**Microwave Physics and Techniques**

**Solutions to Exam1 – June 19, 2003**

## Problem1.

a) For a short dipole antenna of length L carrying a uniform current the radiation resistance is

$$R_{rad} = \frac{\eta}{6\pi}(\beta L)^2 = \left(\frac{2\pi}{3}\right)\eta\left(\frac{L}{\lambda}\right) \approx 20(\beta L)^2$$

At 150 MHz with L=100 mm, we have $R_{rad}$=1.97Ω.

b) The total power radiated is

$$W = \frac{1}{2}R_{rad}|I|^2 \Rightarrow W = 9.85\,mW.$$

c) The dipole does not radiate in the direction of its axis, so the electric field at large distance in the z-direction is zero. Since the z-axis is the direction of the dipole the x-axis is the direction of greatest radiated power density. We can then make use of the known gain of the dipole to find the expression for the power flow per unit area. So

$$\frac{|E|^2}{2\eta} = 9\frac{W}{4\pi r^2}$$

Substituting the values η=120π,g=1.5,W=9.85 mW, and r=1000m gives

$$|E| = 0.942\,mV/m$$

As an alternative method of calculation we may the expression for the electric field of a small dipole,viz

$$E_\theta = \frac{j\omega\mu_0 IL\sin\theta}{4\pi r}e^{-j\beta r}$$

In the equatorial plane, since sinθ=1, substituting the value

$$\omega = 300\pi\times 10^6\ rad/s,\mu_0 = 4\pi\times 10^{-7}\ H/m, I = 100mA, L = 0.1m, and\ r = 100m$$
$$\Rightarrow |E_\theta| = 0.942\,mV/m.$$

d) The skin depth at ω is

$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}} \Rightarrow \delta = 5.4\mu m.$$

## Problem1.Cont.

Thus the surface resistivity is $R_s = \frac{1}{\delta\sigma} = 3.2\ m\Omega/sq$ , the total series loss resistance of the rod of radius a and length L is

$$R_L = \frac{LR_s}{2\pi a} = \frac{0.1\times 3.2\times 10^{-3}}{2\pi\times 10^3} = 50\,m\Omega.$$

This value is significantly less than the radiation resistance and appears to suggest that the dipole could be an efficient radiator. If we assume that the dipole is center fed, we may be tempted to consider instead of the uniform current distribution assumed in the short dipole analysis, a non-uniform current distribution maximum at the center and tapering linearly to zero at the ends. Such a distribution would not, however produce the crowding of the electrostatic field problem in the vicinity of a pointed conductor. Probably the true distribution is somewhere between the uniform and linearly tapering version discussed.

## Problem2.

We use the results for $Z_0$ and $\alpha$.

$$Z_0 = \sqrt{\frac{\mu_0}{\varepsilon_r\varepsilon_0}}\frac{ln(b/a)}{2\pi}$$

$$\alpha = \sqrt{\frac{\omega\varepsilon_r\varepsilon_0}{2\sigma}}\frac{(a+b)}{2ab\,ln(b/a)} \Rightarrow \alpha = \sqrt{\frac{\omega\varepsilon_r\varepsilon_0}{2\sigma}}\frac{1+(b/a)}{2b\,ln(b/a)}$$

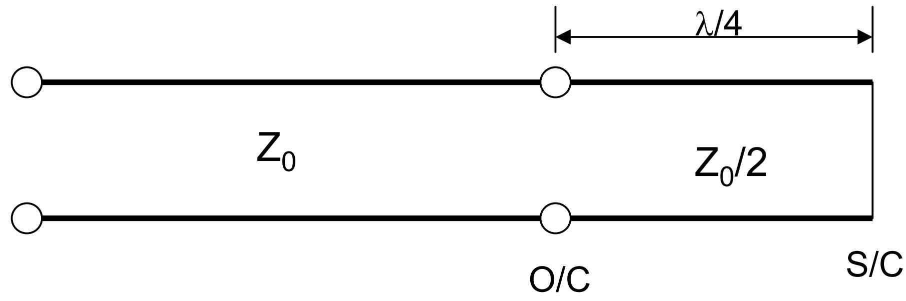
As, for a given value of b, the value of a decreases from its maximum value of b down to zero, the impedance $Z_0$ increases from zero to $\infty$ at a value of b/a=1, down to a minimum value which occurs at about b/a=3.6 and then increases again to $\infty$ as a$\to$0. The minimum value of $Z_0$ is given by

$$Z_0 = \frac{\eta}{2\pi}\frac{ln(3.6)}{\sqrt{\varepsilon_r}}$$

For $\varepsilon_r$=2.25, the characteristic impedance of the minimum loss cable is 51 $\Omega$. We note that rf cables are very commonly of 50 $\Omega$ characteristic impedance.

