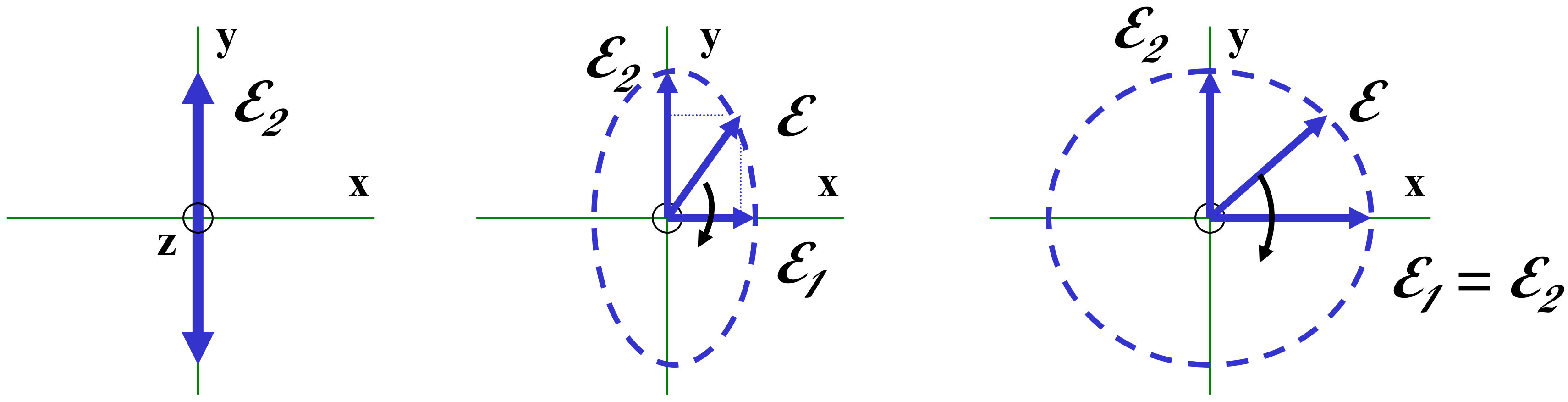
$$\overline{\nabla} \cdot \overline{B} = 0$$



$$\overline{E} = \overline{E}_0 e^{-j\beta z} = \left(\hat{x}E_x + \hat{y}E_y\right)e^{-j\beta z}$$

$$\overline{H} = \mathrm{Y}\overline{E}_0 e^{-j\beta z} = \left(\hat{x}H_x + \hat{y}H_y\right)e^{-j\beta z}$$

The polarization of a plane wave is determined by the orientation of the electric field. Elliptical polarization is the most general case. Linear polarization and circular polarization are the two limiting extremes of elliptical polarization.



Linear polarization **elliptical polarization** **Circular polarization**

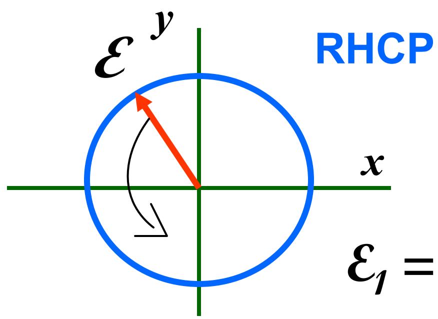
$$\overline{\mathcal{E}}(z,t) = \hat{x}\mathcal{E}_x(z,t) + \hat{y}\mathcal{E}_y(z,t)$$

where

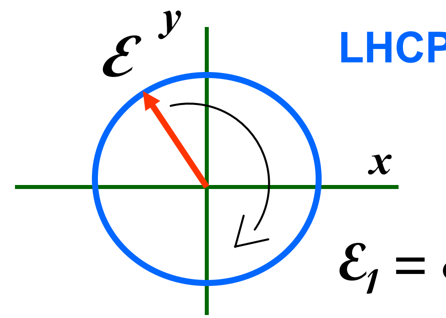
$$\mathcal{E}_x(z,t) = \mathcal{E}_1 \sin(\omega t - \beta z)$$
$$\mathcal{E}_y(z,t) = \mathcal{E}_2 \sin(\omega t - \beta z + \delta)$$

δ **= phase angle by which leads $\mathcal{E}_y$ leads $\mathcal{E}_x$**

RHCP and LHCP



$\mathcal{E}_1 = \mathcal{E}_2 \ , \delta = -90^{o}$



$\mathcal{E}_1 = \mathcal{E}_2 \ , \delta = 90^{o}$

Case 1: RHCP Wave

The phase constant is $\beta^{+} = \omega\sqrt{\varepsilon(\mu+\kappa)}$

$$\overline{E}_{+} = E_0\left(\hat{x} - j\hat{y}\right)e^{-j\beta^{+}z}$$

$$\overline{H}_{+} = \mathrm{Y}_{+}E_0\left(j\hat{x} + \hat{y}\right)e^{-j\beta^{+}z}$$

$$\mathrm{Y}_{+} = \sqrt{\frac{\varepsilon}{\mu+\kappa}}$$ **(Wave admittance)**

Case 2: LHCP Wave

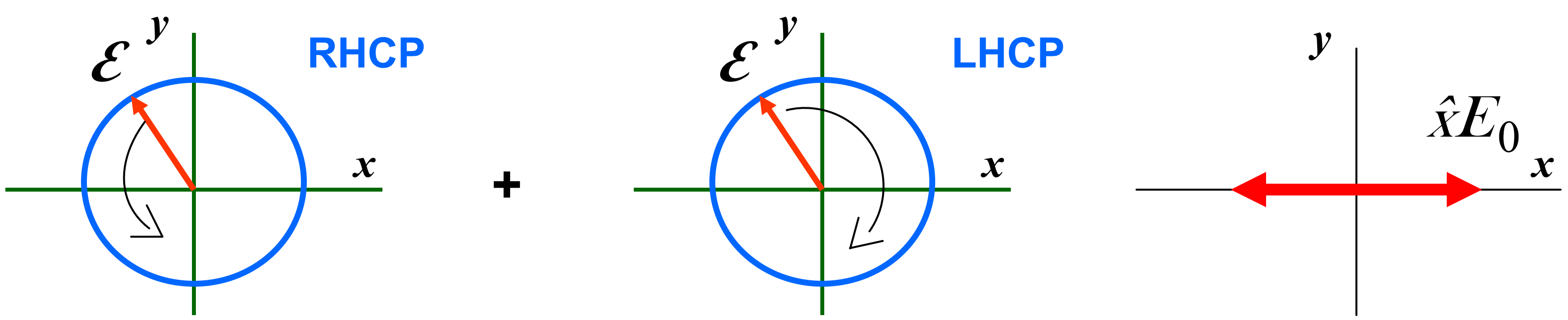
$$\beta^- = \omega\sqrt{\varepsilon(\mu-\kappa)}$$

$$\overline{E}_- = E_0(\hat{x}+j\hat{y})e^{-j\beta^- z}$$

$$\overline{H}_- = Y_- E_0(-j\hat{x}+\hat{y})e^{-j\beta - z}$$

In the z = 0 plane,

$$\overline{E}_{total} = \overline{E}_{RHCP} + \overline{E}_{LHCP} = \frac{1}{2}(\hat{x}-j\hat{y}) + \frac{1}{2}(\hat{x}+j\hat{y}) = \hat{x}E_0$$

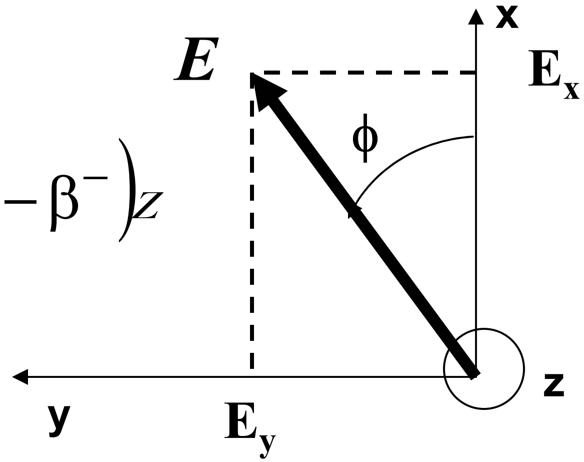


Faradayㅤ Rotation

$$\overline{E}(z) = \frac{1}{2}E_0(\hat{x} - j\hat{y})e^{-j\beta^+ z} + \frac{1}{2}E_0(\hat{x} + j\hat{y})e^{-j\beta^+ z}$$

$$\beta_{av} = \frac{1}{2}\left(\beta^+ - \beta^-\right)$$

$$\overline{E}(z) = E_0\left[\hat{x}\cos\beta_{av}z - \hat{y}\sin\beta_{av}z\right]e^{-j\beta_{av}z}$$



$$\phi = tan^{-1}\left(E_y / E_x\right) = -\beta_{av}z = -\frac{1}{2}\left(\beta^+ - \beta^-\right)z$$

The rotation of the polarization plane in a magnetic medium is called Faraday rotation.

Problem:

Suppose a ferrite medium has a saturation magnetization of M_s = 1400/4π and is magnetically lossless. If there is a z-directed bias field H_0 = 900 Oersted, find the permeability tensor at 8 GHz.

