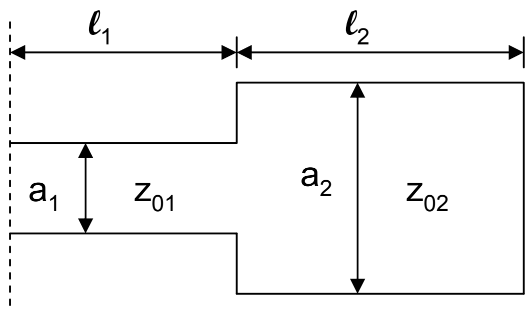
Microwave Physics and Techniques

Solutions to Final Exam – June 20, 2003

Problem1.



$$Z_{01} = a_1\sqrt{\mu/\varepsilon}$$
$$Z_{02} = a_2\sqrt{\mu/\varepsilon}$$

For open CKT load:

$$Z_1 = -jZ_{01}\, cot\left(\frac{\omega\ell_1}{\nu_1}\right)$$

For short CKT load:

$$Z_2 = -jZ_{02}\, tan\left(\frac{\omega\ell_2}{\nu_2}\right)$$

$$\nu_1 = \nu_2 = \frac{1}{\sqrt{\mu\varepsilon}}(= c \quad in\ air)$$

For transverse resonance Z_1+Z_2 =0

$$\Rightarrow jZ_{01}\, tan\frac{2\pi f\ell_1}{c} + jZ_{02}\, tan\frac{2\pi f\ell_2}{c} = 0$$

$$\ell_1 = 0.075m, \ell_2 = 0.05m, a_1 = 0.01cm, a_2 = 0.05cm$$

$$\Rightarrow (0.01)\frac{cot\, 2\pi f(0.075)}{3\times10^8} + (0.05)\frac{tan\, 3\pi f(0.05)}{3\times10^8} = 0$$

$$\Rightarrow cutoff \quad frequency : f_c = 326.21MHz$$

Problem1.Cont.

$$\lambda_g = \frac{\lambda}{\sqrt{1-(\lambda/\lambda_0)}}$$

In the 5m long chamber, λ_g=10m, λ_{g2}=5m, λ_{g3}=3.33m, λ_{g4}=2.5m, …

λ_c=0.9197m

Freq (MHz)	B.C. λ_g (cm)	Rect WG	$\lambda_{\text{fee space}}$(cm)
327.6	1.0	Below cutoff λ_c =2a=0.5 m f_c =600 MHz	0.9158
331.7	5.0		0.9045
338.4	3.33		0.8866
347.6	2.5		0.8631
359	2.0		0.8356

Conductor loss or power dissipation is

$$P_l = \iint R_s |H|^2 ds$$

Without NEG strips, $P_l \approx R_s S$ where R_s is the surface resistance of the aluminum and S is the surface area of the two square chambers. With NEG strips

$$P_l \approx R_s S + R_s \left(\frac{S}{2}\right)\sqrt{10}\cdot 2$$

two sides of NEG

$$P_l \approx 4.2 R_s S$$

Problem2.

$$a)\, S_{12} \neq S_{21} \Rightarrow \text{not reciprocal}$$

$$b)\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix}\begin{bmatrix} S_{11}^* & S_{21}^* \\ S_{12}^* & S_{22}^* \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Rightarrow S_{11}S_{21}^* + S_{12}S_{22}^* = j + 1 \neq 0 \quad \text{Not lossless}$$

Problem3.