Problem3:

a) The transmission line arrangement is shown below:




b) As the slab acts as a shorted λ/4 line, the slab will appear as open circuit at the air-dielectric interface and has a reflection coefficient of 1.

c) In the free space region to the left of the air-dielectric interface, there will be an incident wave and an equal magnitude reflected wave. It is convenient to place the origin of the z-axis in the plane of the air-dielectric interface. The incident, reflected and total electric fields will then be

$$E^{a,i}(x,y,z) = \begin{bmatrix} E_a \\ 0 \\ 0 \end{bmatrix} e^{-j\beta_a z}$$

$$E^{a,r}(x,y,z) = \begin{bmatrix} E_a \\ 0 \\ 0 \end{bmatrix} e^{j\beta_a z}$$

$$E^{a,t}(x,y,z) = 2\begin{bmatrix} E_a \\ 0 \\ 0 \end{bmatrix} cos(\beta_a z)$$

Now in the dielectric there will be a complete reflection at the short circuting plate. If we have in the dielectric a forward wave,

$$E^{d,i}(x,y,z) = \begin{bmatrix} E_d \\ 0 \\ 0 \end{bmatrix} e^{-\beta_d z}$$

Then the reverse wave will be

$$E^{d,r}(x,y,z) = \begin{bmatrix} E_d \\ 0 \\ 0 \end{bmatrix} e^{\beta_d z}$$

## Problem3.Cont.

While the total wave in the dielectric will be

$$E^{d,tot}(x,y,z) = 2\begin{bmatrix} E_d \\ 0 \\ 0 \end{bmatrix} cos(\beta_d z)$$

Matching the electric fields at the air dielectric interface gives $2E_d=2E_a$. Thus

$$E^{d,t} = 2\begin{bmatrix} E_d \\ 0 \\ 0 \end{bmatrix} cos(\beta_d z)$$

The corresponding magnetic fields are obtained as follows

$$H^{d,i}(x,y,z) = \begin{bmatrix} 0 \\ E_a/\eta_d \\ 0 \end{bmatrix} e^{-j\beta_d z}$$

$$H^{d,r}(x,y,z) = \begin{bmatrix} 0 \\ -E_a/\eta_d \\ 0 \end{bmatrix} e^{j\beta_d z}$$

$$H^{d,t}(x,y,z) = 2\begin{bmatrix} 0 \\ E_a/\eta_d \\ 0 \end{bmatrix} 2j\,sin(\beta_d z)$$

It is clear that as neither the dielectric nor the short circuit plat has any mechanism for energy dissipation, all of the incident power will be reflected by the structure. This result will hold for <u>all frequencies</u> , not only the frequency for which the slab is a quarter wave in thickness.

## Problem4.

At 31.8 MHz, $\omega$L=100$\Omega$/m and $\omega$C=0.01S/m. Hence the characteristic impedance is

$$Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{2 + j100}{j0.01}} = 100\sqrt{1 - 0.02j}$$

Using a two-term binomial expansion,

$$Z_0 = (100 - j1)\Omega$$

The complex propagation constant is

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{(2 + j100)(j0.01)} = j\sqrt{1 - j0.02} \approx j + 0.01$$

Hence, the attenuation constant is 0.01 neper/m and the phase constant is 1 rad/m

## Problem5.

a) The shorted circuited-end T.L. will appear at its input end as

i) a s/c at dc;