Solution:

H_0 = 900 Oe, which corresponds to

$$f_0 = 2.8MHz / Oe \times 900Oe = 2.52GHz$$

$$f_m = 2.8MHz / Oe \times 1400G \times 1Oe / G = 3.92GHz$$

$$f = 8GHz$$

$$\therefore \mu = \mu_0 \left[1 + \frac{f_0 f_m}{f_0^2 - f^2} \right] = 0.829\mu_0$$

$$\kappa = \mu_0 \left[\frac{f \; f_m}{f_0^2 - f^2} \right] = -0.544\mu_0$$

$$\Rightarrow \quad [\mu] = \begin{bmatrix} 0.829 & -j0.544 & 0 \\ j0.544 & 0.829 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem:

An infinite lossless ferrite medium has a saturation magnetization of M_s=1000/4π G and a dielectric constant of 6.1. It is biased to a field strength of 350 Oe. At 5 GHz, what is the differential phase shift per meter between a RHCP and a LHCP plane wave propagation along the bias direction? If a linearly polarized wave is propagating in this material, what is the Faraday rotation angle over a distance of 9.423 mm?

Solution:

$$4\pi M_s = 1000G; \varepsilon_r = 6.1; H_0 = 300Oe; f = 5GHz; \lambda = 6cm.$$

$$f_0 = 2.8MHz/Oe \times 300Oe = 840MHz$$

$$f_m = 2.8MHz/Oe \times 1000G \times 1Oe/G = 2800MHz$$

$$K_0 = \frac{2\pi}{\lambda_0} = 104.7m^{-1}$$

$$\mu = \mu_0\left[1 + \frac{f_0 f_m}{f_0^2 - f^2}\right] = 0.903\mu_0 \qquad \kappa = \mu_0 \frac{f\ f_m}{f_0^2 - f^2} = -0.576\mu_0$$

RHCP:

$$\beta^+ = \omega\sqrt{\varepsilon(\mu+\kappa)}$$

$$= k_0\sqrt{\varepsilon_r}\sqrt{0.903 - 0.576} = 147.8m^{-1}$$

LHCP:

$$\beta^- = \omega\sqrt{\varepsilon(\mu-\kappa)}$$

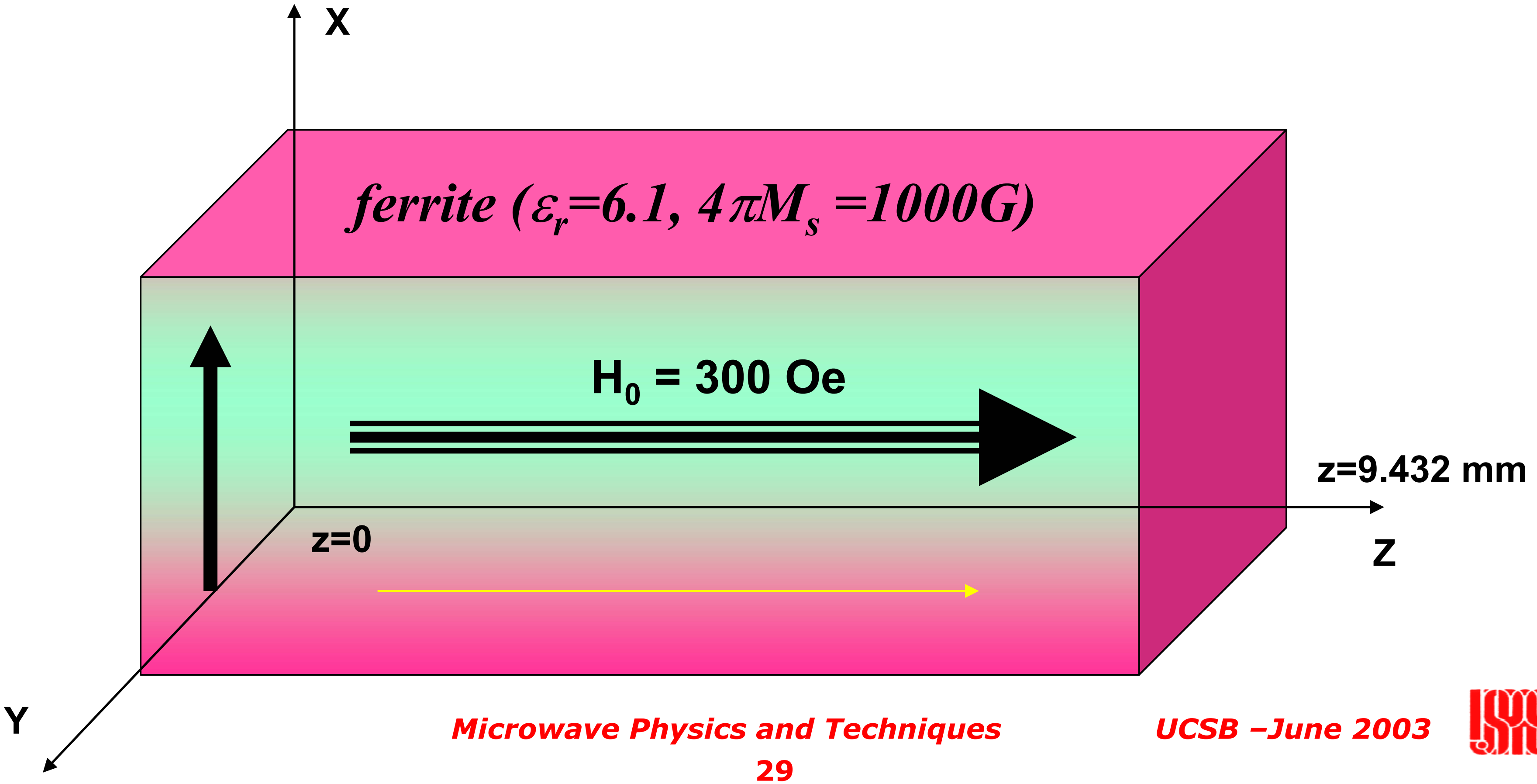
$$= k_0\sqrt{\varepsilon_r}\sqrt{0.903 + 0.576} = 314.5m^{-1}$$

So that

$$\Delta\beta = \beta^{+} - \beta^{-} = -166.7\,m^{-1}$$

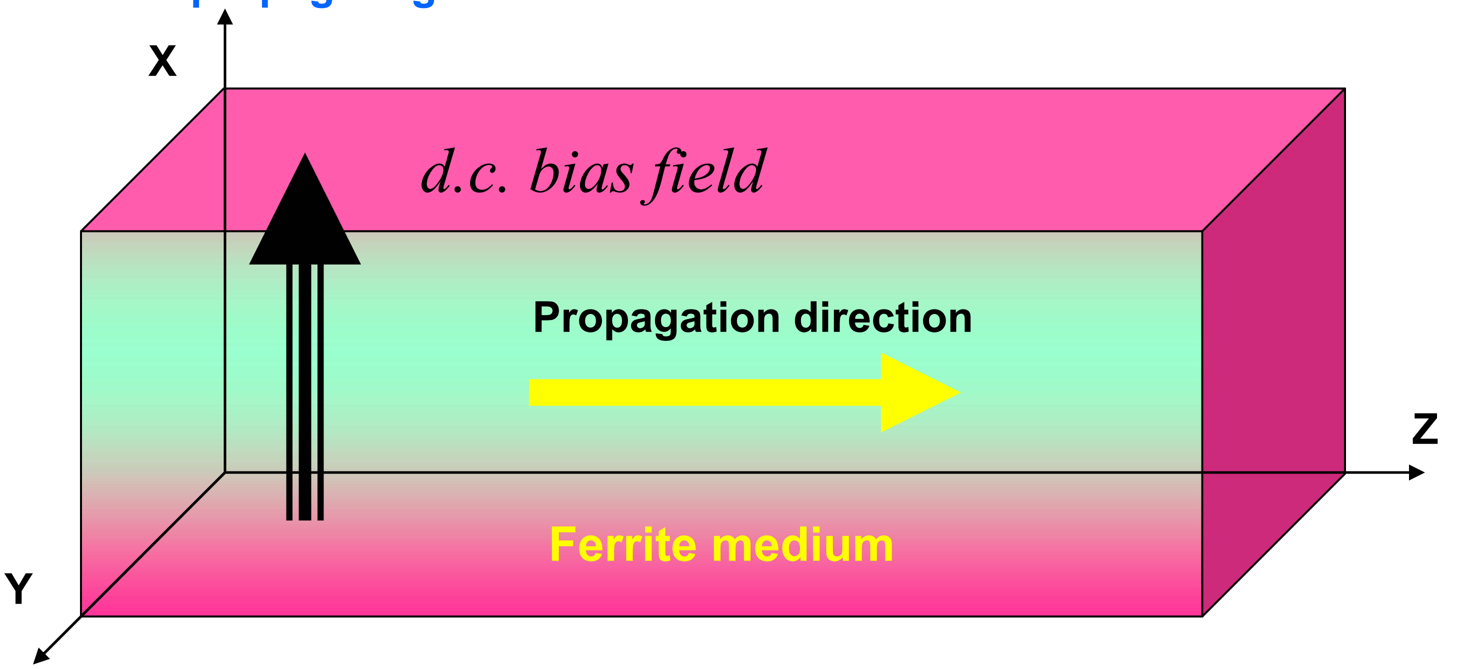
The polarization rotation on an LP wave is

$$\phi = -\frac{\beta^{+} - \beta^{-}}{2} z = (166.7)(9.423) \times 10^{-3} = 1.57\,rad \qquad (90^{0})$$



Propagation transverse to bias field

We now bias the ferrite in the x-direction, e.g. $\overline{H}_0 = \hat{x} H_0$. The rf plane wave is still presumed to be propagating in the Z-direction.