$$\omega_0 = \frac{1}{\sqrt{LC}} = 10^9\ rad/s$$

$$Q_u = \frac{1}{\omega_0 RC} = \frac{\sqrt{LC}}{RC} \approx 6667$$

$$Q_e = \frac{1}{\omega_0 R_L C} = 2000$$

$$Q_L = \left[\frac{1}{Q_L} + \frac{1}{Q_e}\right]^{-1} \approx 1538$$

$$\kappa = \frac{Q_u}{Q_e} = \frac{R_C}{R} = \frac{50}{15} \approx 3.33$$

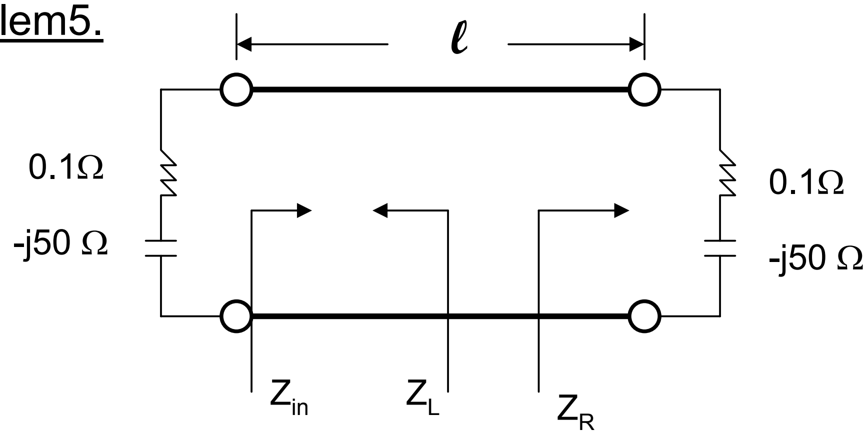
Problem4.

$$f_{cTE_{11}} \approx \frac{1.8412c}{2\pi a} \leq 0.8(6.8)GHz \Rightarrow a_{min} \approx 1.717cm$$

$$f_{cTE_{01}} \approx \frac{3.82c}{2\pi a} \geq 1.1(6.4)GHz \Rightarrow a_{max} \approx 2.6cm$$

∴The waveguide radius must lie in the range 1.771≤ a ≤ 2.6 cm. But since the criteria is to maximize the power delivering capability of the waveguide, we choose a = a_{max}≈2.6 cm.

Problem5.



Since the resonator is symmetric at the midpoint of the line, we must have

$$Z_L = Z_R^* = Z_R \qquad or \;\; \Im m\{Z_R\} = 0$$

Let t = tanβℓ/2 and $Z_L = R_L + jXL$ ($R_1 = 0.1\Omega$, $X_L = -50$)

$$Z_R = Z_0 \frac{Z_L + jZ_0 t}{Z_L + jZ_L t} = Z_0 \frac{R_L + j(X_L + Z_0 t)}{(Z_0 - X_L t) + jR_L t}$$

$$= Z_0 \frac{R_L(Z_0 - X_L t) + jR_L(X_L + Z_0 t) + j(X_L + Z_0 t)(Z_0 - X_L t) - jR_L^2 t}{(Z_0 - X_L t)^2 + (R_L t)^2}$$

$$\Im m\{Z_R\} = 0$$

$$\Rightarrow (X_L + Z_0 t)(Z_0 - X_L t) - R_L^2 t = 0$$

$$-X_L Z_0 t^2 + \left(Z_0^2 - X_L^2 - R_L^2\right)t + Z_0 X_L = 0$$

$$5000t^2 + 7500t - 5000 = 0 \Rightarrow t^2 + 1.5t - 1 = 0$$

$$\Rightarrow t = 0.5 \;\; and \;\; t = -2.0$$

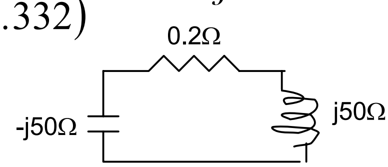
$$\Rightarrow \omega\ell = 53.1^\circ (t = 0.5)$$

$$\omega\ell = -126.9^\circ = 53.1^\circ (t = -2.0)$$

$$\ell = \frac{53.1}{360}\lambda = 0.148\lambda \Rightarrow tan\,\beta\ell = 1.332$$

$$check: \;\; Z_{in} = 100 \frac{(0.1 - j50) + j133.2}{100 + j(0.1 - j50)(1.332)} = 0.1 + j50\Omega$$

$$Q = \frac{\omega_0 L}{R} = \frac{X_L}{R} = \frac{50}{0.2} = 250.$$



Problem6.

Use

$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}}$$

$$R_s = \sqrt{\frac{\omega\mu_0}{2\sigma}}$$

$$\omega = 2\pi\times10^6, \mu_0 = 4\pi\times10^{-7}, \sigma = 5.8\times10^7$$

$$1MHz \rightarrow \delta = 66\mu m, R_s = 2609\mu\Omega / s$$

$$@100MHz \rightarrow reduced\ by\ factor\ 10$$

$$Surface\ Re\,sis\,tan\,ce \rightarrow increased\ by\ 10$$

And so on…