ii) an o/c at 100 MHz

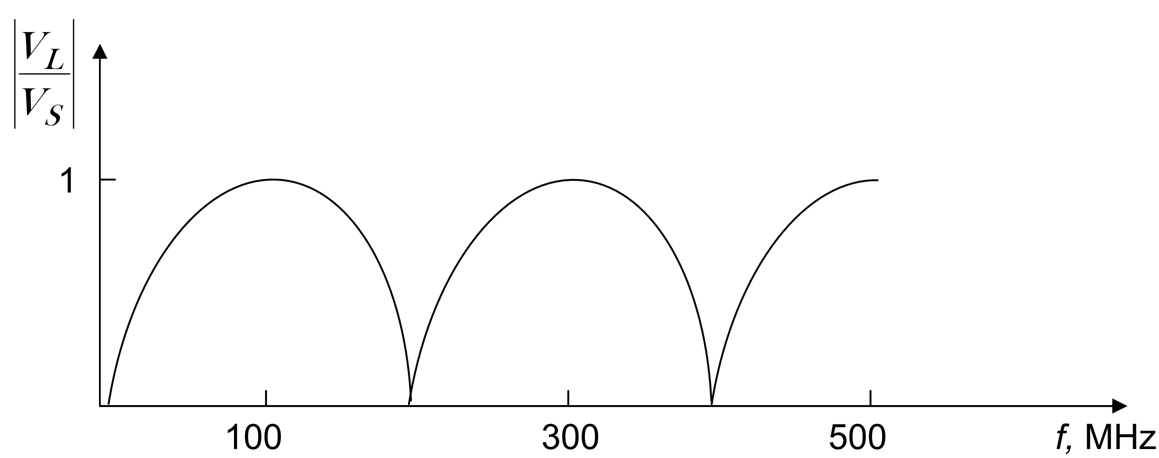
iii) a s/c at 200 MHz

iv) an o/c at 300 MHz

iiv) a s/c at 400 MHz

v) an o/c at 500 MHz

The approximate frequency response is:



Problem5.Cont.

b) The open circuited-end T.L. will appear at its input end as

i) an o/c at dc;

ii) a s/c at 100 MHz

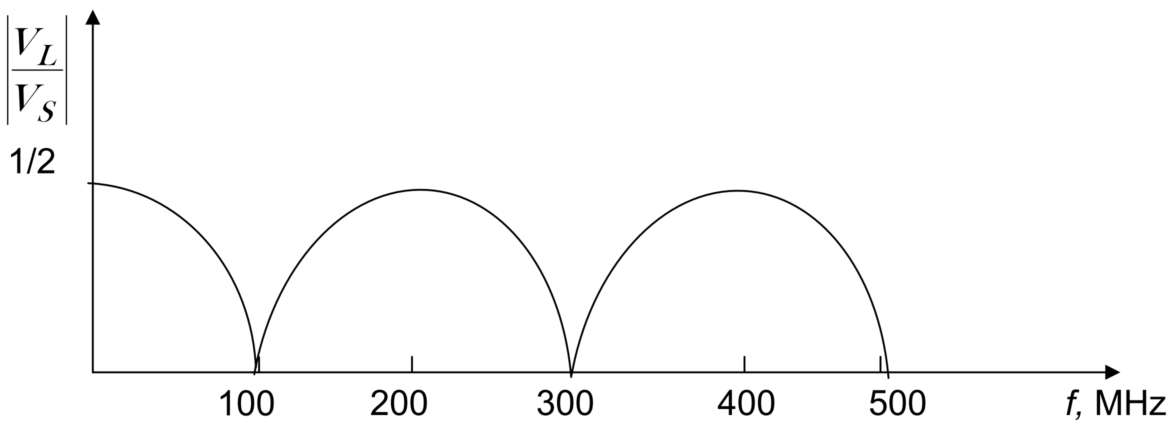
iii) an o/c at 200 MHz

iv) a s/c at 300 MHz

iiv) an o/c at 400 MHz

v) a s/c at 500 MHz

The approximate frequency response is:




## Problem6.

Using

$$\eta_c \approx \frac{\frac{\eta_0}{\sqrt{\varepsilon_r'}}}{\left[1+tan^2\delta_c\right]^{1/2}} e^{j(1/2)tan^{-1}(tan\,\eta_c)}$$

$$\Rightarrow \eta_c \approx \frac{\frac{377}{\sqrt{\varepsilon_r'}}}{\left[1+tan^2\delta_c\right]^{1/2}} e^{j(1/2)tan^{-1}(tan\,\eta_c)} \approx 22.5e^{j37^\circ}\,\Omega$$

The material loss tangent can be found from the phase angle as

$$\frac{1}{2}tan^{-1}\left(tan\,\delta_c\right) \approx 37^\circ \Rightarrow tan\,\delta_c \approx 3.49$$

# Problem6.Cont

Substituting the loss tangent value in the magnitude, we can calculate $\varepsilon_r'$ as;

$$\frac{\dfrac{377}{\sqrt{\varepsilon_r'}}}{\left[1+(3.49)^2\right]^{1/2}} \approx 22.5 \rightarrow \varepsilon_r' \approx 77.4$$

Then, using the value of $\varepsilon_r'$ in the loss tangent expression, we find the value of $\sigma$ as

$$tan\,\delta_c \approx \frac{\sigma}{2\pi\left(2\times10^8\right)\left(77.4\varepsilon_0\right)} \approx 3.49 \Rightarrow \sigma \approx 3.0\,S/m$$