Apply Maxwell's equations to obtain wave equation

1. **Ordinary wave (wave is unaffected by magnetization).**
2. **Extraordinary wave (wave is affected by ferrite magnetization).**

Ordinary wave

$$\overline{H}_o = \hat{x} H_o$$

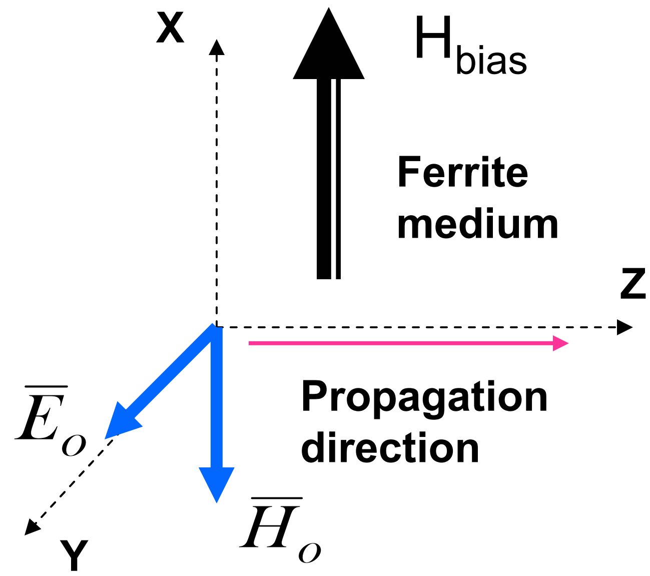
$$\overline{E}_o = \hat{y} E_o e^{-j\beta_o z}$$

$$\overline{H}_o = \hat{x} Y_o E_o e^{-j\beta_o z}$$

where $$Y_o = \sqrt{\frac{\varepsilon}{\mu_o}}$$

and

$$\beta_o = \omega\sqrt{\mu_o \varepsilon}$$



NOTE: **propagation constant β_o is independent of H_{bias} .**

Extraordinary wave

$$\bar{E}_e = \hat{x} E_o e^{-j\beta_o z}$$

$$\bar{H}_e = Y_o E_o \left(\hat{y} + \hat{z} \frac{j\kappa}{\mu} \right) e^{-j\beta_o z}$$

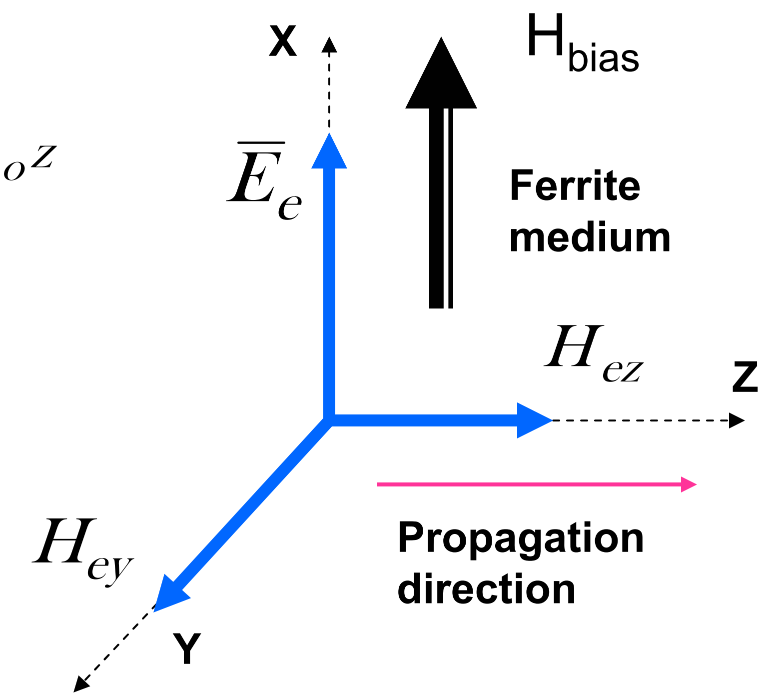
where

$$Y_e = \sqrt{\frac{\varepsilon}{\mu_e}}$$

and

$$\beta_e = \omega \sqrt{\mu_e \varepsilon}$$

$$\mu_e = \frac{\mu^2 - \kappa^2}{\mu}$$



NOTE: **propagation constant β_e dependents of H_{bias} and on propagation direction.**

Problem: **Consider an infinite lossless ferrite medium with a saturation magnetization of $4\pi M_s$ = 1000 G, a dielectric constant of 6.1 and H_{bias} =1500 Oe. At 3 GHz, two plane waves propagate in the +z-direction, one is x-polarized and the other is y-polarized. What is the distance that these two waves must travel so that the differential phase shift is -90 degrees?**

Solution:

$$f = 3.0GHz \quad (\lambda_o = 6cm)$$

$$f_0 = 2.8MHz/Oe \times 1500\,Oe = 4.2GHz$$

$$f_m = 2.8MHz/Oe \times 1000\,Oe = 2.8GHz$$

$$k_0 = 2\pi/\lambda_0 = 104.7\,m^{-1}$$

$$\mu = \mu_0\left[1 + \frac{f_0 f_m}{f_0^2 - f^2}\right] = 1.36\mu_0$$

$$\kappa = \mu_0 \frac{f\ f_m}{f_0^2 - f^2} = 0.972\mu_0$$

Solution: cont.

The y-polarized wave has $\overline{H} = \hat{x} H_x$ and is the **ordinary** wave. Thus

$$\beta_o = \sqrt{\varepsilon_r} K_o = 258.6 m^{-1}$$

Therefore the distance for a differential phase shift of -90 degrees is

$$z = \frac{-\pi/2}{\beta_e - \beta_o} = \frac{\pi/2}{258.6 - 210.9} = 0.0329 m = 32.9 mm$$

Ferrite Isolators

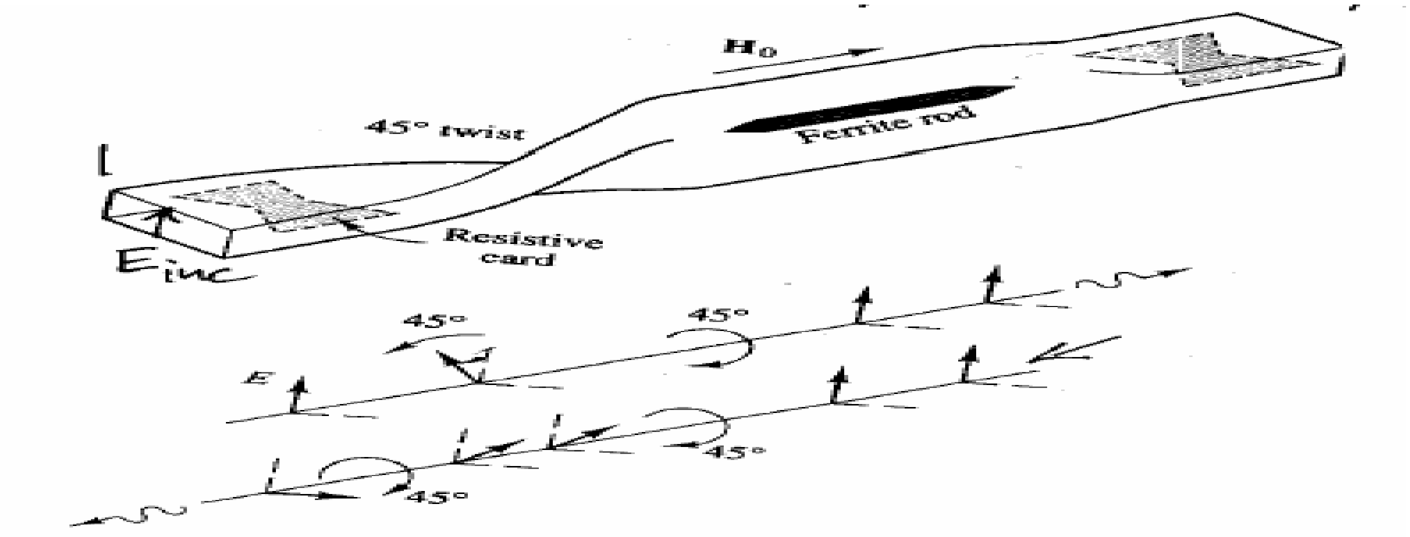
An ideal isolator is a 2-port device with unidirectional transmission coefficients and a scattering matrix given by

$$[S] = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$$

Isolators are lossy and non-reciprocal.

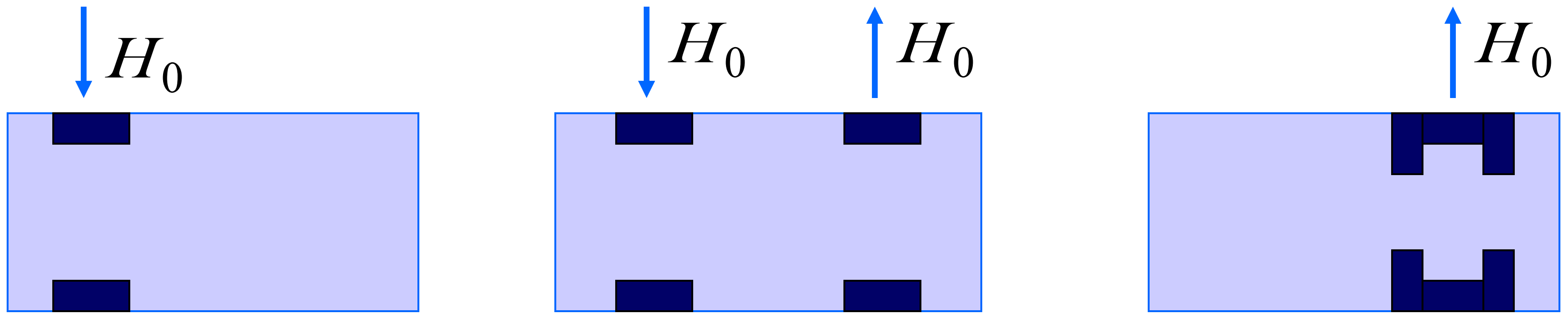
Isolator types

1. *Faraday Rotation Isolator.* This was the earliest type of microwave isolator, but is difficult to manufacture, has inherent power handling limitations due to the resistive cards and is rarely used in modern systems.

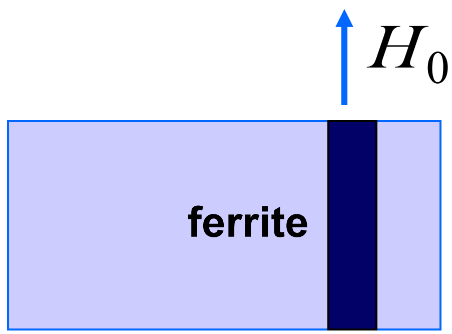


Isolator types

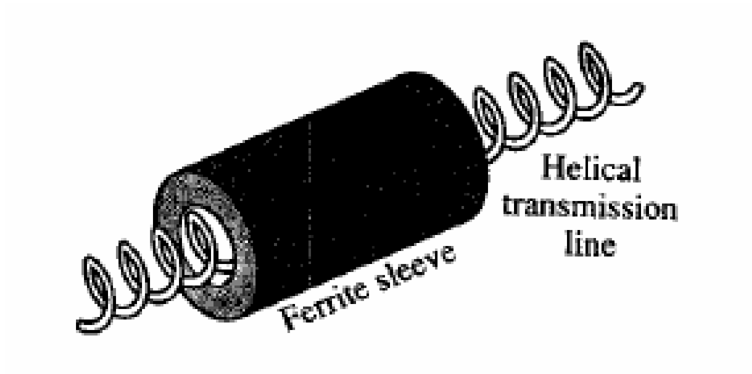
2. *Resonance Isolators.* These must be operated at frequency close to the gyromagnetic resonance frequency. Ideally the rf fields inside the ferrite material should be circularly polarized.



H-plane resonance isolators



E-plane resonance isolator

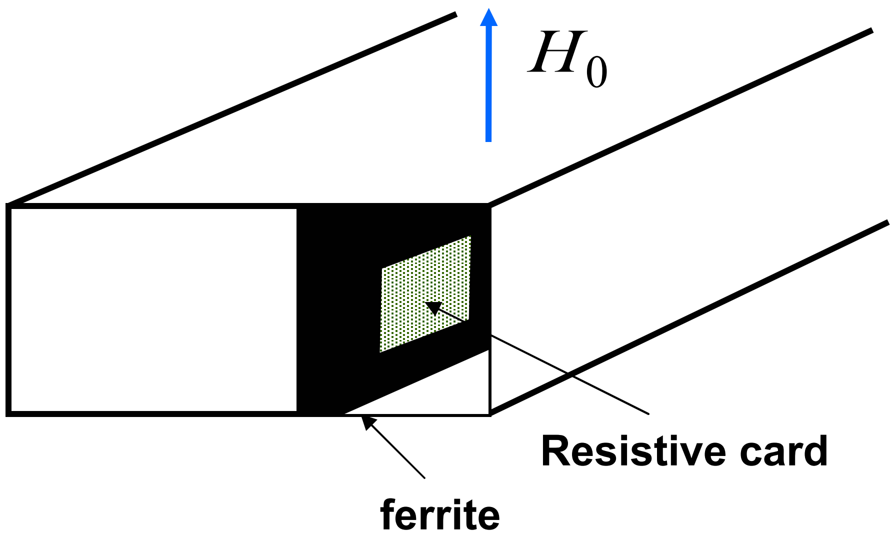


Ferrite loading of helical T.L.

Isolator types

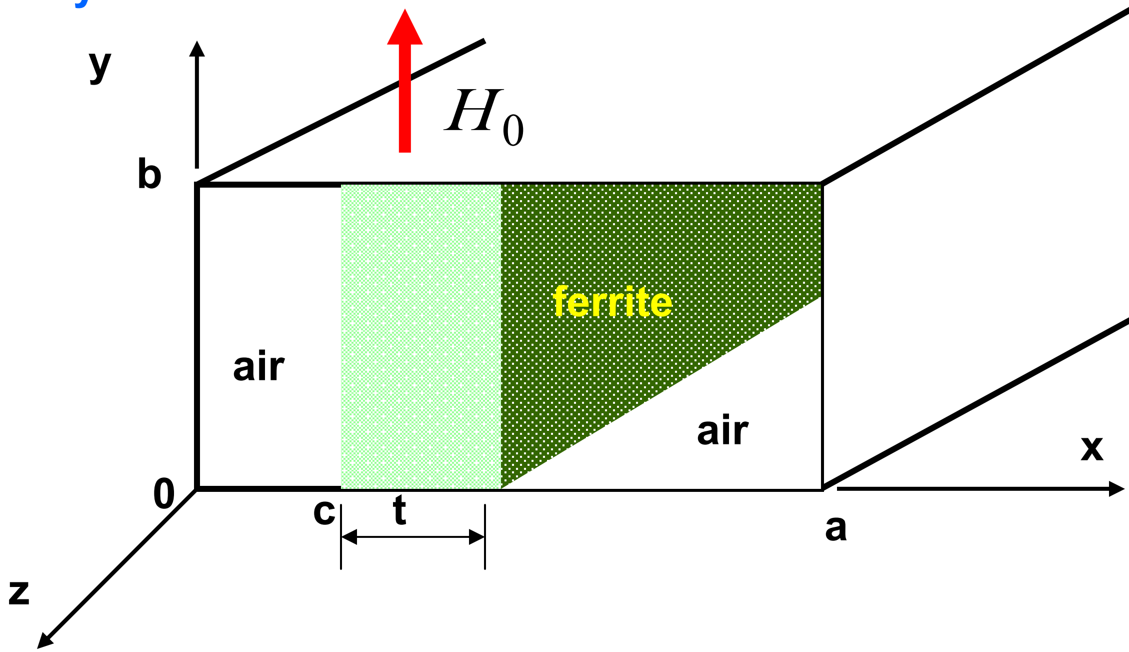
3. *Field Displacement Isolator.* Advantages over resonance isolators:

- much small H_0 bias field required
- high values of isolation, with relatively compact device
- bandwidths about 10%



Propagation in Ferrite Loaded Rectangular Waveguides

Consider a rectangular waveguide loaded with a vertical slab of ferrite which is biased in the y-direction



In the ferrite slab, the fields satisfy Maxwell's equations:

$$\overline{\nabla} \times \overline{E} = -j\omega[\mu]\overline{H}$$

$$\overline{\nabla} \times \overline{H} = j\omega\varepsilon\overline{E}$$

$$[\mu] = \begin{bmatrix} \mu & 0 & -j\kappa \\ 0 & \mu_0 & 0 \\ j\kappa & 0 & \mu \end{bmatrix}$$

Propagation in Ferrite Loaded Rectangular Waveguides

Assume propagation in the +z direction: Let

$$\overline{E}(x,y,z)\{\overline{e}(x,y)+\hat{z}e_z(x,y)\}e^{-j\beta z}$$

$$\overline{H}(x,y,z)\{\overline{h}(x,y)+\hat{z}h_z(x,y)\}e^{-j\beta z}$$

Consider TE_{m0} **modes, i.e.** $E_z=0$ **and** $\frac{\partial}{\partial y}=0$ **.**

$$\kappa_f=\sqrt{\omega^2\mu_e\varepsilon-\beta^2}$$

κ_f **= cutoff wave number for air**

$$\kappa_a=\sqrt{k_0^2-\beta^2}$$

κ_a **= cutoff wave number for ferrite**

$$\mu_e=\frac{\mu^2-\kappa^2}{\mu}$$

Effective permeability

$$\varepsilon=\varepsilon_r\varepsilon_0$$

$$\kappa=2\pi/\lambda_0$$

Propagation in Ferrite Loaded Rectangular Waveguides

$$
e_y = \begin{cases} A \sin k_a x & for(0 < x < 0) \\ B \sin k_f (x - c) + C \sin k_f (c + t - x) & for(c < x < c + t) \\ D \sin k_a (a - x) & for(c + t < x < a) \end{cases}
$$

$$
h_z = \begin{cases} \dfrac{jk_a A}{\omega\mu_0} cos\ k_a x & for(0 < x < c) \\ \dfrac{j}{\omega\mu\mu_e} - \kappa\beta[sin\ k_f (x - c) + C\ sin\ k_f (c + t - x)] & \\ \qquad + \mu k_f\ cos\ k_f (c + t\text{-}x) & for(c < x < c + t) \\ \dfrac{-jk_a D}{\omega\mu_0} cos\ k_a (a - x) & for(c + t < x < a) \end{cases}
$$

Propagation in Ferrite Loaded Rectangular Waveguides

Apply boundary conditions $E_{tan1} = E_{tan2}$ at $x=c$ and $x=c+t$; also $H_{tan1}=H_{tan2}$ at these boundaries. This means we must have match e_y and h_z at the air-ferrite boundaries to obtain the constants A,B,C,D.

Reducing these results give a transcendental equation for the propagation constant β.

$$\sum_{n=1}^{5} T_n = 0$$

$$T_1 = \left(\frac{k_f}{\mu_e}\right)^2, T_2 = \left(\frac{\kappa\beta}{\mu\mu_e}\right)^2, T_3 = -k_a \cot k_a c\left[\frac{k_f}{\mu_0\mu_e}\cot k_f t - \frac{\kappa\beta}{\mu_0\mu\mu_e}\right]$$

$$T_4 = -\left(\frac{k_f}{\mu_0}\right)^2 \cot k_a c \cot k_a d, T_5 = -k_a \cot k_a d\left[\frac{k_f}{\mu_0\mu_e}\cot k_f t + \frac{\kappa\beta}{\mu_0\mu\mu_e}\right]$$

Propagation in Ferrite Loaded Rectangular Waveguides

After solving $\sum_{n=1}^{5} \mathrm{T}_n = 0$ for the roots β, we can then calculate the wave number k_f and k_a.

We can then calculate A,B,C,D by applying B.C's

e_y matched at x=c
e_y matched at x=c+t
h_z matched at x=c
h_z matched at x=c+t

Let A =1, then

$$C = \frac{\sin k_a c}{\sin k_f t}$$

$$B = \frac{\mu_e}{k_f}\left\{\frac{k_a}{\mu_0}\cos k_a c\right\} + \frac{C}{\mu\mu_e}\left[\kappa\beta \sin k_f t + \mu k_f \cos k_f t\right]$$

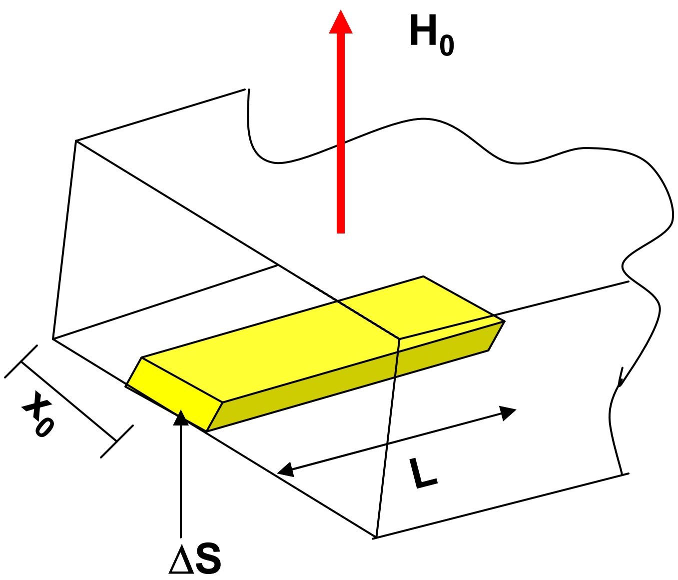
$$D = B\frac{\sin k_f t}{\sin k_a d} \qquad (d = c + t - a)$$

Propagation in Ferrite Loaded Rectangular Waveguides

We can now calculate the fields for each of the three regions.

To design a resonance isolator using a ferrite in a waveguide, we can choose either an E-plane or H-plane configuration (the h-plane version is easier to manufacture).

We need to find the necessary design parameters to give the required forward and reverse attenuation:



1. Cross-sectional area of ferrite (ΔS)
2. Length of the ferrite (L)
3. Saturation magnetization $4\pi M_s$
4. Bias field H_0
5. Location of ferrite (X_0)

Propagation in Ferrite Loaded Rectangular Waveguides

$$R = \frac{\alpha_-}{\alpha_+} = \frac{reverse\ attenuation}{forwarsd\ attenuation}$$

We wish to choose the location X_0 such that R is maximized. If α<<1, we can show that

$$R_{max} = \frac{4}{\alpha^2} = \left(\frac{4H_0}{\Delta H}\right)^2$$

Optimum position X_0 can be found from

$$cos\frac{2\pi x_0}{a} = \frac{\beta_{10}^2\chi''_{xx} - \left(\frac{\pi}{a}\right)^2\chi''_{xy}}{\beta_{10}^2\chi''_{xx} + \left(\frac{\pi}{a}\right)^2\chi''_{xy}}$$

a = waveguide broad wall dimension (m)

X_0 = optimum location for slab (m)

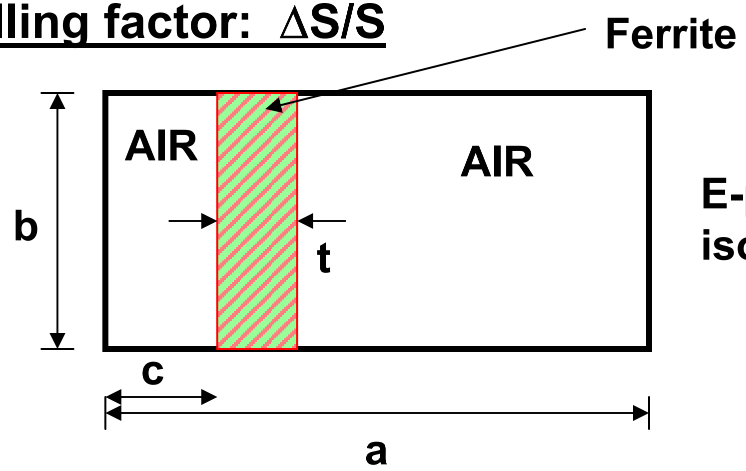
$\beta = k_0\sqrt{1-\left(\frac{\lambda_0}{2a}\right)^2}$ **= phase constant for empty guide (m^{-1})**

χ''_{xx} **= xx-susceptibility, imaginary term**

χ''_{xy} **= xy-susceptibility, imaginary term**

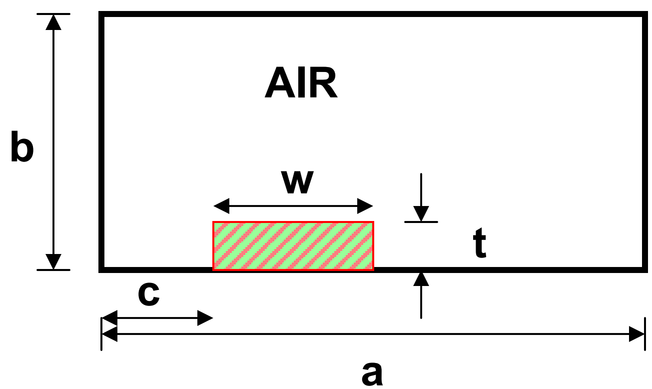
Propagation in Ferrite Loaded Rectangular Waveguides

Filling factor: ΔS/S



E-plane isolator

$$\frac{\Delta S}{S} = \frac{tb}{ab} = \frac{t}{a}$$



H-plane isolator

$$\frac{\Delta S}{S} = \frac{wt}{ab}$$

Propagation in Ferrite Loaded Rectangular Waveguides

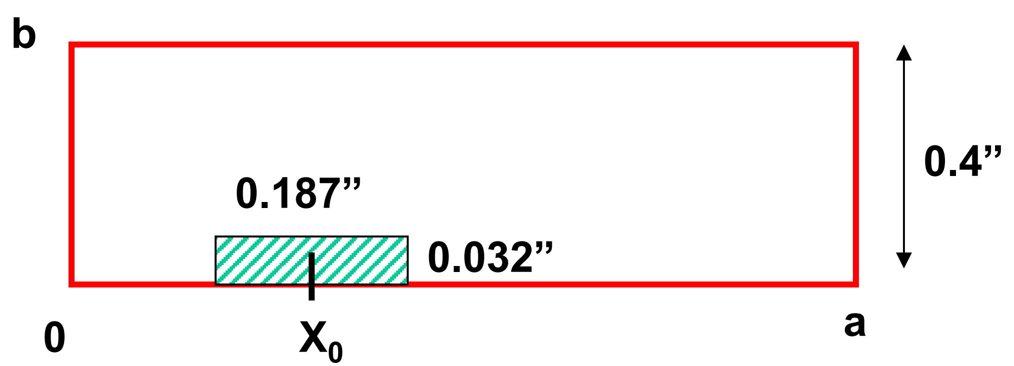
If $\Delta S/S < 0.02$, we can calculate the differential phase shift (RHCP – LHCP) as

$$\beta_+ - \beta_- = \frac{-2k_c \kappa \Delta S}{\mu S} \sin(2k_c c)$$

$$\beta_0 = \sqrt{k_0^2 - k_c^2}$$

$$\alpha_\pm = \frac{\Delta S}{S\beta_0}\left[\beta_0^2 \chi''_{xx} \sin^2 k_c x + k_c^2 \chi''_{zz} \cos^2 k_c x \mp \chi''_{xy} k_c \beta_0 \sin 2k_c x\right]$$

Consider an H-plane resonance isolator to operate at 9 GHz, using a single fettite slab of length L and cross section of 0.187" ×0.032". It is bonded to the lower broad wall of an X-Band waveguide (a=0.90", b=0.40") at X_0 . The ferrite material has a line width ΔH = 250 Oe and a saturation magnetizations $4\pi M_s$ = 1900 G. Find the internal bias field H_0, the external bias field H^e_0, the position X_0 that will yield R_{max} , the value of R_{max}, α_- and α_+ . If the reverse attenuation is 25 dB, find the length L of the slab.



Propagation in Ferrite Loaded Rectangular Waveguides

The internal bias filed is

$$H_0 = \frac{900MHz}{2.8MHz/Oe} = 3214Oe \ \left(A/m\right)$$

The external bias filed is

$$H_0^e = H_0 + 4\pi M_S = 3214 + 1900 = 5114Oe$$

With $\alpha = \Delta H / 2H_0 = 0.039, f_0 = f = 9GHz, f_m = 5.32GHz$ **,**

$$\chi''_{xx} = 7.603, \ \chi''_{xy} = 7.597$$

The free space wavelength is λ_0 =3.4907 in^{-1}, β_{10} =3.2814 in^{-1}

Two solution: X_0/a = 0.260 and X_0/a = 0.740

To get small forward attenuation and large reverse attenuation, we X_0=0.666".

Propagation in Ferrite Loaded Rectangular Waveguides

$$R_{max} = \frac{4}{\alpha^2} = \frac{4}{(0.039)^2} = 2630$$

$$R_{max} = \frac{4}{\alpha^2} = \frac{4}{(0.039)^2} = 2630\frac{\Delta S}{S} = (0.032'')(0.187'')/(0.9'')(0.4'') = 0.0166$$

$$\alpha_{\pm} = 0.4147\,sin^2\frac{\pi x_o}{a} + 0.4693\,cos^2\frac{\pi x_o}{a} \pm 0.4408\,sin^2\frac{2\pi x_0}{a}$$

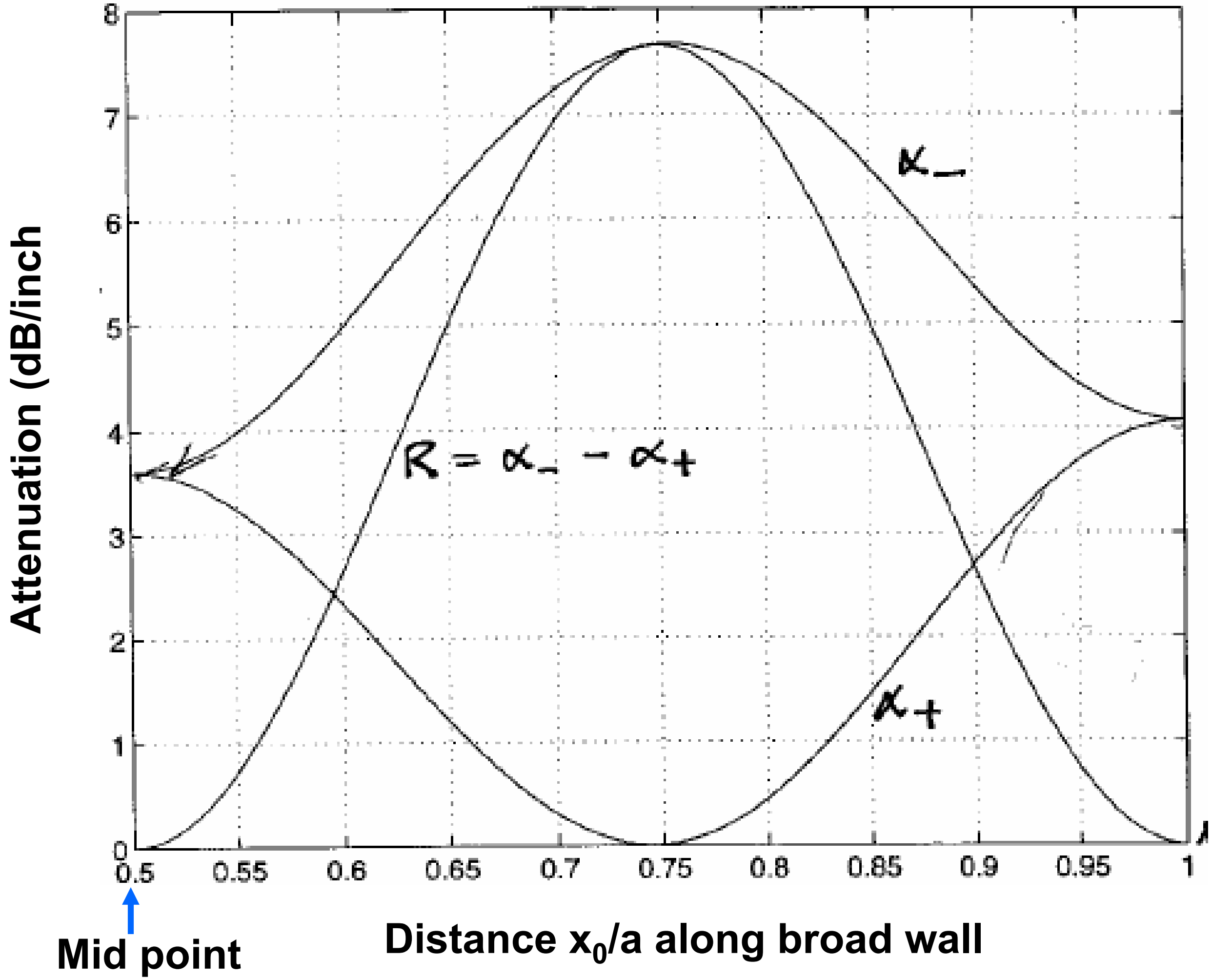
(Neper/inch)

To convert to dB/inch, multiply (Neper/inch) by 8.686

$$\alpha_{\pm} = 3.6021\,sin^2\frac{\pi x_o}{a} + 4.0763\,cos^2\frac{\pi x_o}{a} \pm 3.829\,sin^2\frac{2\pi x_0}{a}$$

(dB/inch)

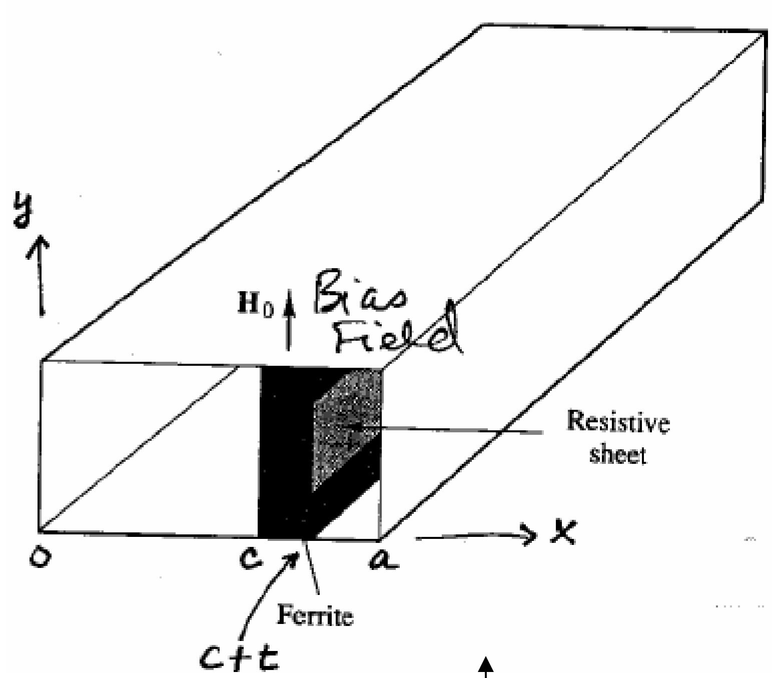
Propagation in Ferrite Loaded Rectangular Waveguides

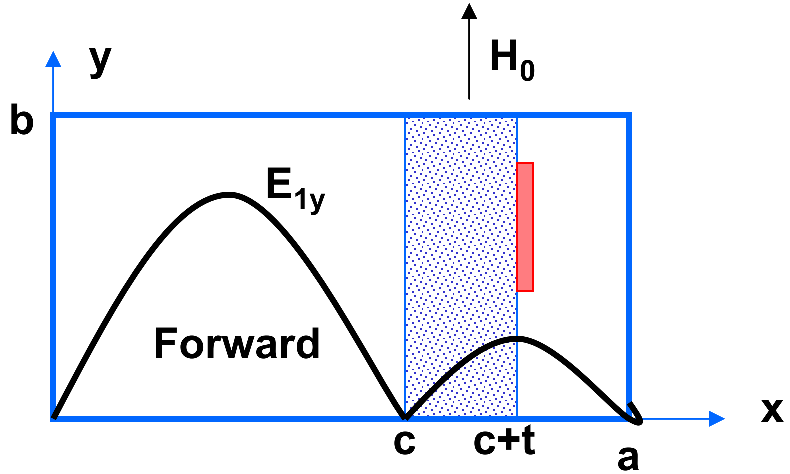


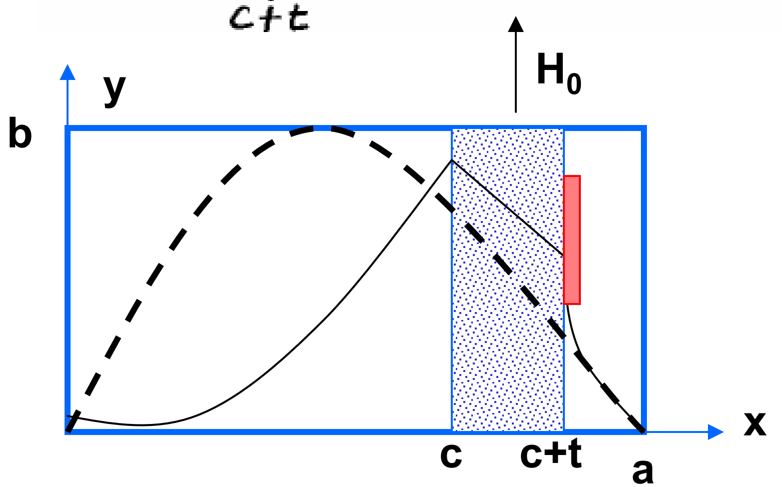
The maximum reverse attenuation α_- is approximately 7.75 dB/inch. Thus the necessary ferrite length is L=25dB/7.75dB/in = 3.23"

Field Displacement Isolator

By placing a ferrite slab in the E-plane with a thin resistive sheet at x=c+t, we can cause the E fields to be distinctly different for forward and reverse propagation. We can make the E field at the slab very small for +z propagation waves but much larger for –z reverse wave.

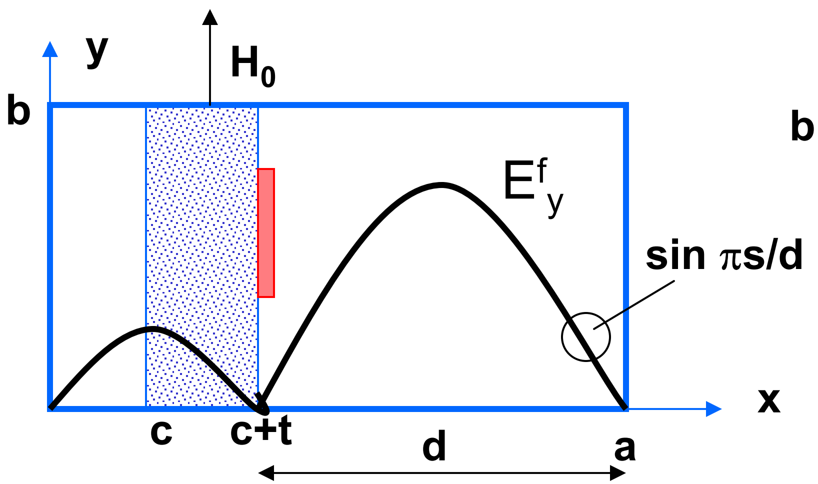




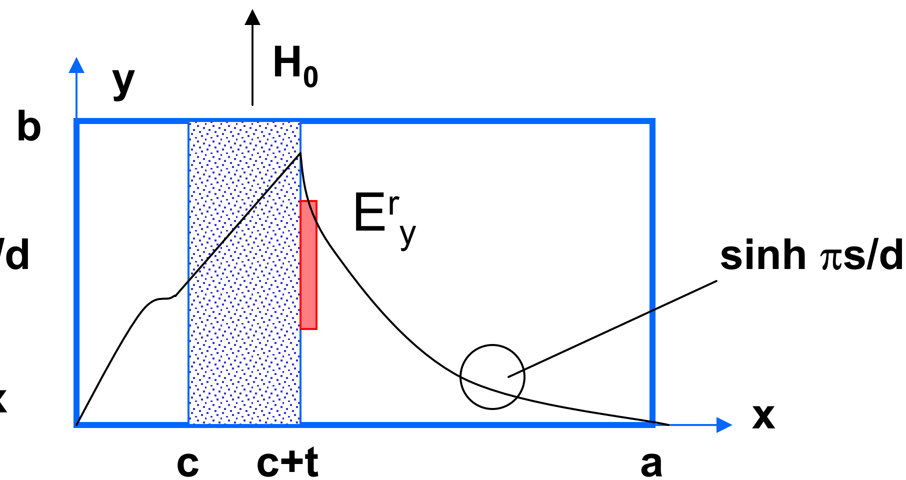


Propagation in Ferrite Loaded Rectangular Waveguides

$$e_y = \begin{cases} A \sin k_a x & 0 < x < c \\ B \sin k_f (x - c) + C \sin k_f (c = t - x) & ferrite \\ D \sin k_a (a - x) & c + t < x < a \end{cases}$$



Forward Wave



Reverse Wave

For E^f_y of forward wave to vanish at x=c+t and to be sinusoidal in x, we require

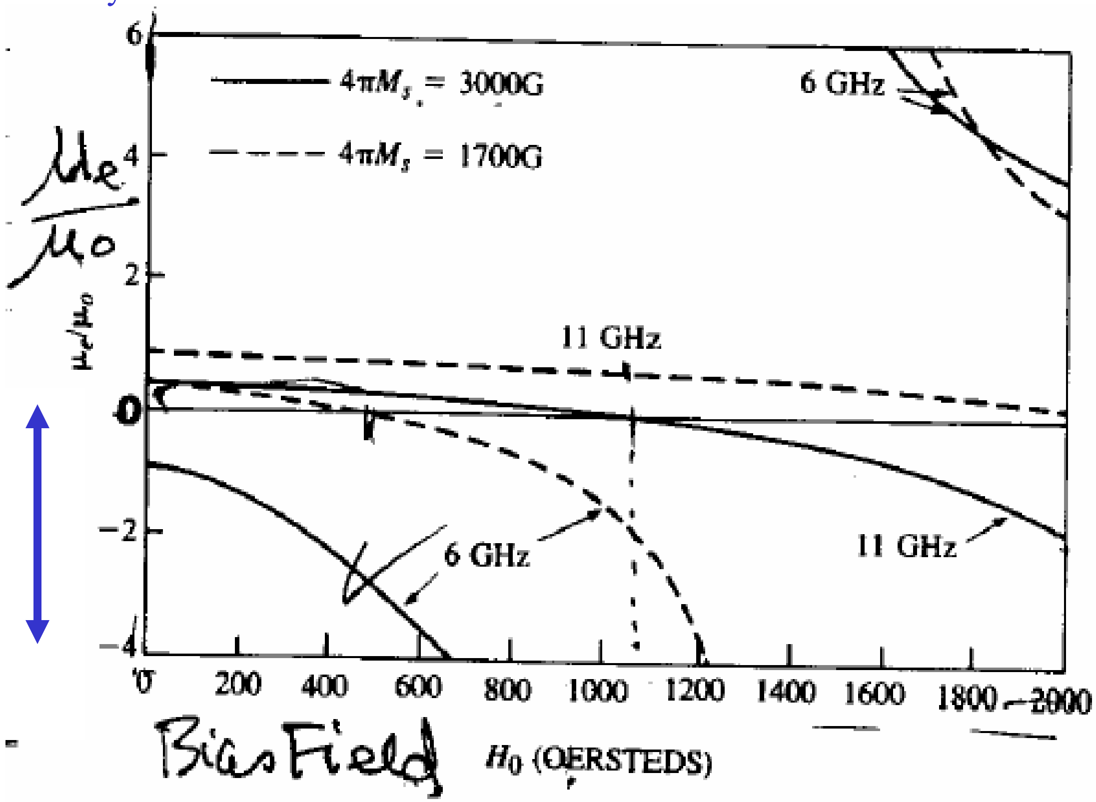
$$\sin\left(k_a^+\left[a - (c + t)\right]\right) = \sin k_a^+ d = 0 \quad \Longrightarrow \quad k_a^+ = \frac{\pi}{d}$$

Propagation in Ferrite Loaded Rectangular Waveguides

For $E^r{}_y$ of the reverse wave to have a hyperbolic sine dependence for c+t <x<a, then the $k^-{}_a$ must be imaginary. Since $k_a^2 = k_0^2 - \beta^2$, this means that $\beta^+ < k_0$ and $\beta^- > k_0$

Very important: **We also require** $\mu_e = \dfrac{\mu^2 - \kappa^2}{\mu}$ **to be negative, if we want to force $E_y = 0$ at $x = c+t$**

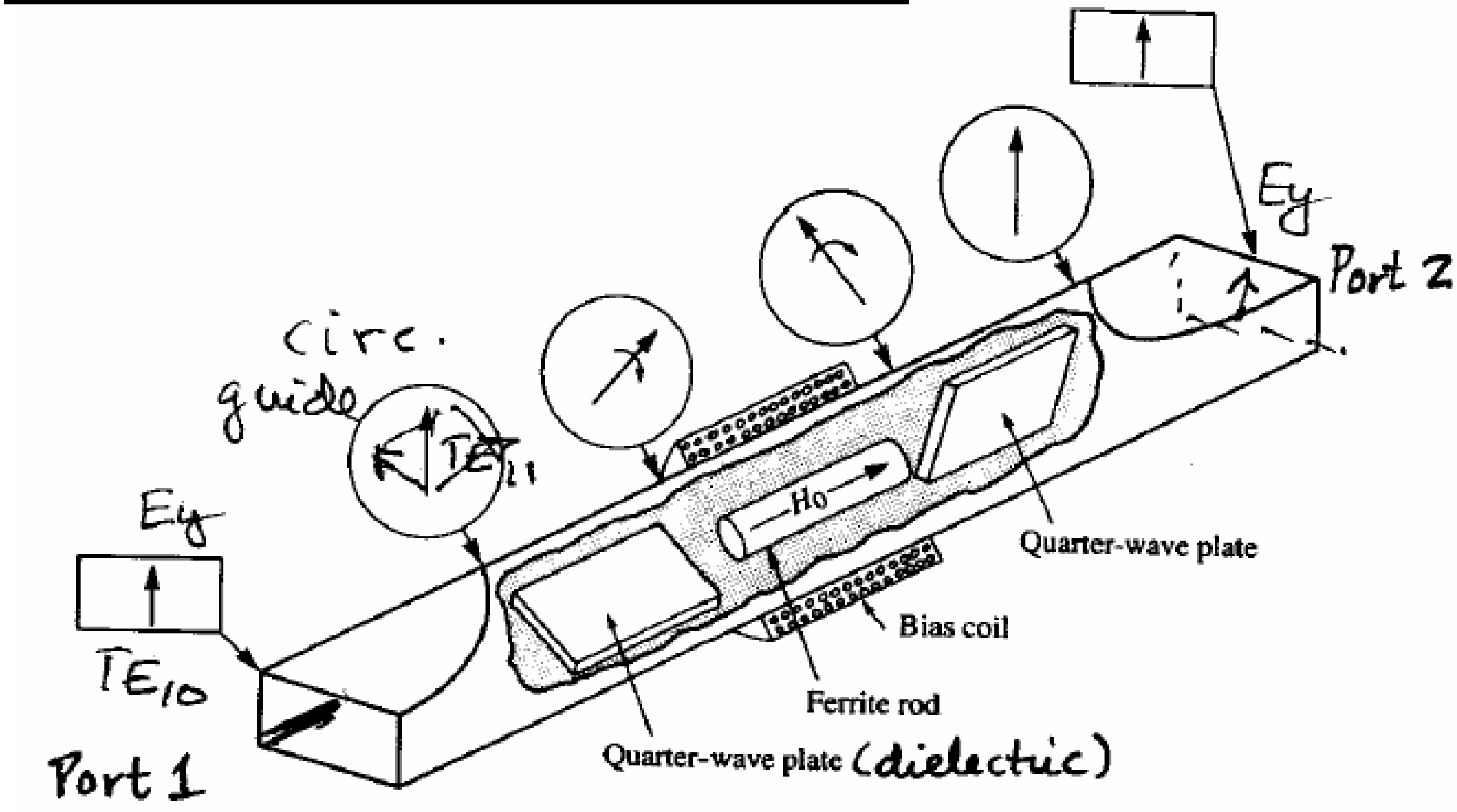
Region of negative effective permeability



Ferrite Phase Shifters

- provide variable phase shift by changing bias field of the ferrite

Nonreciprocal Faraday Rotation Phase Shifter

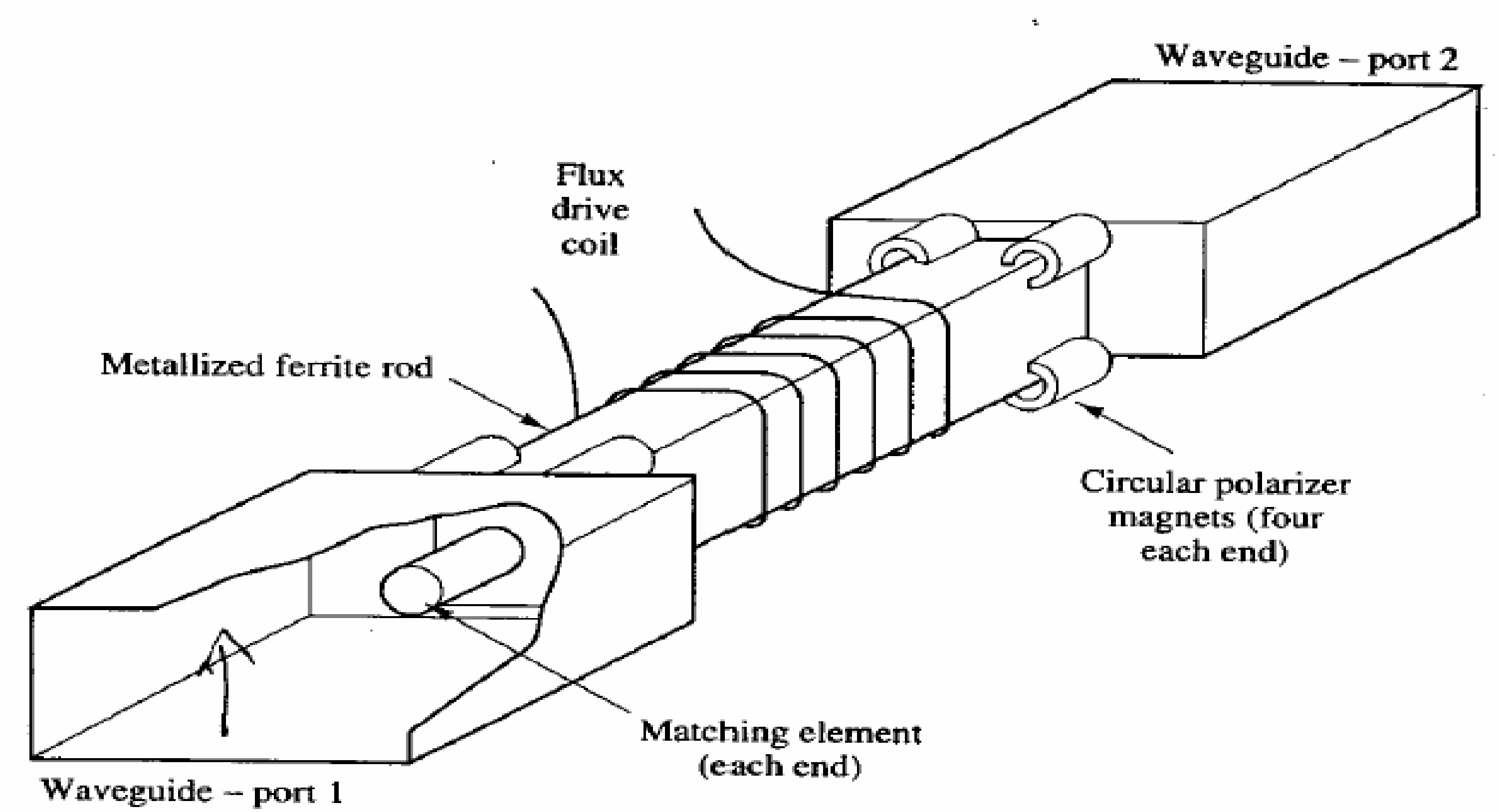


Ferrite Phase Shifters

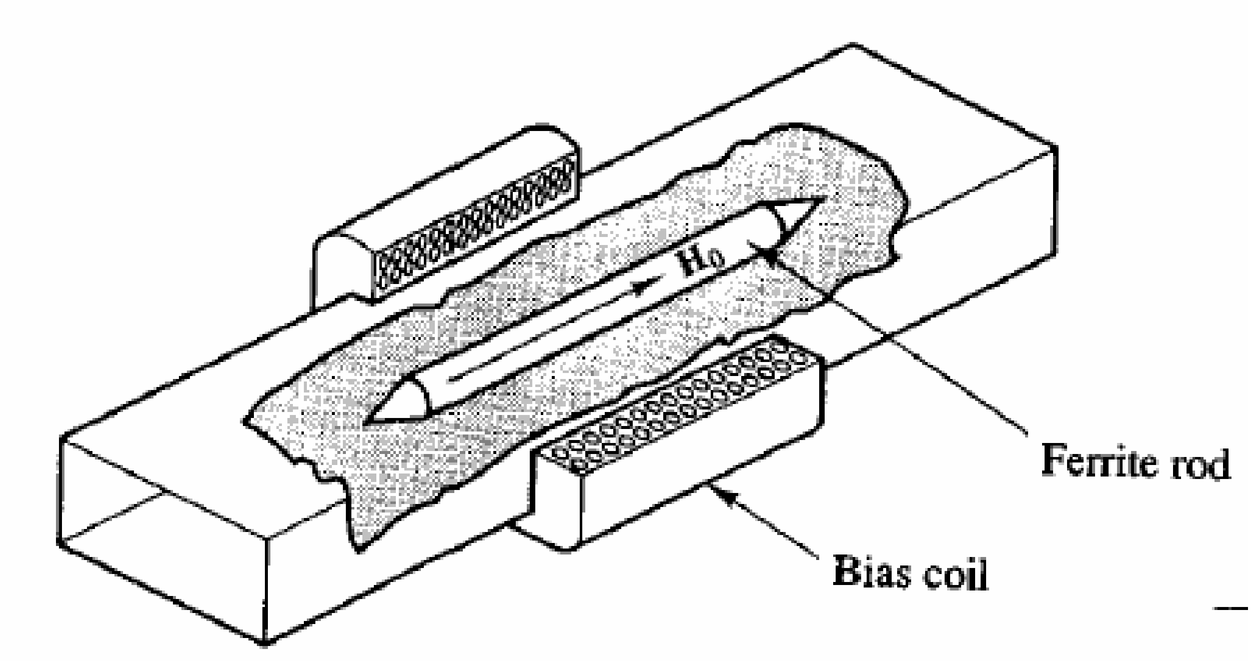
First $\lambda/4$ plate converts linearly polarized wave from input port to RHCP wave; in the ferrite region, the phase delay is $\beta^{+}z$, which can be cancelled by the bias field H_0, the second $\lambda/4$ plate converts RHCP wave back to linear polarization.

Advantages: **Cost, power handling**

Disadvantage: **Bulky**

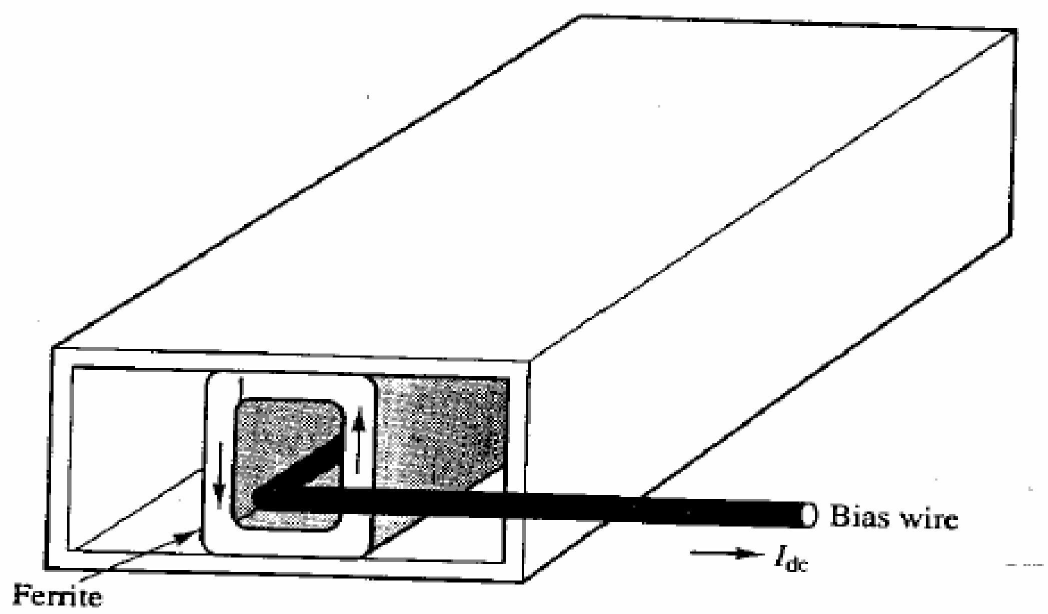


Reggia-spencer Reciprocal Phase Shifter



Reciprocal phase shifters are required in scanning antenna phase arrays used in radar or communication systems, where both transmitting and receiving functions are required for any given beam position. The Reggia-spencer phase shifter is such that a reciprocal device. The phase delay through the waveguide is proportional to the d.c. current through the coil, but independent of the direction of the propagation through the guide.

Nonreciprocal Latching Phase Shifter



Or a simpler version using 2 ferrite slabs: